STUDENT NO : ______________________

Class: ______________________

KILLARA HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT - TASK 1

Year 11 Term 4, 30-11-2011

Time allowed: 1 hour

Maximum marks: 54 marks

INSTRUCTIONS:

- Approved calculators may be used.
- All questions are compulsory and should be attempted.
- Use blue or black biro, pencil for graphs and diagrams.
- Where appropriate, FULL WORKING must be shown.
- Diagrams are NOT TO SCALE.

Format:

Part A: Multiple choice questions (6 marks)
Answer on multiple choice answer sheet

Part B: Four questions (48 marks)
Answer with full working on paper provided.
Start a new sheet for each question.

PART A – Multiple Choice questions (1 mark each)

1. Find the equation of the parabola with coordinates of the vertex (0, 0), equation of the axis $y = 0$ and focal length 9.

   (A) $x^2 = \pm 36y$ (B) $x^2 = -12y$ (C) $y^2 = \pm 36x$ (D) $y^2 = -12x$

2. The focus of the parabola $(x-1)^2 = 8(y-2)$ is at

   (A) (0, 2) (B) (1, 2) (C) (1, 0) (D) (1, 4)

3. If α and β are the roots of $x^2 - 5x - 3 = 0$ then

   (A) $\alpha + \beta = 5$ and $\alpha\beta = -3$ (B) $\alpha + \beta = -5$ and $\alpha\beta = 3$

   (C) $\alpha + \beta = -5$ and $\alpha\beta = -3$ (D) $\alpha + \beta = 5$ and $\alpha\beta = 3$

4. If $Px^2 + (P+Q)x + (P+R) \equiv 9x^2 + 7x + 2$ then

   (A) $P = 9, Q = 7$ and $R = 2$ (B) $P = 9, Q = 2$ and $R = 7$

   (C) $P = 9, Q = -7$ and $R = -2$ (D) $P = 9, Q = -2$ and $R = -7$

5. The fortieth term of the series for which $T_n = 5n - 9$ is?

   (A) 25 (B) 191 (C) 155 (D) 61

6. The number of terms in the series $1 + 1.0825 + 1.0825^2 + ... + 1.0825^{27} + 1.0825^{28}$ is?

   (A) 5 (B) 27 (C) 28 (D) 29

PART B

Question 1 (12 marks)

a. Consider the parabola with equation $x^2 = 4(y-1)$.

(i) Find the coordinate of the vertex of the parabola. 1

(ii) Find the coordinate of the focus of the parabola. 1

b. Show that

$x^2 - 2x + y^2 + 4y + 1 = 0$ and $x^2 - 2x + y^2 + 4y - 4 = 0$ are concentric (have the same centre). 2

c. Let $R\left(\frac{1}{5}, -2\right)$ be a point on the parabola $y^2 = 20x$.

(i) Show the equation of the focal chord passing through R is $5x - 12y - 25 = 0$ 2

(ii) Find the coordinates of the point Q where this chord cuts the directrix. 2

(iii) Find the perpendicular distance from the focal chord to the point $P(-1,-7)$. 2

(iii) Hence find the area of ΔPQR. Or use alternate methods. 2

Question 2 (12 marks)

a. Find the values of k for which the quadratic equation $3x^2 + 2x + k = 0$ has no real roots. 2

b. Let α and β be the solutions of $x^2 - 3x + 1 = 0$. Find

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ 1

(ii) $\alpha^2 + \beta^2$. 1

c. Prove that $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ 2

d. Find values of k in the equation $x^2 + (k+1)x + (\frac{k+1}{4}) = 0$ if:

(i) roots are equal in magnitude but opposite in sign 1

(ii) one root is 1 1

e. (i) Show that the x – coordinates of the points of intersection of a circle $x^2 + y^2 = 4$ and the line $y = x + 1$ satisfy the equation $2x^2 + 2x - 3 = 0$. 2

(ii) Evaluate the discriminant Δ and explain why this shows that there are two points of intersection. 2

Question 3 (12 marks)

a. The first term of an arithmetic series is 15 and the third term is 31. Find the

(i) Common difference 1

(ii) 10^{th} term of the series. 1

b. Evaluate $\sum_{3}^{17} 5n-6$. 2

c. (i) Show that $(x+1)+(2x+4)+(3x+7)+\ldots$ is an arithmetic series. 1

(ii) Find the sum of the first 50 terms of the series, in terms of x. 1

d. The 20^{th} term of an arithmetic series is 131 and the sum of the 6^{th} to 10^{th} terms inclusive is 235. Find first 3 terms. 2

e. For what value of n is the sum of the arithmetic series $5+9+13+\ldots$ equal to 152? 2

f. The n^{th} term of an arithmetic sequence is $T_n = 7+4n$. Prove that $5T_1 + 4T_2 = T_{27}$. 2

Question 4 (12 marks)

a. Which term of the series $8 - 4 + 2 - \ldots$ is equal to $\frac{1}{128}$? 2

b. The common ratio of a geometric series is 4 and the sum of the first 5 terms is 3069. Find the first term. 2

c. A bouncy ball drops from a height of 9 metres and bounces continually, each successive height being $\frac{2}{3}$ of the previous height.

(i) Show that the first distance travelled down-and-up is 15 metres, and show that the successive down-and-up distances form a geometric series. 2

(ii) Through what distance does the ball eventually travel? 1

d. Consider the geometric series $1+(\sqrt{5}-2)+(\sqrt{5}-2)^2+\ldots$

(i) Explain why the geometric series has a limiting sum. 1

(ii) Find the exact value of the limiting sum. Write your answer with a rational denominator. 2

e. Pam deposited \$2000 into an account that earned interest monthly. For 2 years the interest rate was 12% p.a., then it fell to 7% p.a. How much money will there be in Pam's account after 5 years? 2

Part A Answer Sheet **STUDENT NO :** ____________

Total marks: 6

Attempt question 1 – 6

Select the alternative A, B, C and D that best answers the question and indicate your choice by shading the ellipse.

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭

Part A. Multiple choice: 1.(C) 2.(D) 3.(A) 4.(D) 5.(B) 6.(D).

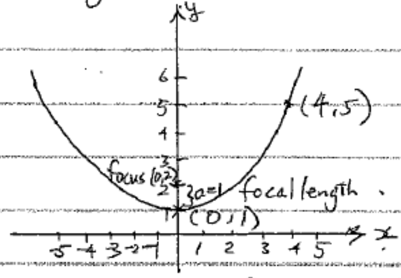
Part B (1)

(a) $x^2 = 4(y-1)$, $(x-h)^2 = 4a(y-k)$

giving $(x-0)^2 = 4 \times 1 \times (y-1)$

(i) The vertex is at $(h,k) = (0,1)$

(ii) Focal length = 1 unit ($\because a = 1$)



From the diagram the focus is at $(0, 2)$

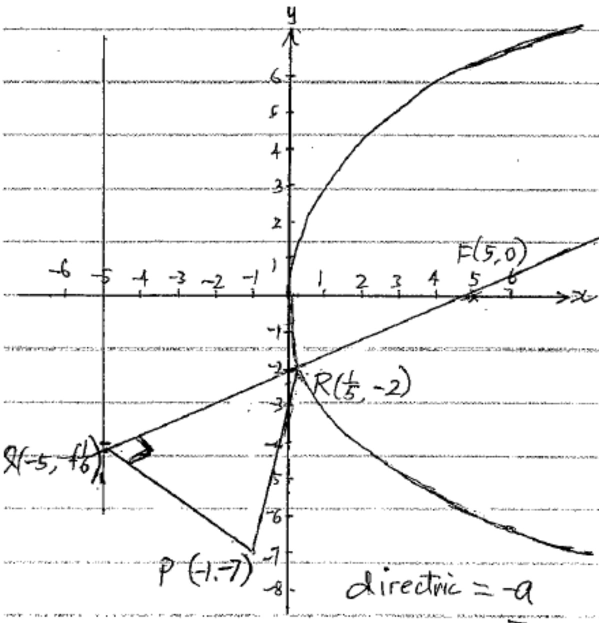
(b) $x^2-2x+1+y^2+4y+4=-1+1+4$ & $x^2-2x+1+y^2+4y+4=4+4+1$

$(x-1)^2+(y+2)^2=2^2$ & $(x-1)^2+(y+2)^2=3^2$

$x=1$ & $y=-2$ radius=2 & $x=1$, $y=-2$ radius=3

$\therefore$ Centre $(1,-2)$, radius $r=2$ & centre $(1,-2)$ radius=3

(c) $R(\frac{1}{5},-2)$ $y^2=20x$



directric = $-a$

$\therefore x = -5$.

(i) $4a = 20$ $a = \frac{20}{4} = 5$

$\therefore$ focus $(5, 0)$.

Gradient $= \frac{-2-0}{\frac{1}{5}-5} = \frac{2}{\frac{24}{5}} = \frac{5}{12}$

$\therefore$ equation of line

$y-0 = \frac{5}{12}(x-5)$

$12y = 5x-25$

$5x-12y-25=0$

(ii) directrix $= -a$ $\therefore x = -5$

$12y-5(-5)+25=0$

$12y=-50$

$y=-4\frac{1}{6}$ $\therefore (-5, -4\frac{1}{6})$

(iii) $P(-1,-7)$ $\perp$ distance of PQ

$\therefore \perp$ dist of PQ $= \frac{|-5(-1)+12(-7)+25|}{\sqrt{5^2+12^2}}$

$= \frac{54}{13}$ units

(iv) Find the area of $\triangle PQR$.

$R(\frac{1}{5},-2)$ $Q(-5,-4\frac{1}{6})$

$PQ = \frac{54}{13}$ units from Q to P.

$QR = \sqrt{(-5-\frac{1}{5})^2+(-4\frac{1}{6}-2)^2}$

$= \frac{169}{30}$

$\therefore$ Area of $\triangle PQR = \frac{1}{2} \times (PQ \times QR)$

$= \frac{1}{2} \times \frac{54}{13} \times \frac{169}{30}$

$= 11.7$ units^2

2(a) $3x^2+2x+k=0$, $\Delta=b^2-4ac$

$\Delta<0$ So $\Delta=4-4\times3\times k$

$=\underline{4-12k}$.

For no real roots $\Delta<0$

$\therefore 4-12k<0$

$12k>4$

$\underline{k>\frac{1}{3}}$

(b) $x^2-3x+1=0$

$\alpha+\beta=-\frac{b}{a}=-\frac{3}{1}=-3$

$\alpha\beta=\frac{c}{a}=\frac{1}{1}=1$

(i) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{-3}{1}$

$=\underline{-3}$

(ii) $\alpha^2+\beta^2$

$(\alpha+\beta)^2-2\alpha\beta$

$=(-3)^2-2(1)$

$=9-2$

$=\underline{7}$

(c) Prove that $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

proof:

LHS $(\alpha-\beta)=(\alpha-\beta)(\alpha-\beta)$

$=\alpha^2-2\alpha\beta+\beta^2$

$=\alpha^2+2\alpha\beta+\beta^2-4\alpha\beta$

$=(\alpha+\beta)^2-4\alpha\beta$

$=$ RHS

(d) $x^2+(k+1)x+\left(\frac{k+1}{4}\right)=0$.

(i) $\alpha+\beta=0$ $\quad\alpha-\alpha=\frac{-(k+1)}{1}$

So $-(k+1)=0$

$-k-1=0$

$-k=1$

$\underline{k=-1}$

(ii) $x=1$

So $1+k+1+\frac{k+1}{4}=0$

$4+4k+4+k+1=0$

$5k=-9$

$k=-\frac{9}{5}$

$k=\underline{-1.8}$

(e)(i) $x^2+y^2=4$ & $y=x+1$

$y^2=4-x^2$ $\quad y^2=(x+1)^2$

$4-x^2=(x+1)^2$

$4-x^2=x^2+2x+1$

$2x^2+2x-3=0$ is satisfy the equation.

$\Delta=b^2-4ac$

(ii) $=2^2-4\times2\times(-3)$

$=4+24$

$=28$

Since $\Delta>0$, the quadratic equatio has two roots means there are two points of intersection.

(3)(a) $T_n = a+(n-1)d$ $a=15$

(i) $T_1 = a+0d = 15$

$T_3 = a+2d = 31$

$2d = 16$

$\underline{\underline{d = 8}}$ ✓

(ii) 10^{th} term of the series.

$T_{10} = 15+(10-1)8$

$= 15 + 9\times 8$

$= \underline{\underline{87}}$ ✓

(b) $\sum_{3}^{17} 5n-6$ $n=3, \therefore 5(3)-6=9$

$n=4,$ $5(4)-6=14$

$n=5,$ $5(5)-6=19$

$\therefore a = 9$ $d = 5$. ✓

$\vdots$

$n=17,$ $5(17)-6=79$

$T_{15} = \frac{15}{2}(9+79)$

$= \underline{\underline{660}}$ ✓

(c)(i) $(x+1)+(2x+4)+(3x+7)+\cdots$

$d = (2x+4)-(x+1) = x+3$

$(3x+7)-(2x+4) = x+3$

common difference equalled each other

$\therefore (x+1)+(2x+4)+(3x+7)+\cdots$

is an arithmetic Series. ✓

(ii) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{50} = \frac{50}{2}[2(x+1)+49(x+3)]$

$= 25[2x+2+49x+147]$

$= 25[51x+149]$

$= \underline{\underline{1275x+3725}}$ ✓

(d)

$T_{20} = a+19d = 131$

$S_{10} = \frac{10}{2}[2a+(9)d]$

$= 10a+45d$

$S_5 = \frac{5}{2}[2a+4d]$

$= 5a+10d$

$\therefore (10a+45d)-(5a+10d) = 235$ ✓

$5a+35d = 235$

$a+7d = 47$ — ①

$a+19d = 131$ — ②

②−① $12d = 84$

$d = 7$

$\therefore a+49 = 47$

$a = -2$ ✓

First 3 terms $\underline{\underline{-2, 5, 12 \cdots}}$

$5+9+13+\cdots+ = 152$

(e) $S_n = \frac{n}{2}[2a+(n-1)d]$

$152 = \frac{n}{2}[10+(n-1)4]$

$152 = \frac{n}{2}[4n+6]$ ✓

$152 = n(2n+3)$

$2n^2+3n-152 = 0$

$(2n-19)(n-8) = 0$ ✓

$\therefore \underline{\underline{n = 8}}$

(f) $T_n = 7+4n$.

Prove that $5T_1+4T_2 = T_{27}$.

LHS $5(7+4)+4(7+8) = 7+4(27)$ ✓

$35+20+28+32 = 7+108$

$115 = 115$ ✓

$= RHS$

$\therefore 5T_1+4T_2 = T_{27}$

LHS = RHS.

(4) (a)

$8 - 4 + 2 - \cdots = \frac{1}{128}$

$a = 8, \quad r = \frac{-4}{8} = \frac{2}{-4} = \frac{1}{-2}$

$T_n = ar^{n-1}$

$8\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{128}$

$\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1024}$ ✓

$(-1)^{n-1}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{10}$

$n - 1 = 10$

$n = \underline{\underline{11}}$ ✓

(b) $r = 4 \quad a = ?$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_5 = \frac{a(4^5 - 1)}{4 - 1} = 3069$ ✓

$a(1023) = 3(3069)$

$a = \frac{9207}{1023}$

$a = \underline{\underline{9}}$ ✓

show ✓

(c)(i) 1^{st} distance $= 9 + 9\frac{2}{3} = 9 + 6 = 15m$

The successive down-and-up distances form a GP with $a = 15$ and $r = \frac{2}{3}$ ✓

(ii) $S_\infty = \frac{a}{1 - r}$

$= \frac{15}{1 - \frac{2}{3}}$

$= \frac{15}{\frac{1}{3}}$

$= \underline{\underline{45 \text{ metres}}}$ ✓

4(d) $1 + (\sqrt{5} - 2) + (\sqrt{5} - 2)^2 + \cdots$

(i) Limiting sum exists when $|r| < 1$

$r = \sqrt{5} - 2, \quad a = 1$

$\therefore |r| = |\sqrt{5} - 2| = 0.236\cdots < 1$ ✓

$\therefore$ Limiting sum exists.

(ii) $S_\infty = \frac{a}{1 - r}$

$= \frac{1}{1 - (\sqrt{5} - 2)}$ ✓

$= \underline{\underline{\frac{1}{3 - \sqrt{5}}}}$ ✓

$n = 2 \times 12 = 24$ months $\qquad n = 3 \times 12 = 36$ months

(e) $A = \underbrace{2000\left(1 + \frac{1}{100}\right)^{24}}_{\text{1st 2yrs was 12\%}} + \underbrace{(2539.4692)\left(1 + \frac{\frac{7}{12}}{100}\right)}_{\text{After 2nd yr interest fell to 7\% p.a.}}$ ✓

$= \underline{\underline{\$3130.98}}$ ✓

$\boxed{A = P\left(1 + \frac{r}{100}\right)^n}$