Killara HIGH SCHOOL

Student Number

2018

Mathematics

HSC ASSESSMENT TASK 2
TUESDAY 20TH MARCH

General Instructions
- Reading time – 5 minutes
- Working time 3 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question
- Diagrams are not to scale

Total Marks – 100

Section I - Pages 3 - 5
10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 7 - 18
90 marks
- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 30% of the Higher School Certificate Course Assessment

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Section I

10 marks
Attempt Questions 1 – 10 **Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

1 What is 0.02095 rounded to 3 significant figures?

(A) 0.020

(B) 0.021

(C) 0.0209

(D) 0.0210

2. What is/are the solution(s) to the equation $x(x-1) = x$?

(A) $x = 0$

(B) $x = 1$

(C) $x = 0$ and $x = 2$

(D) $x = 1$ and $x = 2$

3 Given $f(x) = x^6 + 3x^3 - 4x + 2$,

$f''(x) =$

(A) $6x^5 + 3x^2 - 4$

(B) $6x^5 + 9x^2 - 4$

(C) $30x^4 + 6x$

(D) $30x^4 + 18x$

4 If $f'(x) = x^2 - 5x - 6$ then stationary points may occur when:

(A) $x = 1, -6$

(B) $x = -2, -3$

(C) $x = -1,\ 6$

(D) $x = -3,\ 2$

5 For what values of x is the curve $y = x^3 + 6x^2 - x + 4$ concave up?

(A) $x < -2$

(B) $x > 0$

(C) $x < 0$

(D) $x > -2$

6 Find $\int (2x+1)^{\frac{1}{3}}\, dx$.

(A) $-\frac{2}{3}(2x+1)^{-\frac{1}{3}} + C$

(B) $\frac{3}{4}(2x+1)^{\frac{4}{3}} + C$

(C) $\frac{3}{8}(2x+1)^{\frac{4}{3}} + C$

(D) $\frac{8}{3}(2x+1)^{\frac{4}{3}} + C$

7 Evaluate $\int_0^9 (\sqrt{x}+1)^2\, dx$.

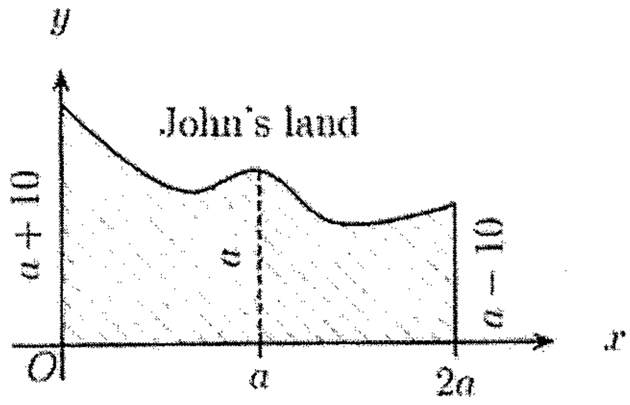
(A) $29\frac{1}{4}$

(B) $40\frac{1}{2}$

(C) $49\frac{1}{2}$

(D) $85\frac{1}{2}$

8 The shaded area shown in the diagram below represents John's land. Its dimensions are given in terms of ***a*.**
Given that the area of this land is 3 200 m^2, use Simpson's rule with 3 function values to find an estimate for the value of ***a***.



(A) $20\sqrt{2}$ m

(B) $40\sqrt{2}$ m

(C) 40 m

(D) 1600 m

9 Differentiate $2\tan 3x$.

(A) $2\sec^2 3x$

(B) $6\sec^2 3x$

(C) $\frac{2}{3}\sec^2 3x$

(D) $6\sec 3x$

10 Evaluate $\int_0^{\frac{\pi}{8}} 4\cos 4x\, dx$

(A) 1

(B) $\frac{1}{\sqrt{2}}$

(C) -4

(D) 4

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Section II

90marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Factorise completely $2c^2 - 50d^2$ **2**

(b) Rationalise the denominator of $\dfrac{6}{\sqrt{17}-\sqrt{11}}$ **2**

(c) Solve $|3x - 7| = 17$ **2**

(d) Simplify $\dfrac{5}{y+2} - \dfrac{2}{y}$ **2**

(e) The midpoint of the points A(3,-4) and B(x, y) is M(5,-1). Find the coordinates of B. **2**

(f) Solve the simultaneous equations **2**

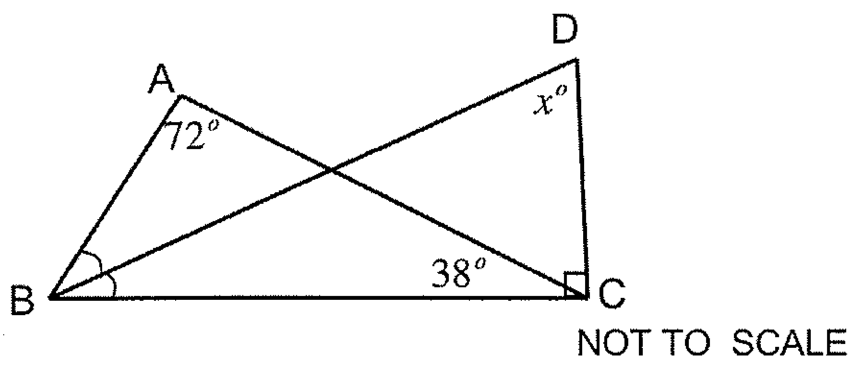
$$2x + y = 2$$

$$5x - y = 19$$

(g) In the diagram below (not drawn to scale), BD bisects $\angle$ ABC,

DC $\perp$ BC, $\angle BAC = 72^0$, $\angle ACB = 38^0$ and $\angle BDC = x^0$.

Copy the diagram into your writing booklet. Find the value of x, giving reasons for each step in your calculation. **3**



End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Indicate and shade the region in which $4 < x^2 + y^2 \leq 9$ **3**

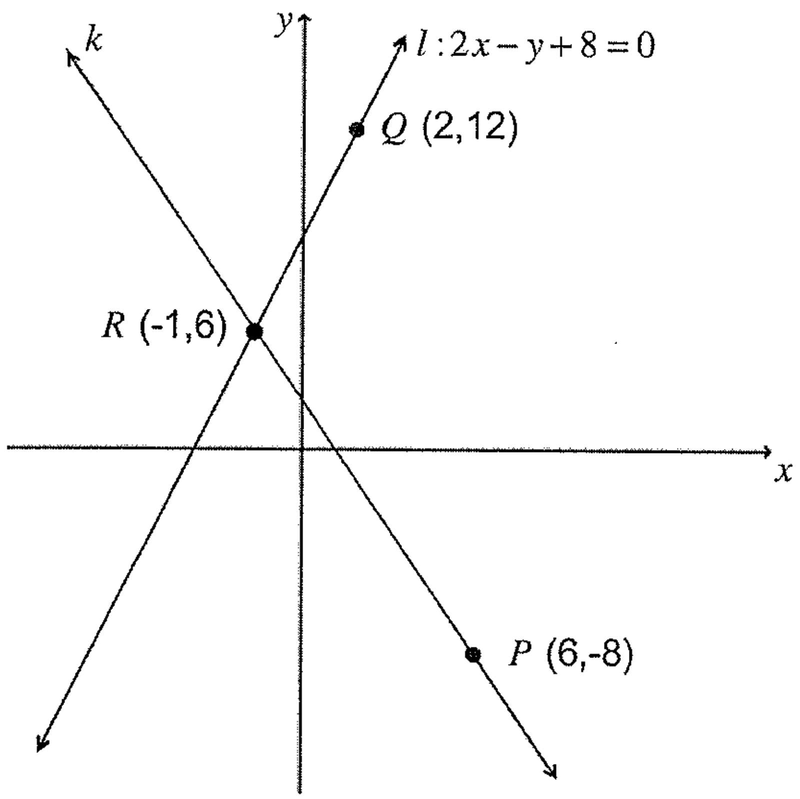
(b) A function $f(x)$ is given by **2**

$f(x) = \{x - 5,\ for\ x \leq 3\}$

$\{x^2 - 16, for\ x > 3\}$

Find $f(3) - f(5)$

(c)



The diagram above shows the line $l: 2x - y + 8 = 0$ and the point $Q\ (2,12)$ on it. The line k has gradient -2 and passes through the point $P\ (6,-8)$. The lines k and l intersect at R.

↑ (6,-8)

(i) Show that the equation of the line k is $2x + y - 4 = 0$. **1**

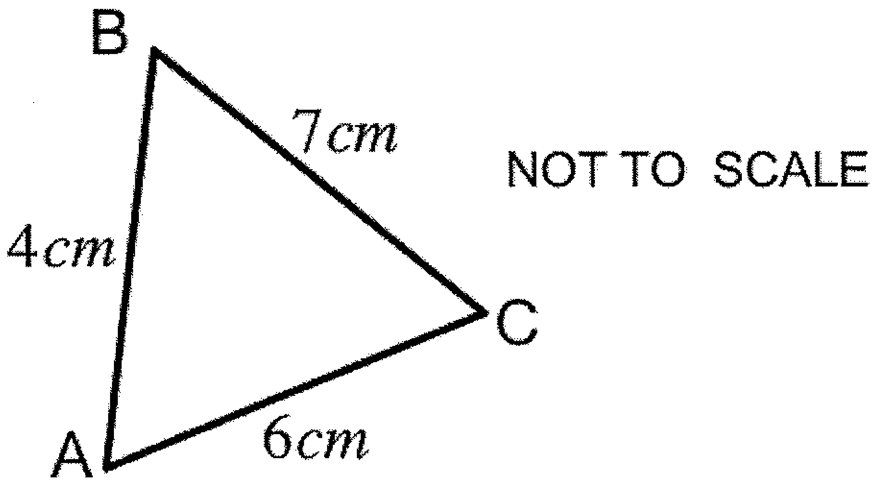
(ii) Show that the length of QR is $3\sqrt{5}$. **1**

(iii) Find the perpendicular distance from P to the line l. Leave your answer in surd form. **2**

(iv) Find the area of the triangle PQR. **1**

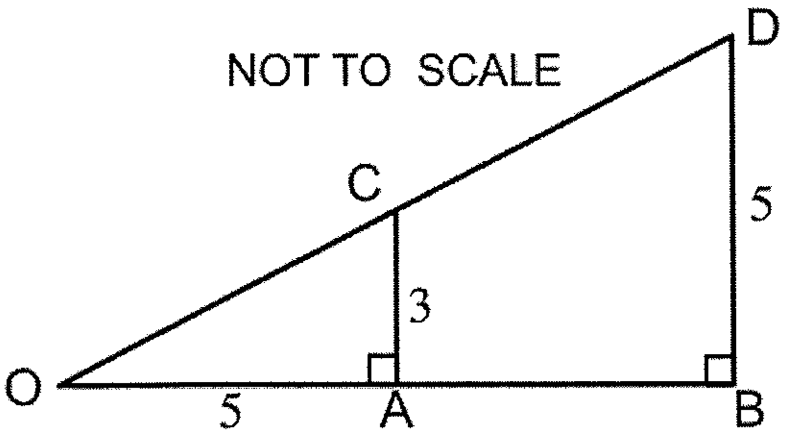
Question 12 (continued)

(d) 1



In the diagram above, ABC is a triangle in which AB = 4cm, BC = 7cm and CA = 6cm. Use the cosine rule to show that $\cos C = \frac{23}{28}$.

(e)



In the diagram above, AC is parallel to BD, OA = 5cm, DB = 5cm and AC = 3cm.

(i) Show that triangles OCA and ODB are similar. 2

(ii) Hence, find the length of AB, giving reasons. 2

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

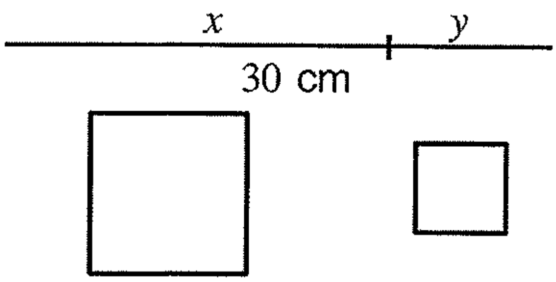
a. Consider the curve $y = 3x^4 - 16x^3 + 24x^2$.

i. Find the stationary turning points. **2**

ii. Hence determine the nature of the stationary points. **3**

iii. Sketch the curve, showing the intercepts with axes, stationary turning points and points of inflexion. **2**

b. A 30 cm length of wire is cut into 2 pieces and each piece bent to form a square.



i. Show that $y = 30 - x$ **1**

ii. Show that the total area of the 2 squares is given by $A = \frac{x^2 - 30x + 450}{8}$ **2**

c. A truck travels 1 500 km at an hourly cost given by $s^2 + 9000$ cents where s is the average speed of the truck.

i. Show that the cost for the trip is given by $C = 1500\left(s + \frac{9000}{s}\right)$. **2**

ii. Find the speed that minimises the cost of the trip. **3**

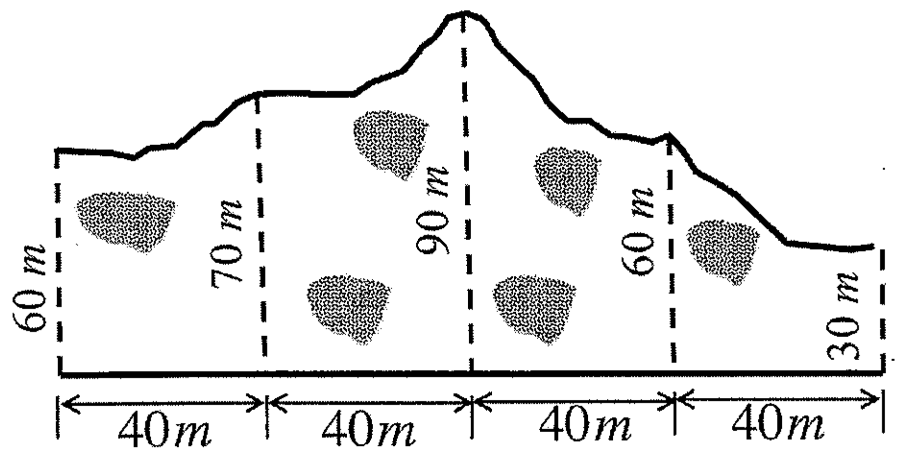
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

a. Find indefinite integral of $\int (x-1)(x-2)\,dx$. **2**

b. Evaluate $\int_0^1 \frac{x^3+2x^2}{x}dx$ $= \int_0^1 x^2 + 2x \; dx.$ **2**

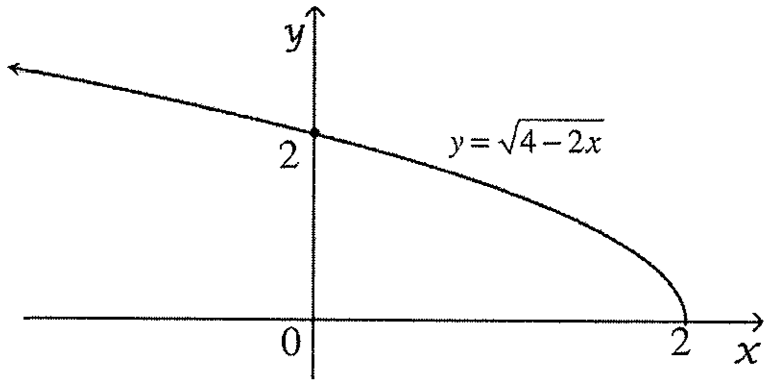
c. The diagram shows the land that Jane bought near a lake.



i. Use the trapezoidal rule with 5 function values to find an estimate for the area of this land. **2**

ii. Is the area obtained in the previous part smaller or larger than the exact area? Give reasons for your answer. **1**

d. The diagram below shows the graph of the curve $y = \sqrt{4 - 2x}$.

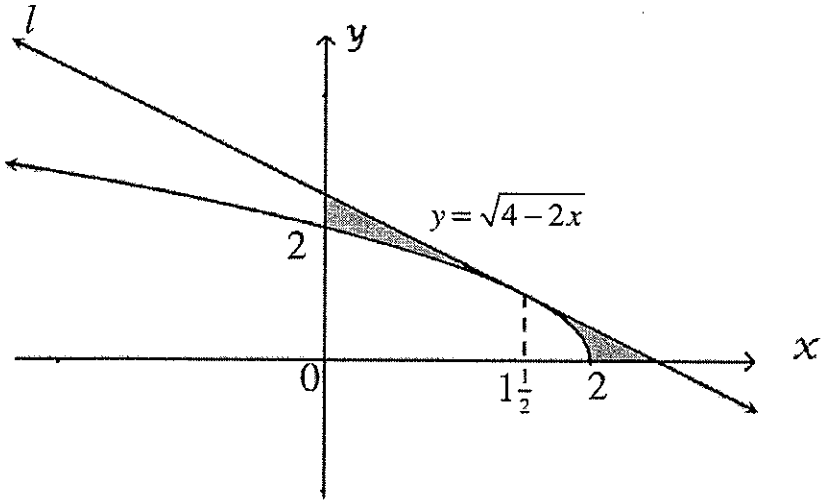


Question 14 Continued

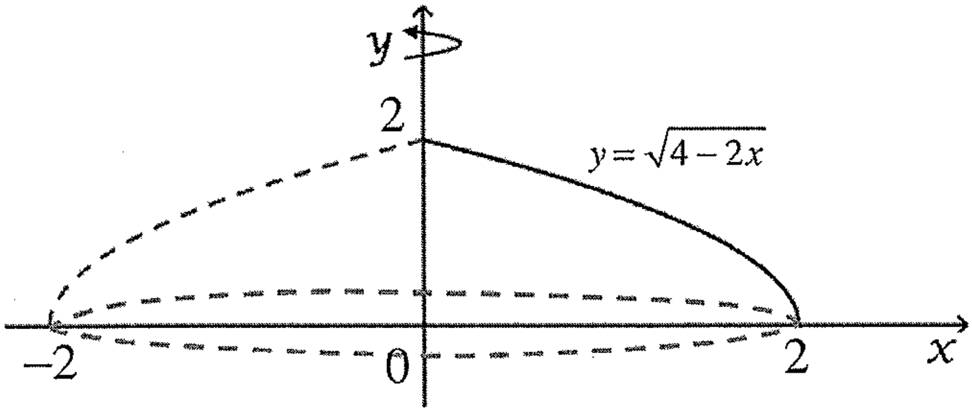
i. Show that the equation of the tangent l at the point on the curve where $x = 1\frac{1}{2}$ is $2x+2y-5=0$ **2**

ii. Show that the area bounded by the curve, the x axis and the y axis is $2\frac{2}{3}$ square units. **2**

iii. Using the result above, or otherwise, find the area of the region shaded between $y = \sqrt{4 - 2x}$, the tangent l, and the x and y axes as shown. **2**



iv. A spaceship is obtained by rotating part of the curve $y = \sqrt{4 - 2x}$ between the points (2, 0) and (0, 2) about the y axis as shown. **2**



Find the volume of the spaceship.

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$ **2**

(b) Let α and β be the roots of $3x^2 - 8x + 3 = 0$.

(i) Find $\alpha\beta$ **1**

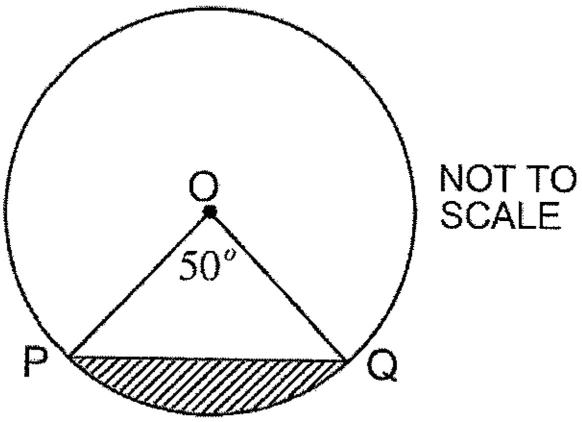
(ii) Hence find $\alpha + \frac{1}{\alpha}$ **1**

(c) A parabola has equation $4y = x^2 - 4x + 8$

(i) Find the coordinates of its vertex. **2**

(ii) Find the equation of its directrix. **1**

(d)

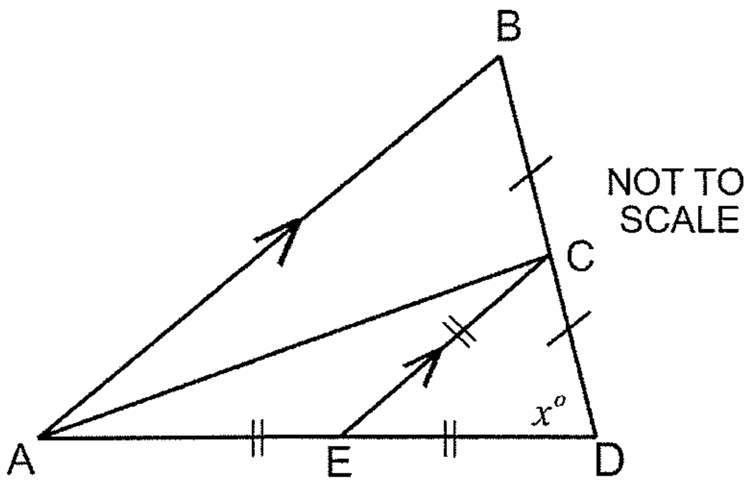


The diagram above, shows an arc PQ, of length 25π cm subtending an angle of $\frac{5\pi}{18}$ radians at O, the centre of the circle.

(i) Show that the radius of the circle is 90cm. **2**

(ii) Calculate the shaded area in the diagram. (Give your answer to 1 decimal place) **2**

(e) In the diagram below, ABD is a triangle, the point C is the mid-point of BD and the point E is the mid-point of AD. Also, AE = CE = DE, AB || EC and $\angle$ EDC = x



(i) Show that $\angle$ACB = 90^0 **3**

(ii) Express $\angle$ BAD in terms of x. **1**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) (i) Prove that $\frac{d}{d\theta}(tan^3\theta) = 3\sec^4\theta - 3\sec^2\theta$ **2**

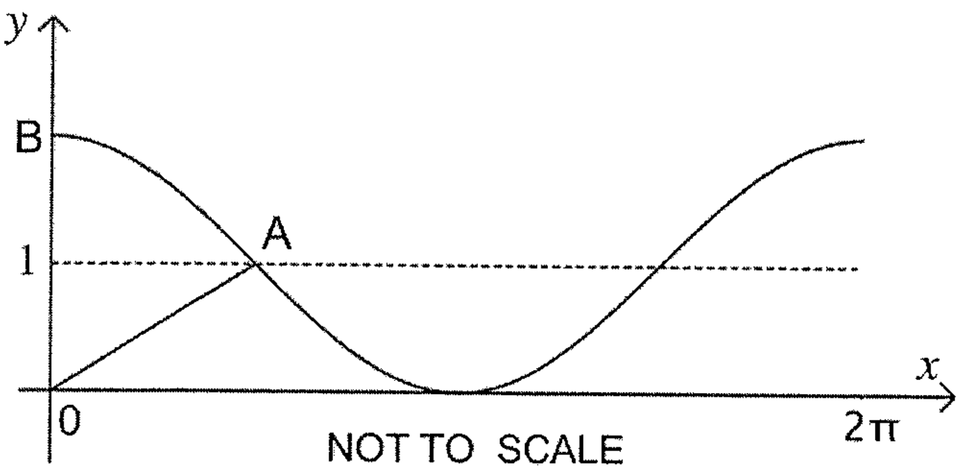
(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} sec^4\theta \, d\theta$ **2**

(b) Given the function $y = -2sin3x$

(i) State the period of the function **1**

(ii) Sketch the graph of $y = -2sin3x$ for $0 \leq x \leq \pi$. **2**

(c)



The diagram shows the graph of $y = 1 + cosx$, for $0 \leq x \leq 2\pi$

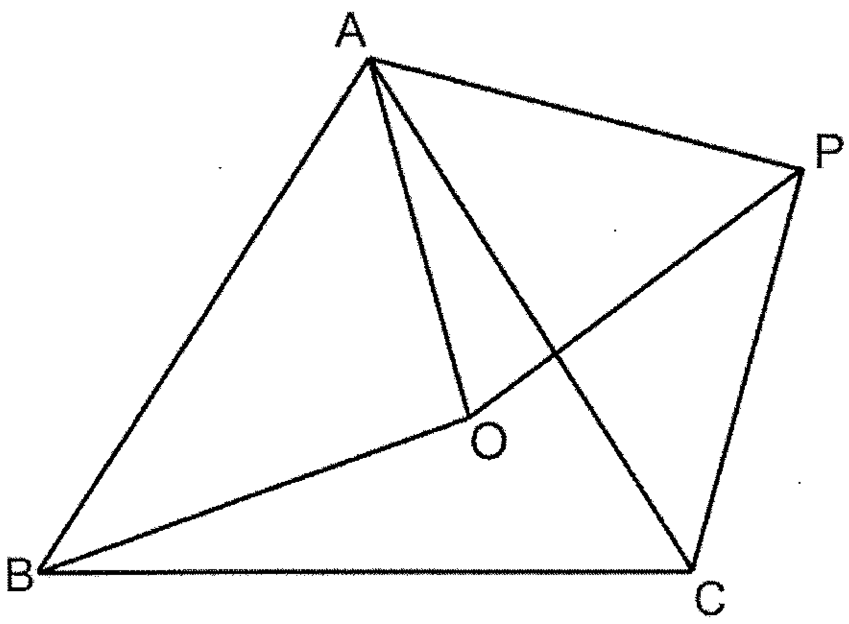
The graph crosses the line $y = 1$ at A, and the y-axis at B.

(i) Show that the y coordinate of A is $\frac{\pi}{2}$. **1**

(ii) Show that the equation of OA is $y = \frac{2}{\pi}x$. **1**

(iii) Hence find the area bounded by the arc AB, the y-axis, and the line OA. **2**

(d)



NOT TO SCALE

In the figure above, triangles ACB and APO are equilateral.

(i) Copy this diagram into your answer booklet and include all the given information.

(ii) Explain why $\angle BAO = \angle PAC$ **2**

(iii) Prove that $\Delta AOB \equiv \Delta APC$ **2**

End of Examination

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Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ✗ B ✗ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Sdn

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Start Here →

1. A ○ B ○ C ○ D ●
2. A ○ B ○ C ● D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ○ C ● D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ○ C ● D ○
7. A ○ B ○ C ○ D ●
8. A ○ B ○ C ● D ○
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○

Multiple-Choice Solutions

1. 0.0210 D

2. $x(x-1)=x$

$x^2-x=x$

$x^2-2x=0$

$x(x-2)=0$

$x=0$ and $x=2$ C

3. $f(x)=x^6+3x^3-4x+2$

$f'(x)=6x^5+9x^2-4$

$f''(x)=30x^4+18x$ D

4. $f'(x)=x^2-5x-6$

$f'(x)=0$

$x^2-5x-6=0$

$(x+1)(x-6)=0$

$x=-1,\ x=6$ C

5. $y=x^3+6x^2-x+4$

$y'=3x^2+12x-1$

$y''=6x+12$

The graph concave up when $y''>0$

$6x+12>0$

$6x>-12$

$x>-2$ D

6. $\int(2x+1)^{\frac{1}{3}}\,dx=\dfrac{(2x+1)^{4/3}}{4/3\times 2}+C$

$=\dfrac{(2x+1)^{4/3}}{8/3}+C$

$=\frac{3}{8}(2x+1)^{4/3}+C$

C

7. $\int_0^9(\sqrt{x}+1)^2dx=\int_0^9(x+2x^{\frac{1}{2}}+1)dx$

$=\left[\frac{x^2}{2}+\frac{4}{3}x^{3/2}+x\right]_0^9$

$=\left[\frac{81}{2}+\frac{4}{3}(9)^{3/2}+9\right]-0$

$=85\frac{1}{2}$ D

8.

x	0	a	$2a$
y	$a+10$	a	$a-10$
	y_0	y_1	y_2

$A\doteqdot\frac{h}{3}(y_0+4\Sigma y_{odd}+2\Sigma y_{even}+y_l)$

$3200\doteqdot\frac{a}{3}((a+10)+4(a)+(a-10))$

$3200\doteqdot\frac{a}{3}(6a)$

$2a^2\doteqdot 3200$

$a^2\doteqdot 1600$

$\therefore a\doteqdot\pm 40$ C

9. Differentiate $2\tan 3x$

$\frac{d}{dx}(2\tan 3x)=2\times 3\sec^2 3x$

$=6\sec^2 3x$

B

10. $\int_0^{\frac{\pi}{8}}4\cos 4x\,dx=[\sin 4x]_0^{\pi/8}$

$=\sin\left(\frac{4\pi}{8}\right)-\sin 0$

$=1-0$

$=1$ A

Q 11

(a) $2c^2 - 50d^2 = 2(c^2 - 25d^2)$

$= 2[c^2 - (5d)^2]$

$= 2(c-5d)(c+5d)$

1 mark for partial factorisation

Another mark for complete factorisation

(b) $$\frac{6}{\sqrt{17}-\sqrt{11}} \times \frac{\sqrt{17}+\sqrt{11}}{\sqrt{17}+\sqrt{11}}$$ ← 1 mark for showing this step.

$$= \frac{6(\sqrt{17}+\sqrt{11})}{17-11}$$

$$= \sqrt{17}+\sqrt{11}$$ ← 1 mark for correct answer.

(c) $|3x-7| = 17$

$3x-7 = \pm 17$ ← 1 mark

$3x = 7 \pm 17$

$x = \frac{7 \pm 17}{3}$

$= 8, -\frac{10}{3}$ ← 1 mark

(d) $$\frac{5}{y+2} - \frac{2}{y} = \frac{5y-2(y+2)}{y(y+2)}$$ ← 1 mark for working

$$= \frac{3y-4}{y(y+2)}$$

or $$\frac{3y-4}{y^2+2y}$$ ← 1 mark for answer.

Q11 (continued)

(e) $\frac{x+3}{2} = 5$ $\qquad \frac{y-4}{2} = -1$

$x = 7$ 1mark $\qquad y = 2$ 1mark.

$\therefore M(7,2)$

(f) $5x - y = 19$

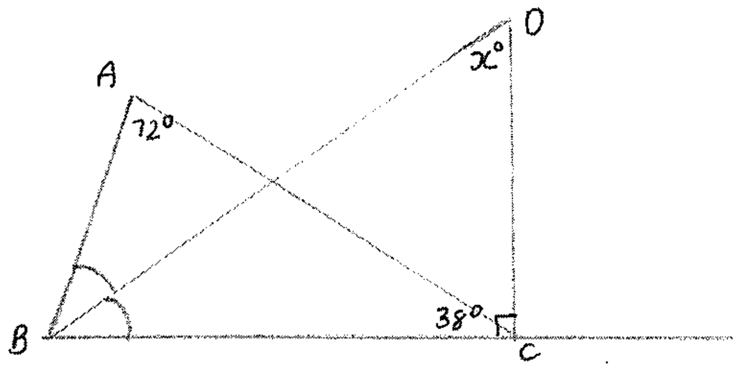
$2x + y = 2$

$7x = 21 \qquad 6 + y = 2$ 1mark for working

$x = 3 \qquad y = -4$

$\therefore$ pt of intersection is $(3,-4)$ 1 for correct answer.

(g)



$\angle ABC + 38 + 72 = 180$ (Angle sum ΔABC) 1mark

$\angle ABC = 70°$

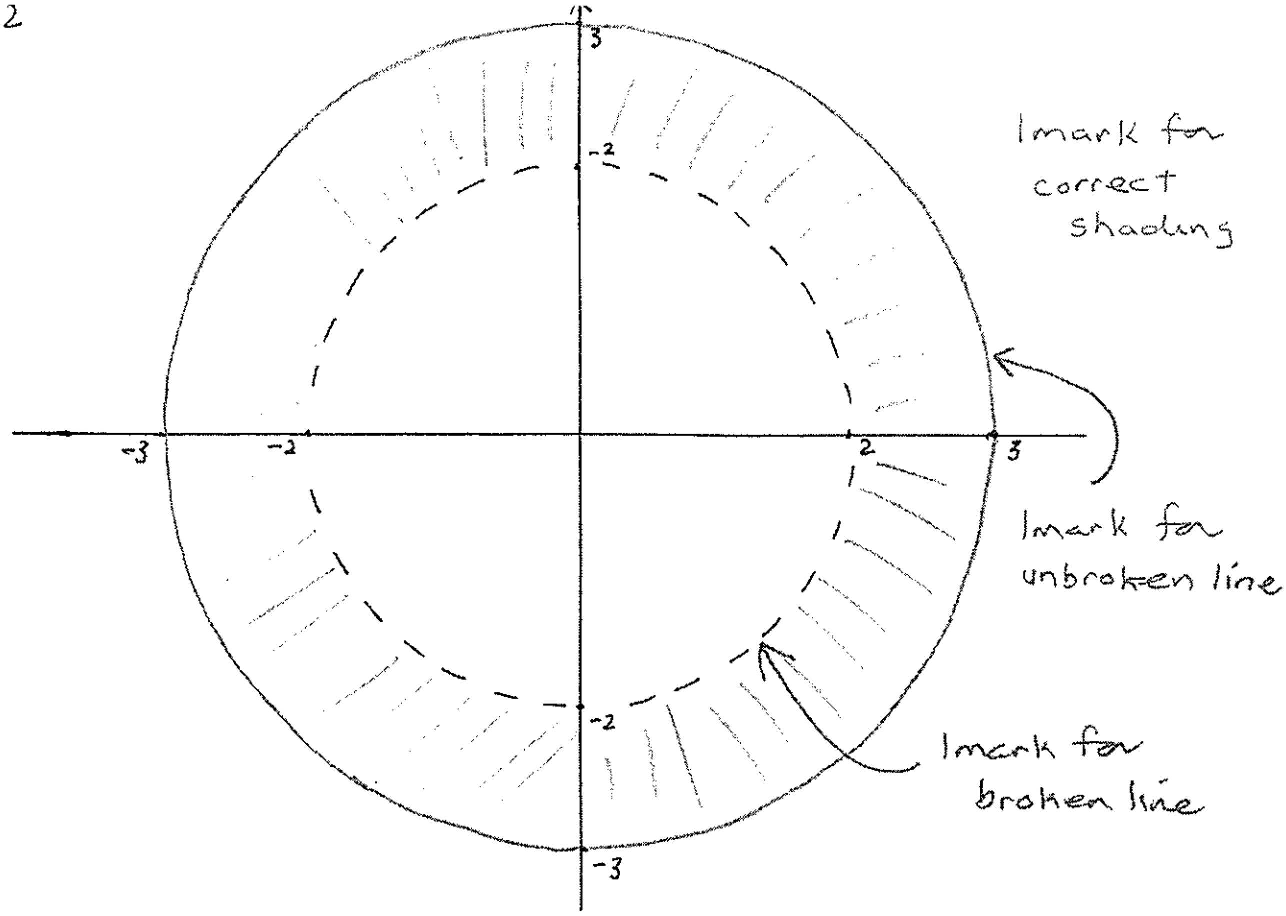
$\angle DBC = 35°$ (DB bisects $\angle ABC$) 1mark

$x + 90 + 35 = 180$ (Angle sum ΔBDC) 1mark

$x = 55°$

Q 12

a)



b)

$$f(3) - f(5)$$

$$= (3-5) - (5^2-16)$$

$$= -2 - 9$$

$$= -11$$

1mark for correct working

1 for correct answer.

Q 12 (continued)

(c) (i) $y + 8 = -2(x - 6)$

$y + 8 = -2x + 12$

$2x + y - 4 = 0$

Must show full working to receive mark.

(ii) $QR = \sqrt{(2 - -1)^2 + (12 - 6)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

must show working

(iii) Distance $P\ell = \dfrac{|2 \times 6 + -1 \times -8 + 8|}{\sqrt{4+1}}$

$= \dfrac{12 + 8 + 8}{\sqrt{5}}$ ✓ 1 mark for working

$= \dfrac{28}{\sqrt{5}}$ ✓ 1 mark for correct answer.

(iv) Area of $\triangle PQR = \frac{1}{2} \times 3\sqrt{5} \times \dfrac{28}{\sqrt{5}}$

$= 42\,u^2$ ✓ correct answer

(d) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{7^2 + 6^2 - 4^2}{2 \times 7 \times 6}$

$= \dfrac{49 + 36 - 16}{84}$

$= \dfrac{23}{28}$

Must apply Cosine rule showing sufficient working.

Q 12 (continued) SOLUTIONS

(e)

(i) $\angle OAC = \angle OBD = 90^\circ$ (Given) ✓

$\angle COA = \angle DOB$ (common angle) ✓

1 mark

$\therefore \triangle OCA \,|||\, \triangle ODB$ (equiangular) 1 mark

(ii) $$\frac{AC}{BD} = \frac{OA}{OB}$$ (sides of similar Δ's in same ratio)

Award 1 mark

$$\therefore \frac{3}{5} = \frac{5}{5+AB}$$

$$15 + 3AB = 25$$

$$3AB = 10$$

$$\therefore AB = \frac{10}{3}\text{ cm}$$ ← Award 1 mark

13(a) $y = 3x^4 - 16x^3 + 24x^2$

(i) $f(x) = 3x^4 - 16x^3 + 24x$

$f'(x) = 12x^3 - 48x^2 + 48x$

$= 12x(x^2 - 4x + 4)$ ← 1 mark for differentiation.

$= 12x(x-2)^2$

$f'(x) = 0$

$12x(x-2)^2 = 0$

$x = 0, \quad x = 2.$

$f(0) = 0$

$f(2) = 3(2)^4 - 16(2)^3 + 24(2)^2$

$= 48 - 128 + 96$

$= 16$ ← 1 mark for finding Stationary points.

$\therefore (0,0)$ and $(2,16)$ are the stationary points.

(ii) For $x = 0$, $y = 0$

$x = 2$, $y = 16$

x		0		2	
$\frac{dy}{dx}$	$-$	0	$+$	0	$+$
y	$\searrow$	$(0,0)$	$\nearrow$	$(2,16)$	$\nearrow$

← 2 marks to determine the nature of the points

$(2,16)$ is a horizontal point of inflexion ← 1 mark for H.P.O.I. and identify $(0,0)$ is a minimum turning point.

$(0,0)$ is minimum turning point

(iii) Points of inflexion occur when $\frac{d^2y}{dx^2} = 0 = f''(x)$

$f''(x) = 36x^2 - 96x + 48$

$= 12(3x^2 - 8x + 4)$

$= 12(3x-2)(x-2)$

$= 0$

iii) Continue

$x = \frac{2}{3}$ or $x = 2$

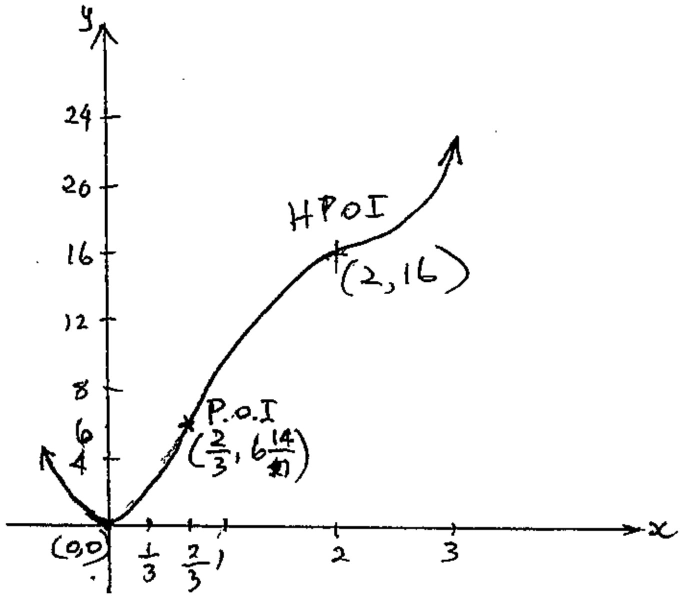
$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^4 - 16\left(\frac{2}{3}\right)^3 + 24\left(\frac{2}{3}\right)^2$

$= 6\frac{14}{27}$

$f(2) = 3(2)^4 - 16(2)^3 + 24(2)^2$

$= 16$

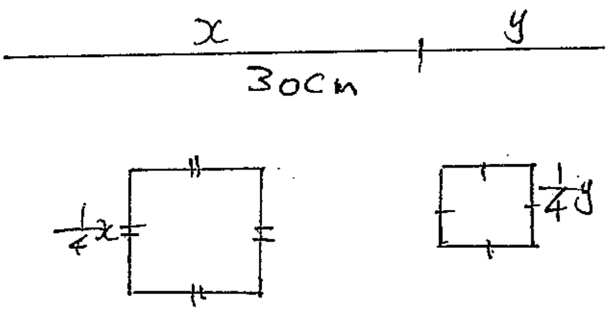
x	0	$\frac{2}{3}$	1	2	3
$\frac{d^2y}{dx^2}$	+	0	−	0	+
y	∪	$6\frac{14}{27}$	∩	16	∪



← 1 mark (2 points must show full working to receive mark)

← 1 mark for correct graph.

b)



i) $x + y = 30$

$y = 30 - x$

← 1 mark for showing $y = 30 - x$.

ii) Total area of the two squares

$= \left(\frac{1}{4}y \times \frac{1}{4}y\right) + \left(\frac{1}{4}x \times \frac{1}{4}x\right)$

$= \left(\frac{1}{4}y\right)^2 + \left(\frac{1}{4}x\right)^2$

$= \frac{1}{16}y^2 + \frac{1}{16}x^2$ since $y = 30 - x$

1 mark for correct substitution.

$\therefore A = \frac{(30-x)^2}{16} + \frac{1}{16}x^2$

$= \frac{900 - 60x + x^2 + x^2}{16}$

$= \frac{900 - 60x + 2x^2}{16}$

$= \frac{450 - 30x + x^2}{8}$

← 1 mark for finding the total area.

$= \frac{x^2 - 30x + 450}{8}$

Hourly Cost $= S^2 + 9000$ cents
Distance $d = 1500$ km

Show $C = 1500\left(S + \frac{9000}{S}\right)$

$$t = \frac{\text{Distance}}{\text{speed}} = \frac{1500\text{km}}{S}$$

Total cost for the trip = hourly Cost × time t

$$C = \left(S^2 + 9000\text{Cents}\right)\frac{1500\text{km}}{S}$$ ← 1 mark for correct substitution

$$= 1500\left(\frac{S^2}{S} + \frac{9000}{S}\right)$$ ← 1 mark for working

$$= 1500\left(S + \frac{9000}{S}\right)$$

Find the speed that minimizes the cost of the trip.

Since $C = 1500\left(S + \frac{9000}{S}\right)$

$$\therefore C = 1500S + \frac{13500000}{S}$$

$$\frac{dc}{dS} = 1500 - \frac{13500000}{S^2}$$ ← 1 mark for differentiation.

$$\frac{dc}{dS} = 0$$

$$1500S^2 = 13500000$$ ← 1 mark for correct working.

$$S^2 = 9000$$

$$S = 94.87$$

$$S \doteqdot 95\ \text{km/h}$$ ← 1 mark for correct answer

14 a) $\int (x-1)(x-2)\,dx = \int (x^2-3x+2)\,dx$ ← 1 mark for expanding

$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x + C$ ← 1 mark for integrating.

b) $\int_0^1 \frac{x^3+2x^2}{x}\,dx = \int_0^1 (x^2+2x)\,dx$

$= \left[\frac{x^3}{3} + x^2\right]_0^1$ ← 1 mark for integrating

$= \left[\frac{1}{3}+1\right] - 0$

$= \frac{4}{3}$

$= 1\frac{1}{3}$ ← 1 mark for correct answer.

c)

(i)

x	0	40	80	120	160
y	60	70	90	60	30
	y_0	y_1	y_2	y_3	y_4

Trapezoidal rule

$A \doteqdot \frac{h}{2}(y_0 + 2(y_1+y_2+y_3) + y_4)$ ← 1 mark for substituting 5 function values.

$\doteqdot \frac{40}{2}(60 + 2(70+90+60) + 30)$

$\doteqdot 10600\text{ m}^2$ ← 1 mark for correct answer.

(ii) It can be seen that the trapezoidal Rule gives an area greater than the actual area. (as the sum of the areas of the 4 trapeziums is greater than the actual area.) ← 1 mark for giving the reasons.

d) $y = \sqrt{4-2x} = (4-2x)^{\frac{1}{2}}$

(i) $m = \frac{dy}{dx} = \frac{1}{\cancel{2}}(4-2x)^{-\frac{1}{2}} \times (-\cancel{2}) = -\frac{1}{\sqrt{4-2x}}$

at $x = 1\frac{1}{2} = \frac{3}{2}$

$m_{tangent} = -\frac{1}{\sqrt{4-\cancel{2}(\frac{3}{\cancel{2}})}} = -1$

When $x = \frac{3}{2}$ $y = (4-\cancel{2}(\frac{3}{\cancel{2}}))^{\frac{1}{2}} = 1$ ← 1 mark for finding the co-ordinate $x=\frac{3}{2}$, $y=1$.

$\therefore x = \frac{3}{2}, y = 1$

d)

i) Continue

Using the point-gradient formula

$$(y-1) = -1\left(x-\frac{3}{2}\right)$$

$$y-1 = -x+\frac{3}{2}$$

$$y = -x+\frac{5}{2}$$

$$2y = -2x+5$$

$$\therefore\ 2x+2y-5=0$$

1 mark for using the point-gradient formula to show the equation of the tangent l.

i) To find the area $\int_0^2 (4-2x)^{\frac{1}{2}}\,dx$

$$= \left[-\frac{1}{3}(4-2x)^{3/2}\right]_0^2$$

1 mark for integrating

$$= -\frac{1}{3}\left(0-4^{3/2}\right)$$

$$= -\frac{1}{3}(-8)$$

$$= \frac{8}{3}$$

$$= 2\frac{2}{3}\ \text{units}^2$$

1 mark correct working.

ii) The equation of the tangent is

$$2x+2y-5=0$$

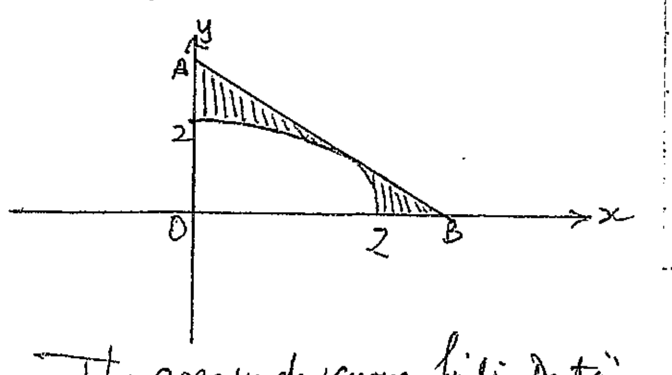
To find the coordinates of A and B.

Let $x=0$ to give $y=\frac{5}{2}$

$y=0$ to give $x=\frac{5}{2}$

$$\therefore\ \text{Area of } \triangle OAB = \frac{1}{2}\times\frac{5}{2}\times\frac{5}{2} = \frac{25}{8}$$

1 mark for finding the area of $\triangle OAB$.



The area under curve find in Part (ii) is

$$A = \frac{8}{3}$$

$\therefore$ The required area is

$$A_{\triangle OAB} - A_{\ldots} = \frac{25}{8} - \frac{8}{3} = \frac{11}{24}\ \text{units}^2$$

1 mark to find the area of the shaded region.

d) Find the volume of the spaceship.

iv) We need to write x^2 in terms of y. Changing the subject to x^2

$$y = \sqrt{4-2x}$$

$$y^2 = 4-2x$$

$$2x = 4-y^2$$

$$x = 2-\frac{y^2}{2}$$

$$x^2 = \left(2-\frac{y^2}{2}\right)^2$$

$$= 4-2y^2+\frac{1}{4}y^4$$

The volume of the spaceship is

$$V = \pi\int_0^2 x^2\,dy$$

$$= \pi\int_0^2 \left(4-2y^2+\frac{1}{4}y^4\right)dy$$

$$= \pi\left[4y-\frac{2}{3}y^3+\frac{1}{20}y^5\right]_0^2$$ ← 1 mark for integrating.

$$= \pi\left[8-\frac{16}{3}+\frac{32}{20}\right]$$

$$= \frac{64\pi}{15} \text{ units}^3$$ ← 1 mark correct answer.

Q 15

SOLUTIONS

(a)

$$\underline{LHS} \qquad\qquad \underline{RHS}$$

$$(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$$

$\underline{LHS}$

$$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$$ 1mark awarded

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\cancel{(1+\cos\theta)}(1+\cos\theta)}{\cancel{(1+\cos\theta)}(1-\cos\theta)}$$ 1mark awarded

$$= \frac{1+\cos\theta}{1-\cos\theta}$$

$\therefore$ LHS = RHS

(b) (i) $\alpha\beta = \frac{3}{3}$

$= 1$ 1mark

(ii) $\alpha\beta = 1$

$\beta = \frac{1}{\alpha}$

$\therefore \alpha + \frac{1}{\alpha} = \frac{8}{3}$ 1mark

Q15 (continued) Solutions

(c)

(i) $x^2 - 4x + 8 = 4y$

$x^2 - 4x = 4y - 8$

$x^2 - 4x + (-2)^2 = 4y - 8 + 4$ 1 mark awarded

$(x-2)^2 = 4(y-1)$

$\therefore$ Vertex is $(2, 1)$ 1 mark for correct answer

(ii) Since the focal length $= 1$,
the equation of directrix is $y = 0$ 1 mark

(d)

(i) $l = r\theta$

$25\pi = r \times \frac{50\pi}{180}$ ← 1 mark

$r = \frac{25 \times 180}{50}$ ← 1 mark

$= 90$ cm

(ii) Shaded area $= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 90^2\left(\frac{5\pi}{18} - \sin\left(\frac{5\pi}{18}\right)\right)$ ✓ 1 mark

$= 431.8 \text{ cm}^2$ (1 decimal pl)

↑ 1 mark for correct rounding

Q 15 continued Solutions

e)(i) In ΔECD, $\angle ECD = x$ (angles opposite equal sides in isosceles Δ.)

$\therefore \angle CEA = 2x^\circ$ (exterior angle of ΔCED) 1mark

In ΔAEC, $\angle ECA = \angle EAC$ (angles opposite equal sides in isosceles Δ)

$= \frac{180-2x}{2}$ (Angle sum of ΔAEC)

$= 90 - x$ 1mark

Now $\angle ACB = 180 - (\angle AEC + \angle ECD)$ (Straight angle)

$= 180 - (90 - x + x)$

$= 90^\circ$ 1mark

(ii) $\angle BAD = \angle CED$ (Corresponding angles, $AB \parallel EC$)

and $\angle CED = 180 - 2x$ (Angle sum of ΔCED) ✓

$\therefore \angle BAD = 180 - 2x$

1mark

Question 16 Solutions

a) (i) $\frac{d}{d\theta}(\tan^3\theta) = 3\tan^2\theta . \sec^2\theta$ 1mark

$= 3(\sec^2\theta - 1)\sec^2\theta$

$= 3\sec^4\theta - 3\sec^2\theta$ 1mark

(ii) $\int_0^{\frac{\pi}{4}} \sec^4\theta \, d\theta$

$= \frac{1}{3}\int_0^{\frac{\pi}{4}} (3\sec^4\theta - 3\sec^2\theta) + 3\sec^2\theta \quad d\theta$ 1mark

$= \frac{1}{3}\left[\tan^3\theta + 3\tan\theta\right]_0^{\frac{\pi}{4}}$

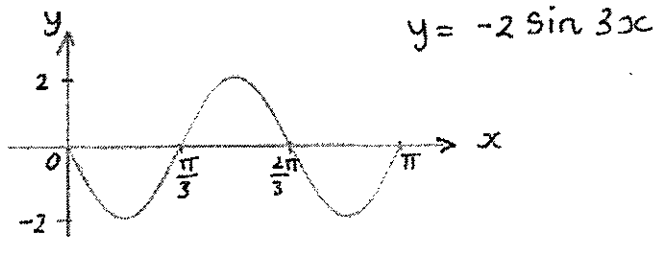
$= \frac{1}{3}\left[\left(\tan^3\frac{\pi}{4} + 3\tan\frac{\pi}{4}\right) - 0\right]$

$= \frac{4}{3}$ 1mark

b) (i) The period x

$3x = 2\pi$

$x = \frac{2\pi}{3}$ 1mark

(ii) 

1mark for correct curve

1mark for correct labelling

Question 16 (continued)

(c) (i) When $y=1$,

$$1+\cos x = 1$$

$$\cos x = 0$$

$$\therefore \quad x = \frac{\pi}{2}$$

Show working for 1 mark

(ii) Gradient $OA = \dfrac{1}{\frac{\pi}{2}}$

$$= \frac{2}{\pi}$$

Show working for 1 mark.

$\therefore$ OA : $\quad y = \dfrac{2}{\pi}x$

(iii) $$A = \int_0^{\frac{\pi}{2}} \left(1+\cos x - \frac{2x}{\pi}\right) dx$$

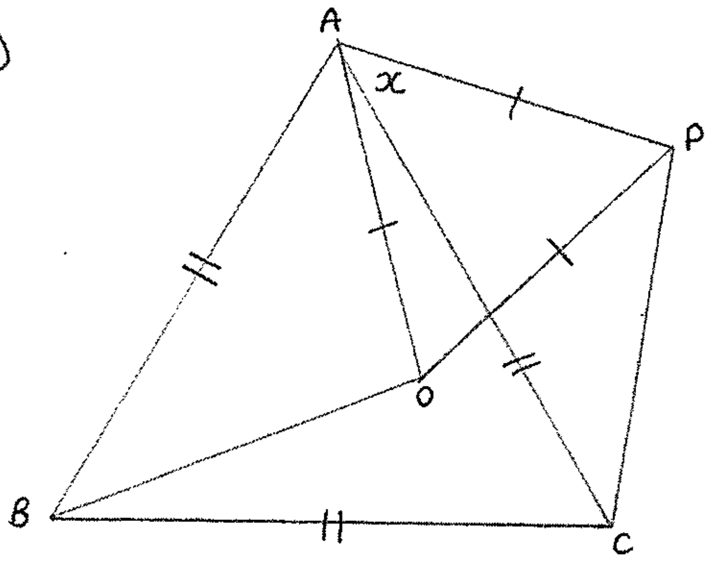
$$= \left[x + \sin x - \frac{1}{\pi}x^2\right]_0^{\frac{\pi}{2}}$$ ✓ 1 mark.

$$= \left[\frac{\pi}{2} + 1 - \frac{1}{\pi} \times \frac{\pi^2}{4}\right] - \left[0+0-0\right]$$

$$= \left(\frac{\pi}{4} + 1\right) \text{unit}^2$$ ✓ 1 mark

Question 16 (continued) Solutions

(d) (i)



ii) let $\angle PAC = x$

$\therefore \angle CAO = 60 - x$ (ΔAPO is equilateral)

$\angle BAC = 60$ (ΔBAC is equilateral)

1 mark

$\angle BAO = 60 - (60 - x)$

$= x$

$\therefore \angle PAC = \angle BAO$

1 mark

(iii) In Δ's AOB & APC :

$AB = AC$ (sides of equilateral ΔABC)

$AO = AP$ (sides of equilateral ΔAOP)

1 mark

$\angle BAO = \angle CAP$ (shown above in (ii))

$\therefore \Delta AOB \equiv \Delta APC$ (SAS)

1 mark

deduct 1 mark if no diagram given.