Name: ______________________________

KILLARA HIGH SCHOOL

HSC MATHEMATICS ASSESSMENT

TERM 2, 2003

Time Allowed: 70 minutes

Maximum Marks: 50 marks

Instructions:

* approved calculators may be used.
* all necessary working must be shown.
* start each question on a new page.
* use one side of page only.
* answer in blue or bl ack biro, pencil for graphs and diagrams.

FORMAT:

Question 1 -	Geometry - Coordinate and Plane	14 marks
Question 2 -	Trigonometric Functions	14 marks
Question 3 -	Exponential and Logarithmic Functions	22 marks
	TOTAL:	50 marks

Question 1 Geometry *(14 marks)*

(a) (i) Neatly draw a number plane, marking the following points: **1**

O, the origin
M, (5, 0)
N, (8, 4)
P, (0, 10)

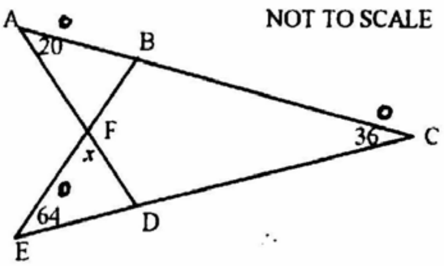
(ii) Show that the equation of MN is $4x - 3y - 2o = 0$ **2**

(iii) Find the gradient of NP. **1**

(iv) Show that MN and NP are perpendicular. **2**

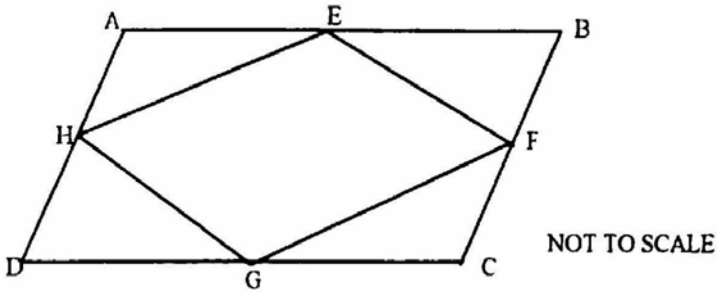
(v) Find the length of MN. **2**

(b) **2**



Find the value of x, giving all reasons.

(c) In the diagram, ABCD is a parallelogram. Points E, F, G and H are the mid points of AB, BC, CD and DA respectively.



(i) Prove that ΔAEH is congruent to ΔFGC **3**

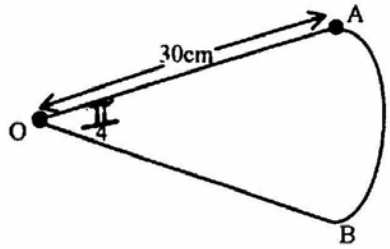
(ii) What other information is needed to show that EFGH is a parallelogram? **1**

Question 2 Trigonometric Functions *(14 marks)*

(a) (i) Write $53° \ 45'$ in radians in terms of π. **1**

(ii) Find to 4 decimal places $\sec\dfrac{3\pi}{4}$. **1**

(b)



NOT TO SCALE

In the diagram, AB is the arc of a circle with centre O. The radius OA is $30cm$.
Angle AOB is $\dfrac{\pi}{4}$ radians.
Find the exact area of sector AOB. **2**

(c) Differentiate with respect to x :

(i) $\tan 6x$ **1**

(ii) $\dfrac{x^3}{\sin x}$ **1**

(iii) $\dfrac{1}{\cos x}$ **1**

(d) Integrate:

(i) $\sin 2x \quad dx$ **1**

(ii) $\int_o^{\pi}(\cos\theta - 1)d\theta$ **1**

(e) (i) Neatly draw the graph of $y = 3\cos 2x$ in the domain $O \le x \le \pi$ **2**

(ii) Find the area bounded by the curve $y = 3\cos 2x$ and the x axis between $x = 0$ and $x = \dfrac{3\pi}{4}$. Give your answer correct to 3 decimal places. **3**

Question 3 Exponential and Logarithmic Functions *(22 marks)*

(a) (i) Write as a fraction in simplest form $125^{-\frac{2}{3}}$ 1

(ii) Simplify $4^{3x} \times 2^{4x} \div 8^{2x}$ 1

(b) Express as a numeral $\log_5 50 + \log_5 10 - \log_5 4$ 1
Give your answer correct to 3 decimal places.

(c) (i) If $\log_6 2 = 0 \cdot 43$ and $\log_6 3 = 0 \cdot 68$, evaluate $\log_6 48$ correct to 2 decimal places. 1

(ii) Find the value of $\log_9 14$ correct to 3 decimal places. 1

(d) On the same number plane, graph: 3

(i) $y = 3^x$

(ii) $y = \log_3 x$

(e) Differentiate:

(i) e^{-2x} 1

(ii) $\dfrac{e^x}{e}$ 1

(iii) $\log_e(5x - 4)$ 1

(iv) $\dfrac{\log e^x}{3x + 1}$ 1

(f) Integrate:

(i) $\int e^{2x}$ 1

(ii) $\int \dfrac{4x}{2x^2 + 3}$ 1

(g) Find $\displaystyle\int_2^5 \frac{x^2}{x^3 - 1}dx$ giving your answer correct to 2 decimal places. 2

(h) Find the area bounded by the curve $y = 3e^x$, the x axis and the lines $x = 1$ and $x = 4$. Give your answer correct to 2 decimal places. 3

Continued over . . . .

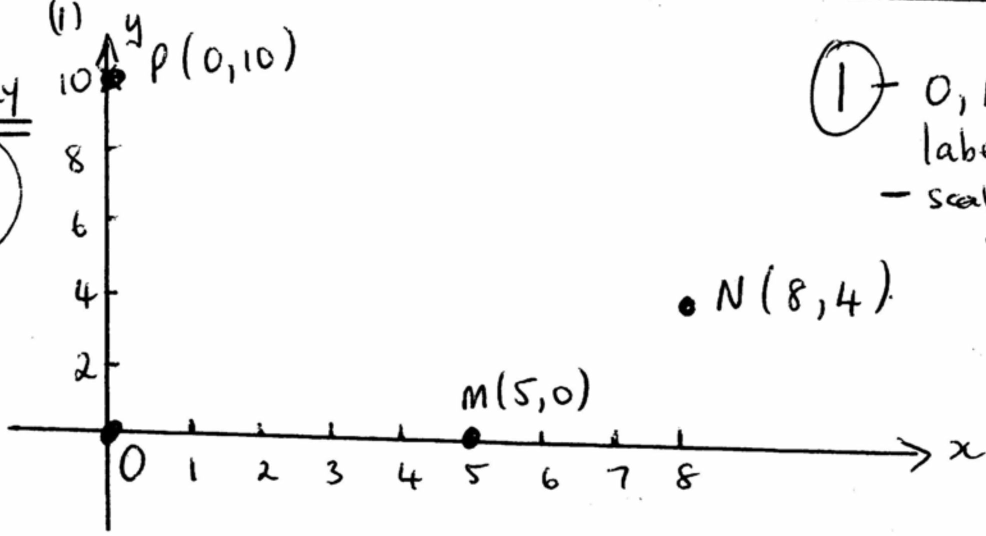
(i) The region bounded by the curve $y = \dfrac{1}{\sqrt{x+3}}$, the x axis, and the lines $x = -2$ and $x = 2$, is rotated around the x axis. Find the volulme of the solid of revolution formed. Give your answer in exact form. **3**

END OF EXAM

YR 12 MSC ASSESSMENT TERM 2 2003 – ANSWERS

Q1. GEOMETRY
$\frac{}{14}$

(i)



(1) – O, M, N & P labelled
– scales on both axes.

(ii) Equation of MN : $\frac{y-0}{x-5} = \frac{4-0}{8-5}$ (1)

$$\frac{y-0}{x-5} = \frac{4}{3}$$

$$3y = 4x - 20$$

$$\therefore 4x - 3y - 20 = 0 \quad (1)$$

(iii) Gradient of NP : $\frac{y_2 - y_1}{x_2 - x_1} = m.$

$$\frac{10-4}{0-8} = -\frac{6}{8} = -\frac{3}{4} \quad (1)$$

(iv) Gradient of MN : $\frac{4-0}{8-5} = \frac{4}{3}.$ (1)

$\therefore$ gradient of NP × gradient of MN $= -\frac{3}{4} \times \frac{4}{3} = -1$ (1)

$\therefore NP \perp MN.$

(v) $MN = \sqrt{(8-5)^2 + (4-0)^2}$ (1)

$= \sqrt{9 + 16} = \sqrt{25}$

$=$ 5 units (1)

(b) $\angle ADC = 124^\circ$ (angle sum ΔACD)
$\angle FDE = 56^\circ$ (straight $\angle EDC$) } (1) must have 2 answers + 2 reasons.
$\therefore\ x = 60^\circ$ (angle sum ΔFED)
(1)

(c) (i) In Δ's AEH and FGC,
(1) $\angle HAE = \angle FCG$ (opp. $\angle$'s, //gram ABCD)
(1) $HA = BF$ (H + F mid points of equal sides //gram ABCD)
(1) $AE = GC$ (E + G mid points of equal sides //gram ABCD)
$\therefore \Delta AEH \equiv \Delta FGC$ (SAS)

(ii) $HG = EF$ (as $HE = GF$ from (i))
(1) (other facts possible - must have one - no reason necessary.

Q2 TRIG. FUNCTIONS

14

(a) (i) $53^\circ 45' = \frac{\pi}{180} \times 53\frac{3}{4} = \frac{43\pi}{144}$ (1)

(ii) $\sec\frac{3\pi}{4} = \frac{1}{\cos\frac{3\pi}{4}} = -1\cdot4142$ (1)

(b) Area $= \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 30^2 \times \frac{\pi}{4}$ (1)
$= \frac{450\pi}{4}\ cm^2. = \frac{225\pi}{2}\ cm^2.$ (1)

(c) (i) $y = \tan 6x$
$\frac{dy}{dx} = 6\sec^2 6x$ (1)

(ii) $y = \frac{x^3}{\sin x}$
$\frac{dy}{dx} = \frac{\sin x . 3x^2 - x^3 . \cos x}{(\sin x)^2}$
$\frac{dy}{dx} = \frac{x^2(3\sin x - x\cos x)}{\sin^2 x}$ } either step (1)

(iii) $y = \frac{1}{\cos x}$
$\frac{dy}{dx} = \frac{\sin x}{\cos^2 x}$
$\frac{dy}{dx} = \sec x \tan x$ } (1) either step

Q2 (continued)

(d) (i) $\int \sin 2x \, dx$

$= -\cos 2x + C$ ①

(ii) $\int_0^{\pi} (\cos\theta - 1) \, dx$

$= \left[\sin\theta - \theta\right]_0^{\pi}$

$= (0 - \pi) - (0 - 0)$

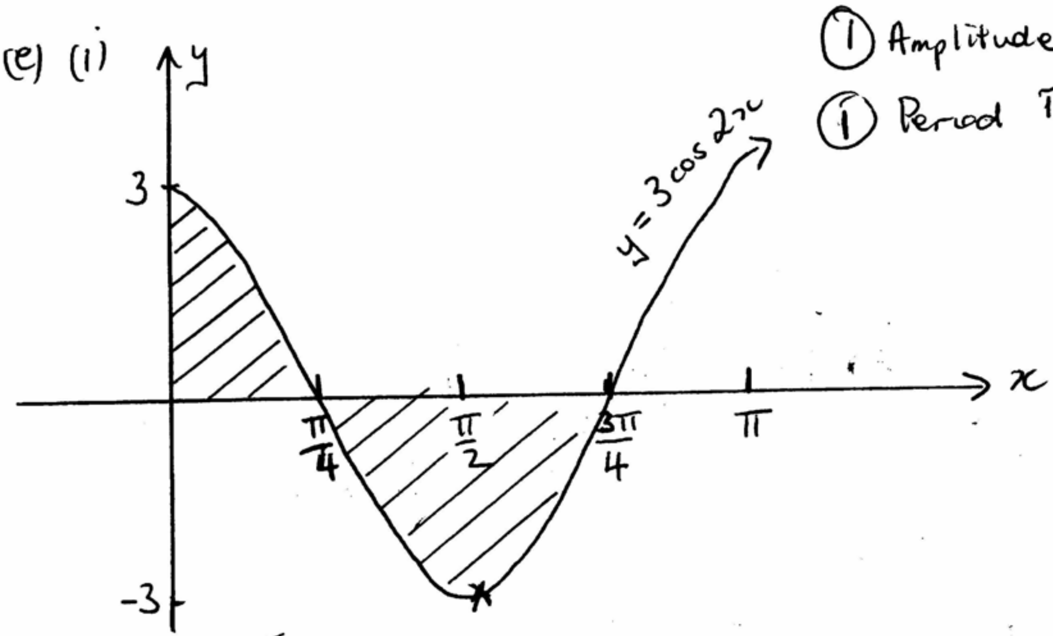
$= -\pi$ ①

Graph must be cos graph.

① Amplitude 3

① Period π

(e) (i)



(ii) Area $= \int_0^{\frac{3\pi}{4}} 3\cos 2x \, dx$

$= \int_0^{\frac{\pi}{4}} 3\cos 2x \, dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 3\cos 2x \, dx$ ① for one of these steps.

$= 3\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} + 3\left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= 3\left[\left(\frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin 0\right] + 3\left[\left(\frac{1}{2}\sin\frac{3\pi}{2} - \frac{1}{2}\sin\frac{\pi}{2}\right]\right.\right.$ ①

$= 1\frac{1}{2} + 3$

$= 4 \cdot 500 \, u^2$ ①

QUESTION 3 (22)

EXP. and LOG FUNCTIONS.

(a) (i) $\dfrac{1}{(\sqrt[3]{125})^2} = \dfrac{1}{25}$ 1

(ii) $(2^2)^{3x} \times 2^{4x} \div (2^3)^{2x}$

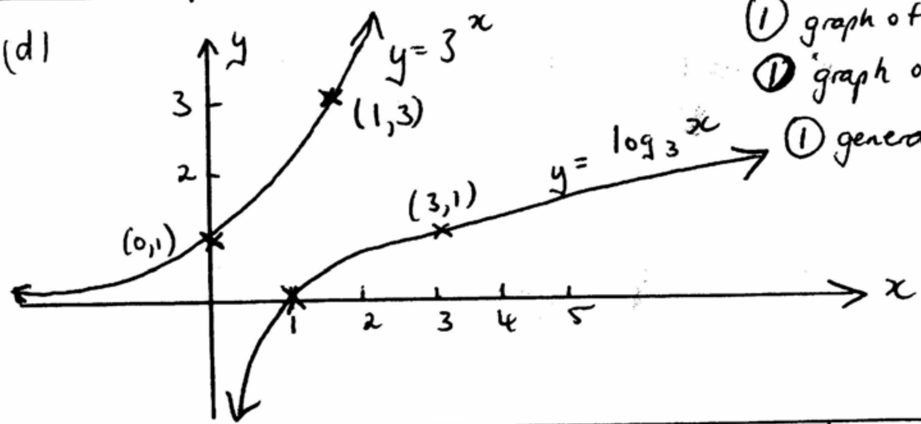
$= 2^{6x} \times 2^{4x} \div 2^{6x} = 2^{4x}$ 1

(b) $\dfrac{\log 50}{\log 5} + \dfrac{\log 10}{\log 5} - \dfrac{\log 4}{\log 5} = 3$ 1 (3.000 ✓)

(c) (i) $\log_6 48 = \log_6(2^4 \times 3) = \log_6 2^4 + \log_6 3 = 4\log_6 2 + \log_6 3 = 2.4$ 1

(ii) $\dfrac{\log 14}{\log 9} = 1.20108\ldots = 1.201$ to 3 dec. pl. 1

(d)



1 graph of $y = 3^x$ (show (0, 1))

1 graph of $y = \log_3 x$ (show (1, 0))

1 general – ruled axes
– graphs labelled

(e) (i) $y = e^{-2x}$

$\dfrac{dy}{dx} = -2e^{-2x}$ 1

(ii) $y = \dfrac{e^x}{e}$

$\dfrac{dy}{dx} = \dfrac{e^x}{e}$ (or e^{x-1}) 1

(iii) $y = \log_e(5x-4)$

$\dfrac{dy}{dx} = \dfrac{5}{5x-4}$ 1

(iv) $y = \dfrac{\log_e e^x}{3x+1}$ $\quad \dfrac{dy}{dx} = \dfrac{(3x+1)\times 1 - x\times(3)}{(3x+1)^2} = \dfrac{1}{(3x+1)^2}$ 1

(f) (i) $\int e^{2x} = \frac{1}{2}e^{2x} + C$ 1

(ii) $\int \dfrac{4x}{2x^2+3} = \log(2x^2+3) + C$ 1

(g) $\displaystyle\int_2^5 \frac{x^2}{x^3-1} = \int_2^5 \frac{1}{3} \times \frac{3x^2}{x^3-1} = \frac{1}{3}\Big[\ln(x^3-1)\Big]_2^5$ 1

$= \frac{1}{3}(\ln 124 - \ln 7)$

$= 0.96$ to 2 dec. pl. 1

(3)

(h) $A = \int_1^4 3e^x$

$= \left[3e^x \right]_1^4$ (1)

$= 3e^4 - 3e^1$

$= 3(e^4 - e)$

$= 155{\cdot}6396\ldots\ldots$ (1)

$= 155{\cdot}64\ u^2$ to 2 dec. pl. (1)

(i) $\left. \begin{aligned} V &= \pi \int_{-2}^{2} y^2\, dx \\ &= \pi \int_{-2}^{2} \left(\frac{1}{\sqrt{x+3}} \right)^2 dx \end{aligned} \right\}$ (1) one of these steps.

$= \pi \int_{-2}^{2} \frac{1}{x+3}$

$= \pi \left[\ln(x+3) \right]_{-2}^{2}$ (1)

$= \pi \left[\ln 5 - \ln 1 \right]$

$= \pi \ln 5\ u^3$ (1)