Trinity Grammar School

Mathematics Department

2016

HALF-YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals (if required) in this course is supplied
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 30%

Board of Studies Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Thursday, 28 April 2016

Total marks – 100

Section I

10 marks

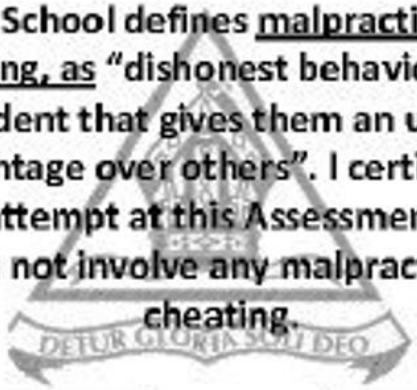
- Attempt all Questions
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about 2 hours 45 minutes for this section

The School defines malpractice, or cheating, as "dishonest behaviour by a student that gives them an unfair advantage over others". I certify that my attempt at this Assessment task does not involve any malpractice or cheating.



Student signature _____ Date _____

Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 The derivative of $x^2(x+2)^3$ is :

(A) $3x^2(x+2)$

(B) $3x^2(x+2)^2$

(C) $3x^2(x+2)^2 + 2x(x+2)^3$

(D) $3x^2(x+2)^3 + 2x(x+2)^2$

2 Let α and β be roots of the equation $x^2 - 2x + 4 = 0$. What is the value of $(\alpha+\beta)\alpha\beta$?

(A) -16

(B) -8

(C) 8

(D) 16

3 The value of $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5}$ is :

(A) 0

(B) 1

(C) 5

(D) 10

4 Given that $V = \frac{1}{3}\pi r^2 h$ and that $r > 0$, find r to 3 significant figures if V = 2000 and $h = 12$.

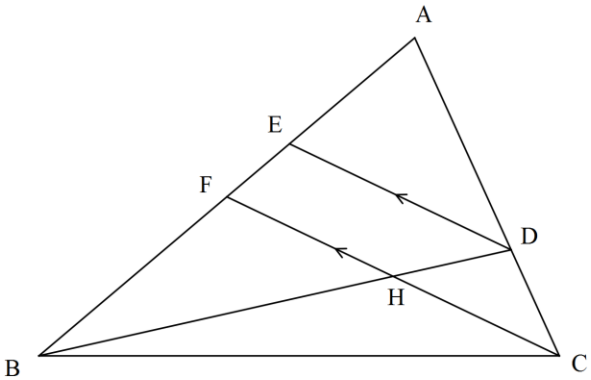
(A) 4.205

(B) 4.21

(C) 12.6

(D) 12.616

5 For the diagram below, F is the mid-point of AB and AD:DC=2:1



NOT TO SCALE

What is the ratio of BH : HD?

(A) 2 : 1

(B) 2 : 3

(C) 3 : 1

(D) 3 : 2

6 What is the solution to the inequality $(x+2)(x-5) < 0$?

(A) $-5 < x < 2$

(B) $x < -5$ and $x > 2$

(C) $-2 < x < 5$

(D) $x < -2$ and $x > 5$

7 What is the value of $f'(\frac{1}{2})$ if $f(x) = 5x - \frac{1}{x}$?

(A) $\frac{1}{2}$

(B) 2

(C) $5\frac{1}{2}$

(D) 9

8 A parabola has focus (7,0) and directrix $x = 1$. What is the equation of the parabola?

(A) $y^2 = 24(x - 7)$

(B) $y^2 = 12(x - 4)$

(C) $y^2 = -24(x - 7)$

(D) $y^2 = -12(x - 4)$

9 What is the solution to $\int_{-1}^{1} (x^2 + x + 2)\ dx$?

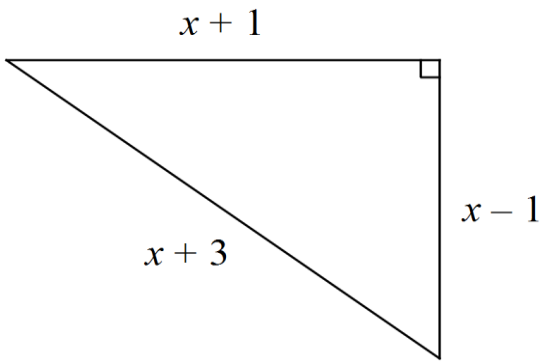
(A) 1

(B) 4

(C) $4\frac{2}{3}$

(D) $5\frac{2}{3}$

10 For the triangle below, what is a possible value for x?



NOT TO SCALE

(A) 3

(B) 5

(C) 7

(D) 9

End of Section I
Section II commences on the next page

Section II 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

a) Factorise : $am + bm + ap + bp + ay + by$ **2**

b) $\dfrac{4}{\sqrt{3}-1}$ can be expressed in the form $a\sqrt{3} + b$. Find the values of a and b. **2**

c) Solve the equations:

(i) $3^{2x+1} = \dfrac{1}{243}$ **2**

(ii) $|x+5| = 3x-1$ **3**

d) Differentiate the following functions:

(i) $(2x + 7)^5$ **2**

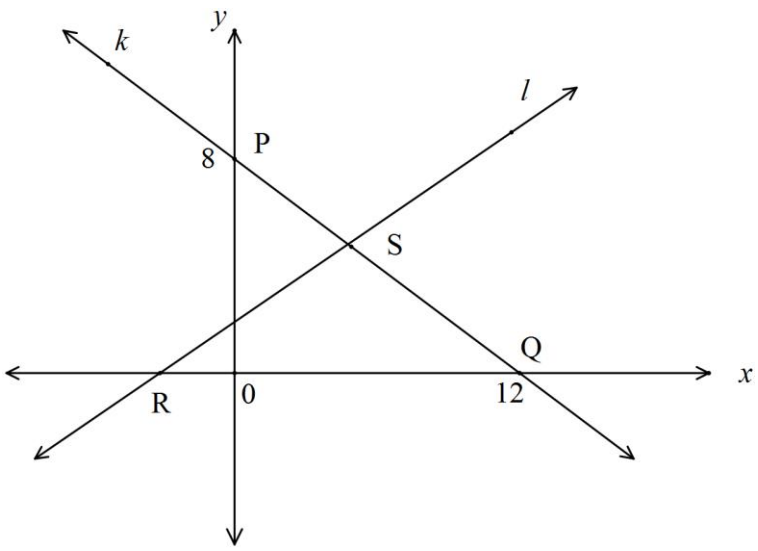
(ii) $\dfrac{1}{\sqrt[4]{x^3}}$ **2**

(iii) $\dfrac{x^2}{3x + 1}$ **2**

End of Question 11.

Question 12 (15 marks) Use a SEPARATE writing booklet

a)



NOT TO SCALE

In the diagram, the line l has equation $3x - 2y + 3 = 0$, and the line k passes through the points P (0,8) and Q (12,0).

Copy the diagram onto your answer sheet.

(i) Line l cuts the x-axis at R. Find the co-ordinates of R. **1**

(ii) Find the gradient of the line k. **1**

(iii) Show that the line l is perpendicular to the line k. **2**

(iv) Show that the equation of the line k is $2x + 3y - 24 = 0$. **2**

(v) Find the co-ordinates of S, the point of intersection of the lines l and k. **3**

(vi) Find the perpendicular distance of the point S from the line PR. **3**

(vii) Hence calculate the area of ΔSPR. **2**

(viii) Shade the region where the following inequalities hold simultaneously:

$2x + 3y - 24 \leq 0$, $3x - 2y + 3 \geq 0$ and $y \geq 0$ **1**

End of Question 12.

Question 13 (15 marks) Use a SEPARATE writing booklet

a) How many terms in the series 88 + 84 + 80 + 76 +……

must be added to give a sum of 1000? **3**

b) The function $y = x^3 - 3x^2 - 9x + 1$ is defined in the domain $-4 \leq x \leq 5$.

(i) Find the co-ordinates of any stationary points and determine their nature. **4**

(ii) Find the co-ordinates of any points of inflection. **2**

(iii) Draw a neat sketch of the curve for the given domain, showing clearly the *y*-intercept and the co-ordinates of the end points. **3**

(iv) Determine the minimum value of the function in the given domain. **1**

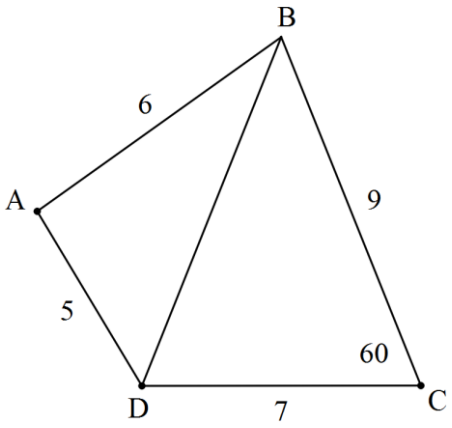
(v) Find the equation of the tangent to the curve at $x = -1$. **1**

c) In 2015, Council Rates increased by $12\frac{1}{2}\%$ p.a.. The new council rate for my property is \$2565. What was the old council rate for my property? Give your answer correct to the nearest dollar. **1**

End of Question 13.

Question 14 (15 marks) Use a SEPARATE writing booklet

a) ABCD is a quadrilateral. AD = 5m, AB = 6m, BC = 9m, CD = 7m and $\angle BCD = 60°$.



NOT TO SCALE

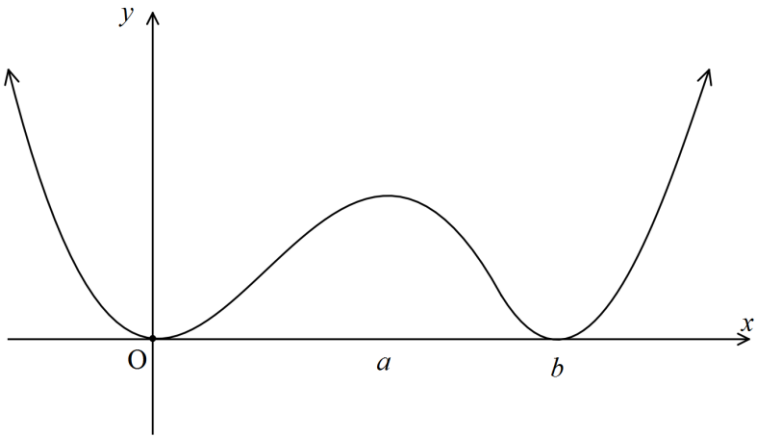
Calculate:

(i) BD. **2**

(ii) $\angle DAB$. **2**

(iii) the area of ABCD. **3**

b) The graph of function $y=f(x)$ is shown below.



Draw a sketch showing a possible graph of $y = f'(x)$. **2**

Question 14 continues on the next page.

c) Evaluate $\sum_{n=3}^{30} 5n - 7$ **2**

d) If $\cos\theta = \frac{7}{25}$ and $\tan\theta < 0$, find $\operatorname{cosec}\theta$ **2**

e) The graph of $y = f(x)$ passes through the point (2,2), and $f'(x) = 2x-5$. Find $f(x)$. **2**

End of Question 14.

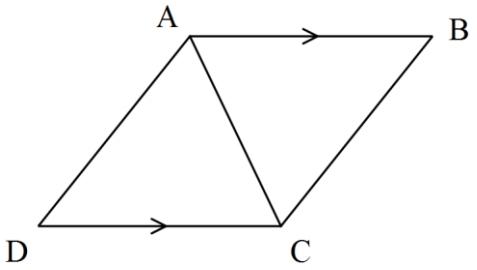
Question 15 (15 marks) Use a SEPARATE writing booklet

a) Blue metal is distributed by one truck operating from a base at the beginning of a road. The first load is deposited 400m from the base, the second load 500m from the base and each subsequent load at intervals of 100m. Find the number of kilometres travelled by the truck in depositing 17 loads, and then returning back to the base. **3**

b) Find all real numbers which satisfy the equation $x^4 = 72 - x^2$. **3**

c) Prove that $1 + \dfrac{\cot^2\theta}{1 + \operatorname{cosec}\theta} = \operatorname{cosec}\theta$ **3**

d) ABCD is a rhombus.



NOT TO SCALE.

Copy the diagram into your writing booklet.

(i) Show that diagonal AC bisects the angles it makes with the sides. **2**

(ii) Show that the diagonals AC and BD are perpendicular. **2**

e) Find the limiting sum of $\dfrac{3}{5} + \dfrac{6}{25} + \dfrac{12}{125} + \ldots\ldots\ldots$ **2**

End of Question 15.

Question 16 (15 marks) Use a SEPARATE writing booklet

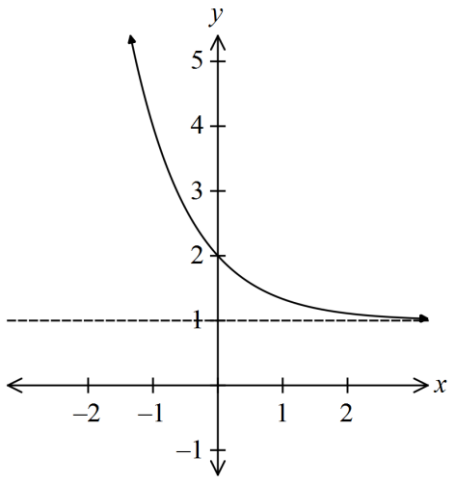
a) The sum of the radii of two circles is 100m. One of the circles has a radius of x cm.

(i) Show that the sum of the areas of the two circles is given by

$A = 2\pi\left(x^2 - 100x + 5000\right)$. **2**

(ii) Find the value of x for which A is least. **3**

b) The function below passes through the point (-1,4). Determine the equation of the function.



3

c) Isaac wants to buy a car. He decides to invest $950 at the beginning of each year into an account which pays 9% p.a., compounded monthly.

(i) What will his investment be worth after 1 year? **1**

(ii) How much will he have to spend on a car at the end of 5 years? **3**

(iii) Isaac decides to leave the investment in the bank for an extra 3 years, compounding at the same rate of interest. He wants to have $15 000 at the end of that time to buy a better car. How much should Isaac invest at the beginning of each of the extra 3 years to make this happen? **3**

End of Question 16.

End of Section II
End of Examination.

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Section I

1. $x^2(3(x+2)^2) + 2x(x+2)^3$

$= 3x^2(x+2)^2 + 2x(x+2)^3$

C

2. $\alpha+\beta = \frac{-b}{a} = 2 \qquad \alpha\beta = \frac{c}{a} = 4$

$\therefore \alpha\beta(\alpha+\beta) = 4 \times 2 = 8$

C

3. $\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} \frac{(x-5)(x+5)}{x-5}$

$= \lim_{x\to 5} (x+5) = 10$

D

4. $2000 = \frac{1}{3}\pi r^2 \times 12$

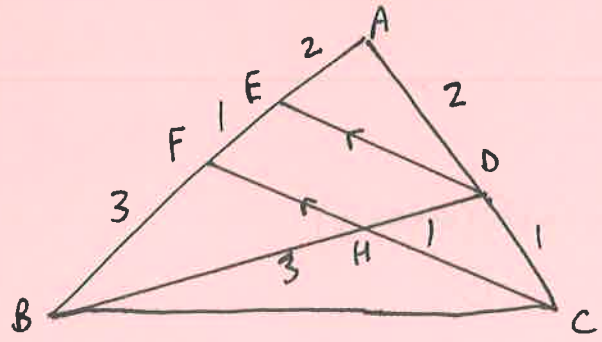
$r^2 = \frac{2000}{4\pi}$

$r = \sqrt{\frac{500}{\pi}} = 12.615$

$= 12.6$

C

5.

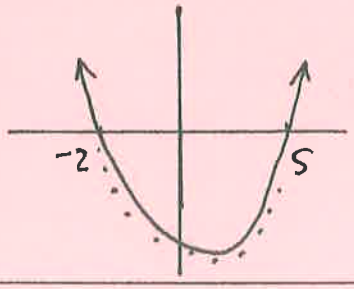


BH : HD

3 : 1

C

6.



$-2 < x < 5$

B

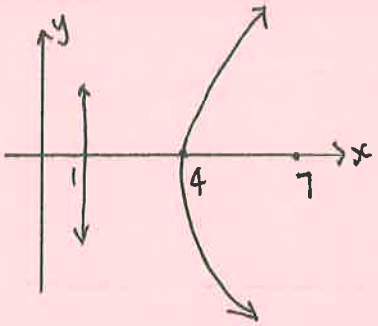
7. $f(x) = 5x - x^{-1}$

$f'(x) = 5 + x^{-2}$

$f'\left(\frac{1}{2}\right) = 5 + \frac{1}{\left(\frac{1}{2}\right)^2}$

$= 5 + 4 = 9$

D

8. 

$y^2 = 4(3)(x-4)$

$y^2 = 12(x-4)$

B

9. $\left[\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{1} = \left(\frac{1}{3} + \frac{1}{2} + 2\right) - \left(-\frac{1}{3} + \frac{1}{2} - 2\right)$

$= 2\frac{5}{6} + \frac{11}{6}$

$= 4\frac{2}{3}$

C

10. $x^2 + 6x + 9 = x^2 + 2x + 1 + x^2 - 2x + 1$

$x^2 + 6x + 9 = 2x^2 + 2$

$0 = x^2 - 6x - 7$

$0 = (x-7)(x+1)$

$x = 7 \quad x \neq -1$

C

Section II

QUESTION 11.

(a) $m(a+b) + p(a+b) + y(a+b) = (a+b)(m+p+y)$ ① ①

b) $\frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4\sqrt{3}+4}{3-1} = \frac{4(\sqrt{3}+1)}{2}$ ①

$= 2\sqrt{3} + 2$

$= a\sqrt{3} + b \quad \therefore a = 2, b = 2$ ①

c) (i) $3^{2x+1} = 3^{-5}$ ①

$2x + 1 = -5$

$2x = -6$

$x = -3$ ①

Question 11 – continued...

c) (ii) $|x+5| = 3x-1$

$x+5 = 3x-1$ | $x+5 = -(3x-1)$

$6 = 2x$ | $x+5 = -3x+1$

$x = 3$ ① | $4x = -4$

$|3+5| = 3(3)-1$ | $x = -1$ ①

$8 = 8$ ✓ | No solution

$\therefore \quad x = 3$ ①

d) (i) $5(2x+7)^4 \times 2 = 10(2x+7)^4$

① ①

(ii) $\frac{d}{dx} x^{-\frac{3}{4}} = -\frac{3}{4}x^{-\frac{7}{4}}$

① ①

(iii) $\frac{(3x+1).2x - x^2.3}{(3x+1)^2}$ ① $= \frac{6x^2+2x-3x^2}{(3x+1)^2}$

$= \frac{3x^2+2x}{(3x+1)^2}$ ①

QUESTION 12

a) (i) $3x - 2(0) + 3 = 0$

$3x = -3$

$x = -1$ $\quad R = (-1, 0)$ ①

(ii) $M_k = -\frac{8}{12} = -\frac{2}{3}$ ①

(iii) $3x - 2y + 3 = 0$

$2y = 3x + 3$

$y = \frac{3}{2}x + \frac{3}{2}$ $\quad \therefore M_l = \frac{3}{2}$ ①

$M_k \times M_l = -\frac{2}{3} \times \frac{3}{2} = -1$ ① $\quad \therefore$ perpendicular.

(iv) $m_K = -\frac{2}{3}$ $(0,8)$

$y - 8 = -\frac{2}{3}(x-0)$ (1)

$3y - 24 = -2x$

$2x + 3y - 24 = 0$ (1)

(v) $2x + 3y - 24 = 0$ — k x3

$3x - 2y + 3 = 0$ — l x2

$6x + 9y - 72 = 0$ —(1)

$6x - 4y + 6 = 0$ —(2) (1)

(1) − (2)

$13y - 78 = 0$

$13y = 78$ (1)

$y = 6$

$2x + 3(6) - 24 = 0$ $S = (3,6)$ (1)

$x = 3$

(vi) $S = (3,6)$ PR: $m = 8$ $b = 8$: $y = 8x + 8$

$8x - y + 8 = 0$ (1)

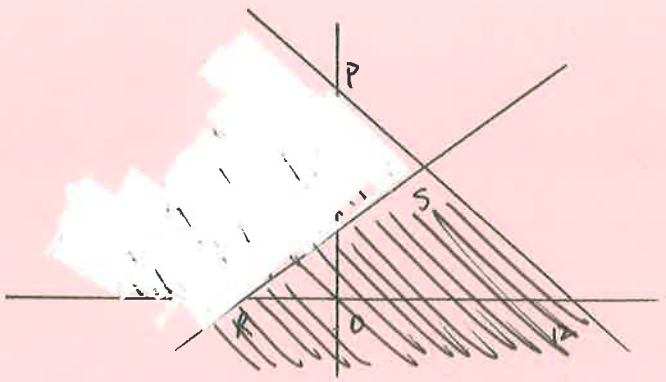
(1)

$$d_h = \frac{|8(3) - 1(6) + 8|}{\sqrt{8^2 + (-1)^2}} = \frac{26}{\sqrt{65}}$$ (1)

(vii) $d_{PR} = \sqrt{8^2 + 1^2} = \sqrt{65}$ (1)

$A = \frac{1}{2} \times \sqrt{65} \times \frac{26}{\sqrt{65}} = 13$ (1)

(viii)



(1)

QUESTION 13

a) $S_n = \frac{n}{2}\left[2(88) - 4(n-1)\right]$ ① $a = 88,\ d = -4$ ①

$1000 = \frac{n}{2}\left(176 - 4n + 4\right)$

$2000 = 180n - 4n^2$

$0 = 4n^2 - 180n + 2000$

$0 = n^2 - 45n + 500$

$0 = (n-20)(n-25)$ ①

$n = 20,\ n = 25$

$\therefore$ The first 20 terms add to 1000.

b) (i) $y = x^3 - 3x^2 - 9x + 1$

$y' = 3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ $\quad$ $x = -1$

$y = -26$ ① $\quad$ $y = 6$ ①

$y'' = 6x - 6$

at $x = 3$ $y'' = 12$ $\quad\therefore$ $(3,-26)$ is min. st. point ①

at $x = -1$ $y'' = -12$ $\quad\therefore$ $(-1,6)$ is max. st. point ①

(ii) $y'' = 6x - 6 = 0$

$6x = 6$

$x = 1,\ y = -10$

check

x	0	1	2
y''	-6	0	6

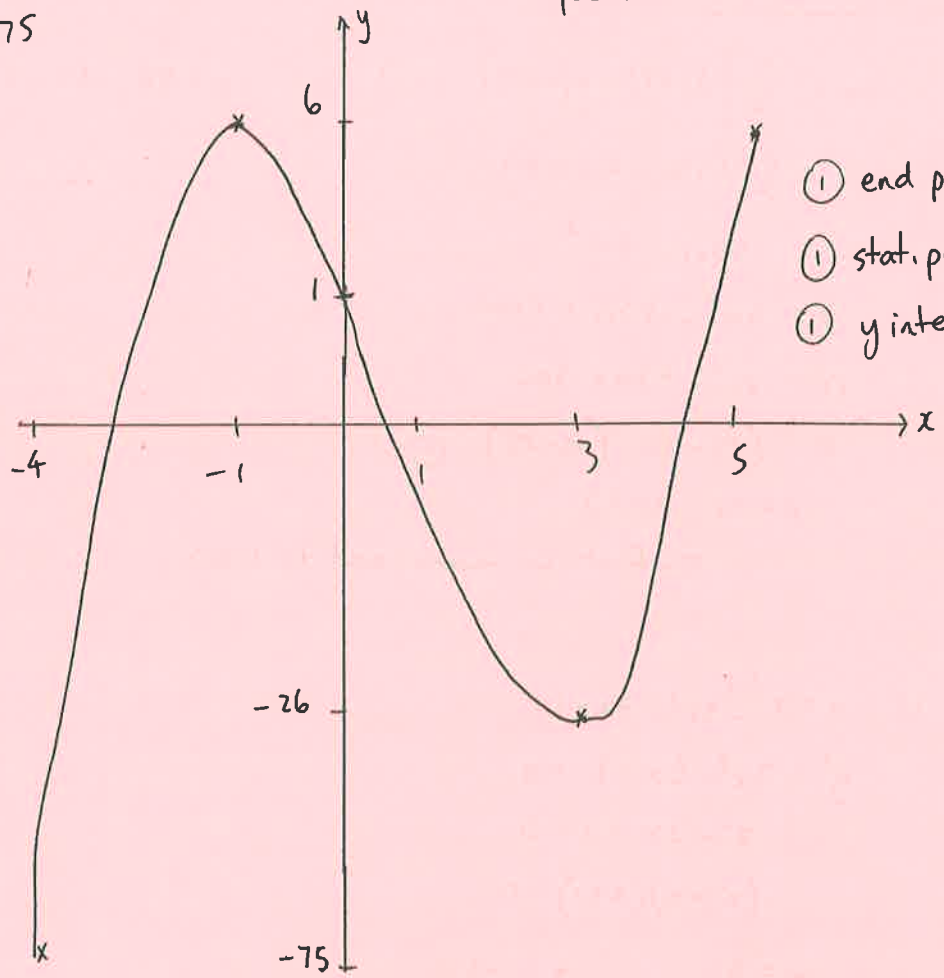
①

concavity changes $\quad\therefore$ $(1,-10)$ is point of inflexion. ①

(iii) $x = -4$, $y = -75$

$x = 5$, $y = 6$

$x = 0$, $y = 1$



① end points
① stat. points
① y intercept

(iv) -75 ①

(v) at $x = -1$, $m = 0$ ∴ horizontal line passing through $(-1, 6)$

∴ equatn of tangent is $y = 6$. ①

c) $\$2565 = 112.5\%$

$\frac{}{112.5} \qquad \frac{}{112.5}$

$22.8 = 1\%$

$\$2280 = 100\%$

∴ old rate was $\$2280$. ①

QUESTION 14

a) (i) $BD^2 = 9^2 + 7^2 - 2(9)(7)\cos 60$ ①

$= 81 + 49 - 126\cos 60$

$BD = \sqrt{67}$ ①

(ii) $\cos\theta = \dfrac{6^2 + 5^2 - (\sqrt{67})^2}{2 \times 6 \times 5} = \dfrac{-6}{60}$ ①

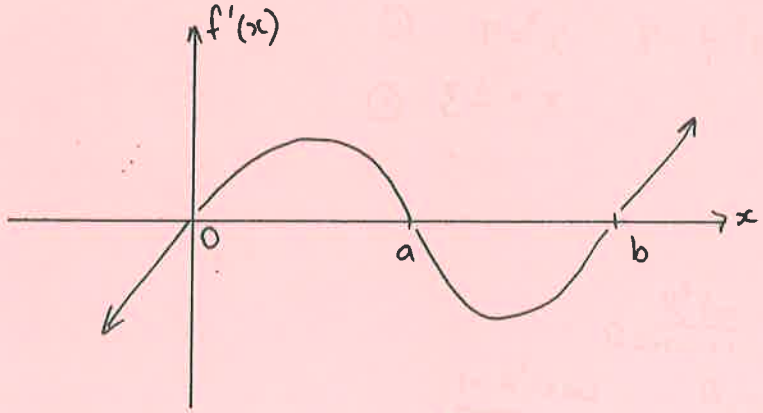
$\theta = \cos^{-1}\left(\dfrac{-6}{60}\right) = 95.739 = 96^\circ$ (nearest degree) ①

(iii) $A = \left(\frac{1}{2} \times 6 \times 5 \times \sin 95.739\right)$ ① $+ \left(\frac{1}{2} \times 9 \times 7 \times \sin 60\right)$ ①

$= 42.197628$

$= 42.2$ (1 dec. pl.) ①

b)

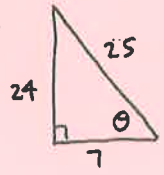


shape ①

roots ①

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{\frac{3}{5}}{1-\frac{2}{5}}$ ① $= \frac{3}{5} \div \frac{3}{5} = 1$ ①

d) $\cos\theta = \frac{7}{25}$ ①



$\cos\theta > 0$, $\tan\theta < 0$ ∴ fourth quadrant

$\therefore \operatorname{cosec}\theta = -\dfrac{25}{24}$ ①

e) $f(x) = x^2 - 5x + C$ ①

(2,2) $2 = 2^2 - 5(2) + C$

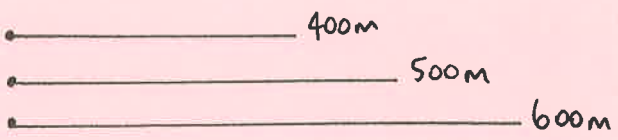
$2 = 4 - 10 + C$

$C = 8$

$\therefore f(x) = x^2 - 5x + 8$ ①

QUESTION 15

a)



800m trip
1000 m trip
1200 m trip

$800 + 1000 + 1200 + \ldots\ldots$ ①

$a = 800 \quad d = 200 \quad n = 17$

$T_{17} = 800 + (17-1)200$ ①

$= 4000\,m$ ①

$= 4km$

b) $x^4 - x^2 - 72 = 0$ Let $M = x^2$

$M^2 - M - 72 = 0$ ①

$(M+8)(M-9) = 0$

$M = -8 \quad M = 9$

$x^2 \neq -8 \quad x^2 = 9$ ①

$x = \pm 3$ ①

c) $(\cot^2\theta = \operatorname{cosec}^2\theta - 1)$

$$LHS = 1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta}$$

$$= \frac{1+\operatorname{cosec}\theta}{1+\operatorname{cosec}\theta} + \frac{\operatorname{cosec}^2\theta - 1}{1+\operatorname{cosec}\theta}$$

$$= \frac{1+\operatorname{cosec}\theta + \operatorname{cosec}^2\theta - 1}{1+\operatorname{cosec}\theta}$$

$$= \frac{\operatorname{cosec}\theta + \operatorname{cosec}^2\theta}{1+\operatorname{cosec}\theta}$$

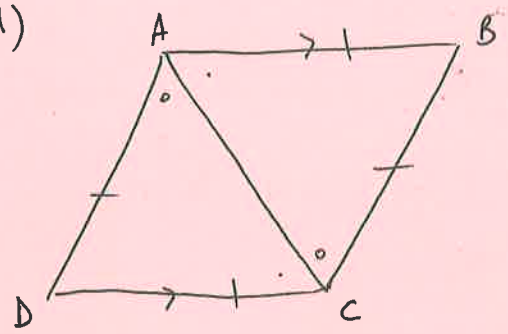
$$= \frac{\operatorname{cosec}\theta(1+\operatorname{cosec}\theta)}{1+\operatorname{cosec}\theta}$$

$$= \operatorname{cosec}\theta$$

$$= RHS.$$

Question 15 continued . . . .

(d)



$\angle BAC = \angle DCA$ (alternate $\angle$'s on parallel lines)

$\angle BCA = \angle DAC$ (" " " ") (1)

$\angle BAC = \angle BCA$ (base angles of isosc. Δ) (1)

$\therefore \angle BAC = \angle BCA = \angle DCA = \angle DAC$

$\therefore$ AC bisects the angles through which it passes. (1)

e) $\sum_{n=3}^{30} 5n-7 = 8+13+18+\ldots\ldots+143$

$a=8 \quad d=5 \quad n=28$ (1)

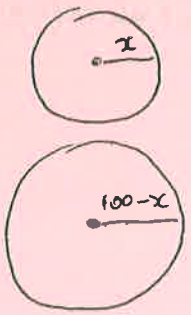
$S_{28} = \frac{28}{2}\left(2(8)+(28-1)(5)\right)$ (1)

$= 14\left(16+27\times5\right)$

$= 2114$

QUESTION 16

a) (i)



$A = \pi x^2 + \pi(100-x)^2$

$= \pi x^2 + \pi(10000-200x+x^2)$ (1)

$= \pi(x^2+10000+200x+x^2)$

$= \pi(2x^2+200x+10000)$

$= 2\pi(x^2-100x+5000)$ (1)

(ii) $A' = 4\pi x - 200\pi = 0$

$4\pi x = 200\pi$ (1)

$x = 50$

$A'' = 4\pi > 0 \therefore$ minimum point. $\therefore x = 50$m (1)

(1)

b) $y = a^{-x} + b$ ①

$(0,2)$ $2 = a^0 + b = 1 + b$

$b = 1$ ①

$(-1,4)$ $y = a^{-x} + 1$

$4 = a^{-(-1)} + 1$

$4 = a + 1$

$a = 3$ ① $\therefore y = 3^{-x} + 1$

c) 9% p.a. = 0.75% p.month = 0.0075

(i) $950(1.0075)^{12} = \$1039.12$ ①

(ii) $950(1.0075)^{60} + 950(1.0075)^{48} + \ldots + 950(1.0075)^{12}$ ①

$= 950\left[1.0075^{12} + 1.0075^{24} + 1.0075^{36} + 1.0075^{48} + 1.0075^{60}\right]$ ①

$= 950\left[\frac{1.0075^{12}(1.0075^{60} - 1)}{1.0075^{12} - 1}\right]$

$= \$6266.15$ ①

(iii) $6266.15(1.0075)^{36} + M\left(1.0075^{36} + 1.0075^{24} + 1.0075^{12}\right) = 15000$

$M\left(1.0075^{36} + 1.0075^{24} + 1.0075^{12}\right) = 15000 - 6266.15(1.0075)^{36}$

$M = \frac{15000 - 6266.15(1.0075)^{36}}{\left(1.0075^{36} + 1.0075^{24} + 1.0075^{12}\right)}$

$= \frac{6799.831809}{3.598865}$

$= \$1889.44$

He should invest \$1889.44 for each of the next 3 years.

END OF PAPER.