NSW GOVERNMENT

NSW Education Standards Authority

2021 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 12 Writing Booklet attached

Total marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–12)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Given that $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, what is $\overrightarrow{PQ}$?

A. $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$

B. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

C. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

D. $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

2 Which of the following integrals is equivalent to $\int \sin^2 3x \, dx$?

A. $\int \frac{1 + \cos 6x}{2} \, dx$

B. $\int \frac{1 - \cos 6x}{2} \, dx$

C. $\int \frac{1 + \sin 6x}{2} \, dx$

D. $\int \frac{1 - \sin 6x}{2} \, dx$

3 What is the remainder when $P(x) = -x^3 - 2x^2 - 3x + 8$ is divided by $x + 2$?

A. -14

B. -2

C. 2

D. 14

4 Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$.

Which of the following equations best represents this relationship between x and y?

A. $y^2 = x^2 + c$

B. $y^2 = \dfrac{x^2}{2} + c$

C. $y = x \ln|y| + c$

D. $y = \dfrac{x^2}{2} \ln|y| + c$

5 For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is known that

$$\overrightarrow{OA} \cdot \overrightarrow{OB} < 0.$$

Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or, $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|$ is negative.

D. The points O, A and B are collinear.

6 The random variable X represents the number of successes in 10 independent Bernoulli trials. The probability of success is $p = 0.9$ in each trial.

Let $r = P(X \geq 1)$.

Which of the following describes the value of r?

A. $r > 0.9$

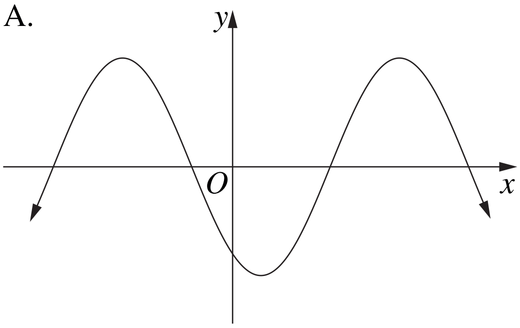
B. $r = 0.9$

C. $0.1 < r < 0.9$

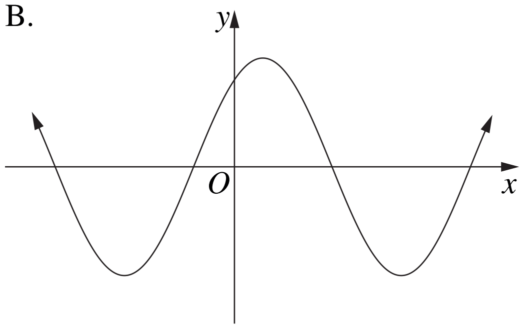
D. $r \leq 0.1$

7 Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive?

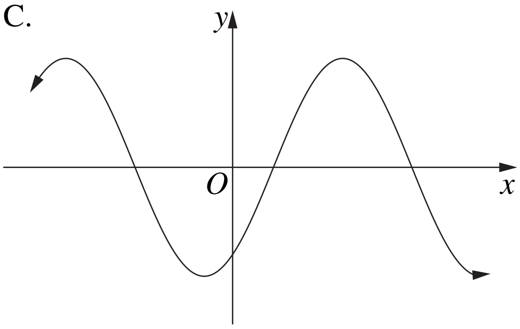
A.



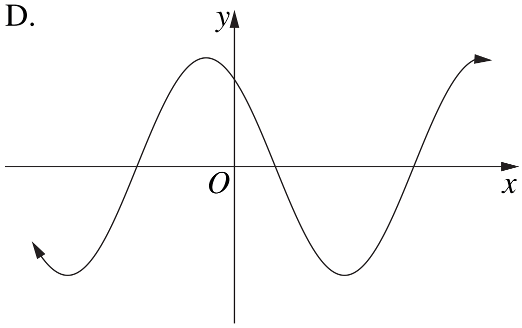
B.



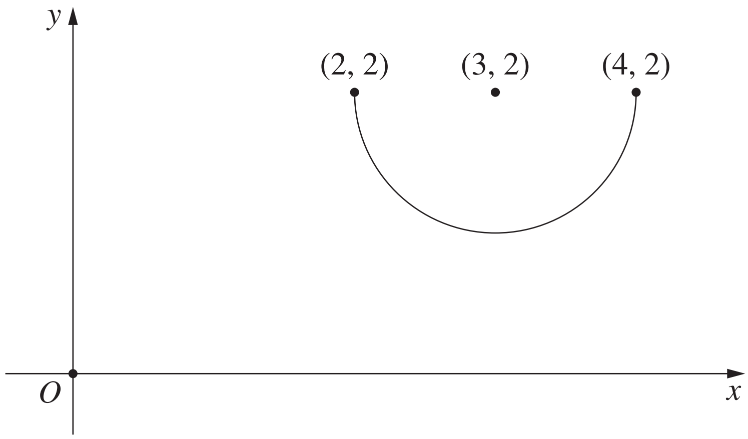
C.



D.



8 The diagram shows a semicircle.



Which pair of parametric equations represents the semicircle shown?

A. $\begin{cases} x = 3 + \sin t \\ y = 2 + \cos t \end{cases}$ for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$

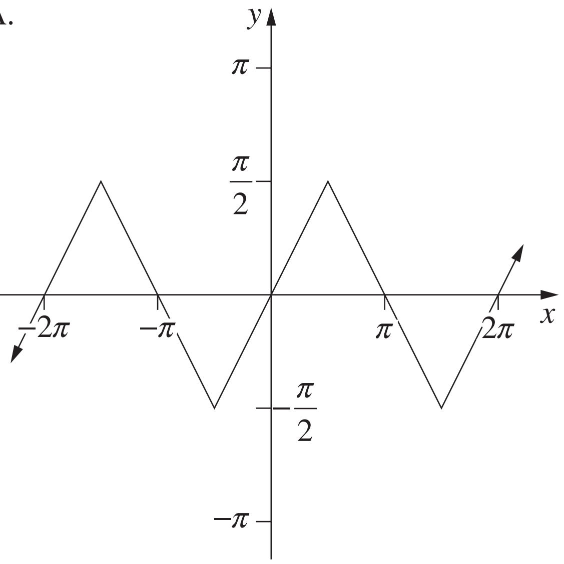
B. $\begin{cases} x = 3 + \cos t \\ y = 2 + \sin t \end{cases}$ for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$

C. $\begin{cases} x = 3 - \sin t \\ y = 2 - \cos t \end{cases}$ for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$

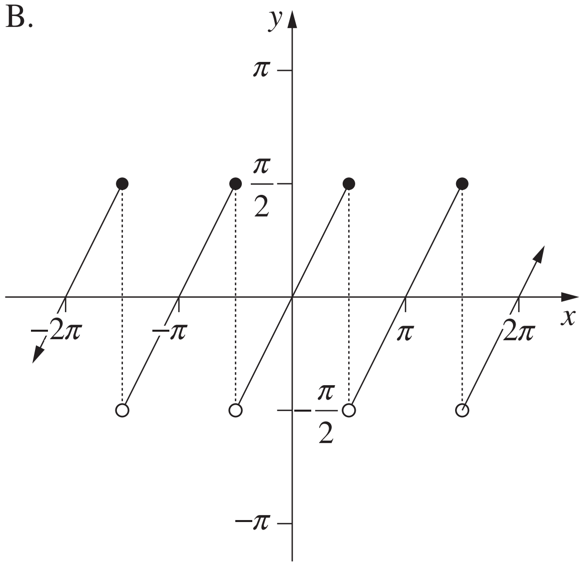
D. $\begin{cases} x = 3 - \cos t \\ y = 2 - \sin t \end{cases}$ for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$

9 Which graph represents the function $y = \sin^{-1}(\sin x)$?

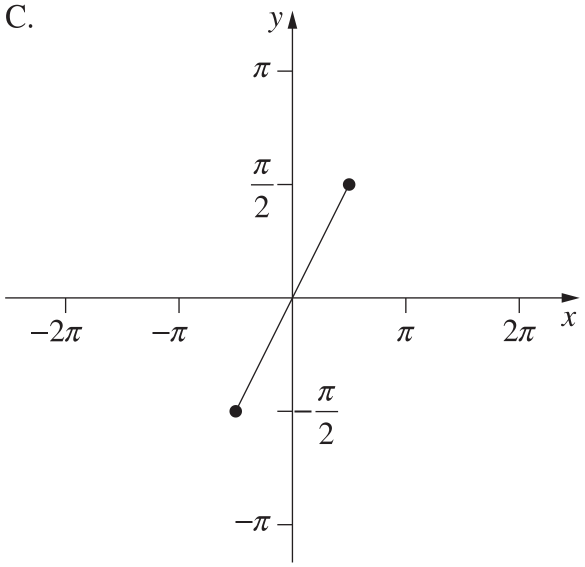
A.



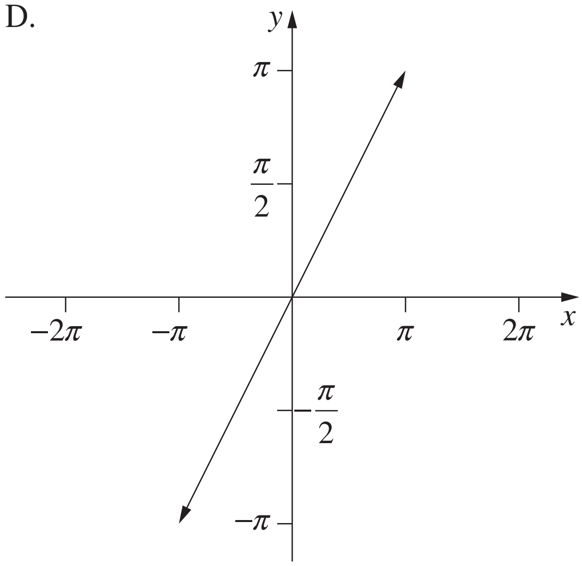
B.



C.



D.



10 The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only.

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A. 236

B. 237

C. 238

D. 239

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) Find $\left(\underset{\sim}{i} + 6\underset{\sim}{j}\right) + \left(2\underset{\sim}{i} - 7\underset{\sim}{j}\right)$. **1**

(b) Expand and simplify $(2a - b)^4$. **2**

(c) Use the substitution $u = x + 1$ to find $\displaystyle\int x\sqrt{x+1}\,dx$. **3**

(d) A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women. **1**

In how many different ways can the committee be formed?

(e) A spherical bubble is moving up through a liquid. As it rises, the bubble gets bigger and its radius increases at the rate of 0.2 mm/s. **2**

At what rate is the volume of the bubble increasing when its radius reaches 0.6 mm? Express your answer in mm^3/s rounded to one decimal place.

(f) Evaluate $\displaystyle\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. **2**

(g) By factorising, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$. **3**

(h) The roots of $x^4 - 3x + 6 = 0$ are α, β, γ and δ. **2**

What is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$?

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) The direction field for a differential equation is given on page 1 of the Question 12 Writing Booklet. **1**

The graph of a particular solution to the differential equation passes through the point P.

On the diagram provided in the writing booklet, sketch the graph of this particular solution.

(b) A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After t minutes, the temperature of the water, in degrees Celsius, is T.

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \qquad \text{(Do NOT prove this.)}$$

where k is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C. **3**

By solving the differential equation, find the value of t when the temperature of the water reaches 20°C. Give your answer to the nearest minute.

(ii) Sketch the graph of T as a function of t. **1**

(c) Use mathematical induction to prove that **3**

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

for all integers $n \geq 1$.

(d) A function is defined by $f(x) = 4 - \left(1 - \frac{x}{2}\right)^2$ for x in the domain $(-\infty, 2]$.

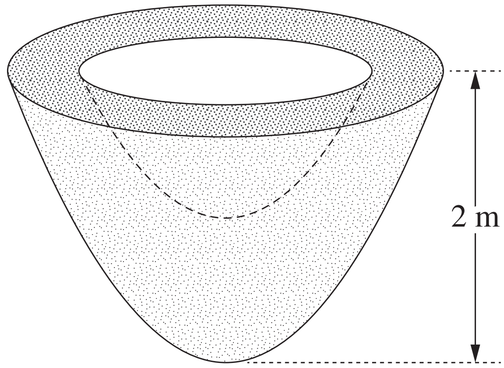
(i) Sketch the graph of $y = f(x)$ showing the x- and y-intercepts. **2**

(ii) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(iii) Sketch the graph of $y = f^{-1}(x)$. **1**

Question 13 (14 marks) Use the Question 13 Writing Booklet

(a) A 2-metre-high sculpture is to be made out of concrete. The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis. **3**



Find the volume of concrete needed to make the sculpture.

(b) When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is **4**

$$\underset{\sim}{r}(t) = \left(Vt\cos\theta\right)\underset{\sim}{i} + \left(-5t^2 + Vt\sin\theta + h\right)\underset{\sim}{j}.$$ (Do NOT prove this.)

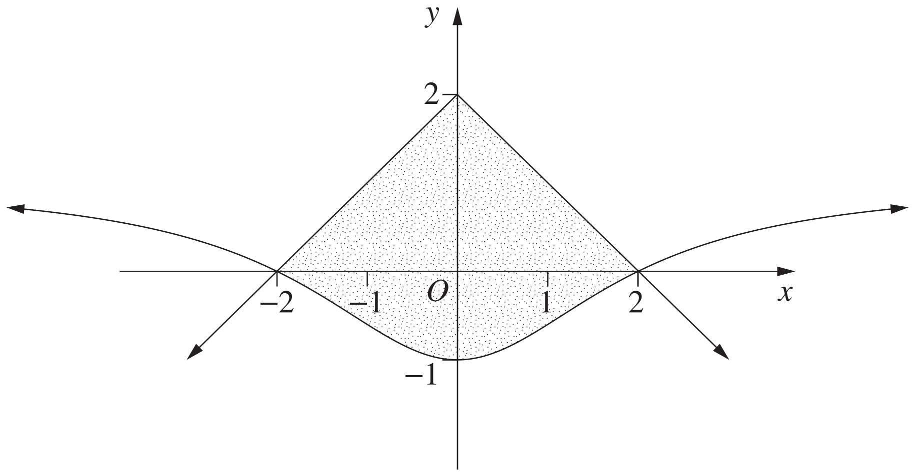
A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

Question 13 continues on page 10

Question 13 (continued)

(c) The region enclosed by $y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is shaded in the diagram. **3**



Find the exact value of the area of the shaded region.

(d) (i) The numbers A, B and C are related by the equations $A = B - d$ and $C = B + d$, where d is a constant. **2**

Show that $\dfrac{\sin A + \sin C}{\cos A + \cos C} = \tan B$.

(ii) Hence, or otherwise, solve $\dfrac{\sin\frac{5\theta}{7} + \sin\frac{6\theta}{7}}{\cos\frac{5\theta}{7} + \cos\frac{6\theta}{7}} = \sqrt{3}$, for $0 \le \theta \le 2\pi$. **2**

End of Question 13

Question 14 (16 marks) Use the Question 14 Writing Booklet

(a) A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h. **3**

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination?

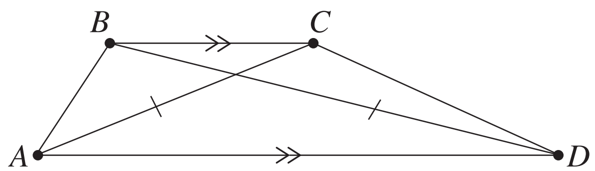
(b) In a certain country, the population of deer was estimated in 1980 to be 150 000. The population growth is given by the logistic equation $\dfrac{dP}{dt} = 0.1P\left(\dfrac{C-P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity. **4**

In the year 2000, the population of deer was estimated to be 600 000.

Use the fact that $\dfrac{C}{P(C-P)} = \dfrac{1}{P} + \dfrac{1}{C-P}$ to show that the carrying capacity is approximately 1 130 000.

(c) (i) For vector $\underset{\sim}{v}$, show that $\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$. **1**

(ii) In the trapezium $ABCD$, BC is parallel to AD and $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$. **3**



NOT TO SCALE

Let $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$.

Using part (i), or otherwise, show $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1-k)\,|\underset{\sim}{b}|^2 = 0$.

Question 14 continues on page 12

(d) At a certain factory, the proportion of faulty items produced by a machine is $p = \dfrac{3}{500}$, which is considered to be acceptable. To confirm that the machine is working to this standard, a sample of size n is taken and the sample proportion $\hat{p}$ is calculated. **3**

It is assumed that $\hat{p}$ is approximately normally distributed with $\mu = p$ and $\sigma^2 = \dfrac{p(1-p)}{n}$.

Production by this machine will be shut down if $\hat{p} \geq \dfrac{4}{500}$.

The sample size is to be chosen so that the chance of shutting down the machine unnecessarily is less than 2.5%.

Find the approximate sample size required, giving your answer to the nearest thousand.

(e) The polynomial $g(x) = x^3 + 4x - 2$ passes through the point $(1, 3)$. **2**

Find the gradient of the tangent to $f(x) = xg^{-1}(x)$ at the point where $x = 3$.

End of paper

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NSW GOVERNMENT

Centre Number

Student Number

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Writing Booklet

Question 12

Instructions

- Use this Writing Booklet to answer Question 12.
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).
- Write your Centre Number and Student Number at the top of this page.
- Write using black pen.
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

12

of

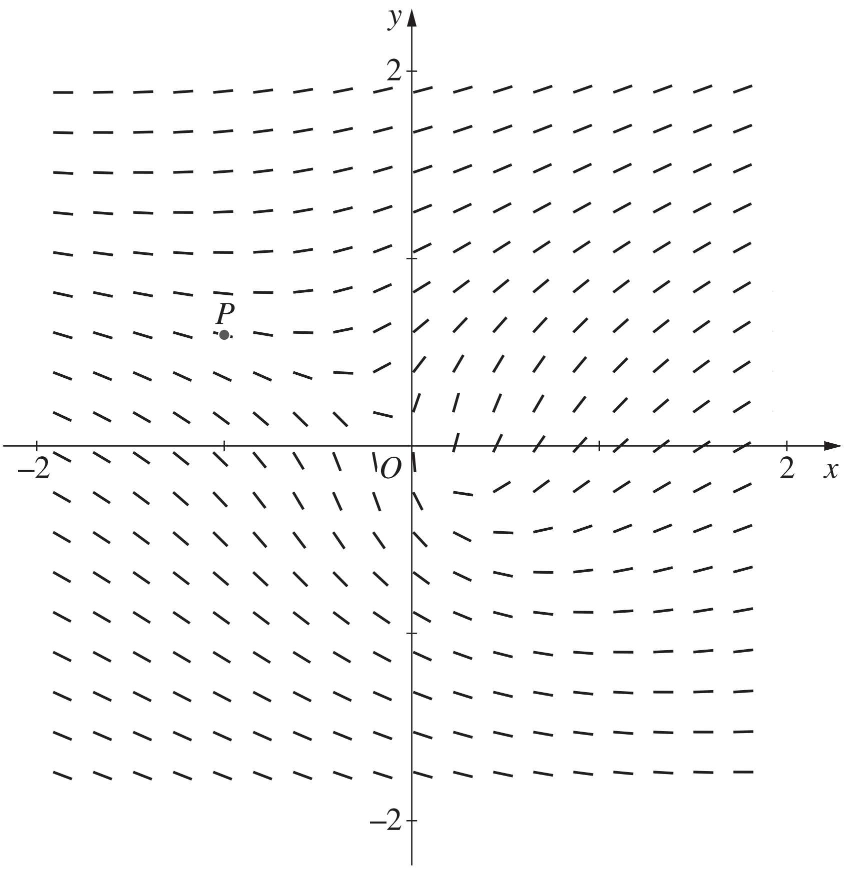
this booklet

number of booklets for this question

Office Use Only – Do NOT write anything, or make any marks below this line.

Start here for Question Number: **12**

(a) Sketch the graph of the particular solution that passes through the point P.



Additional writing space on back page.

Tick this box if you have continued this answer in another writing booklet.

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2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

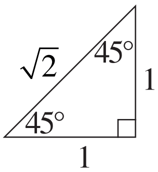
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

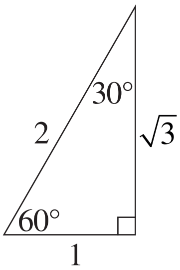


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

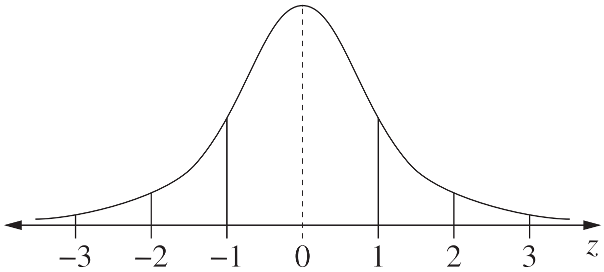
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	D
4	A
5	B
6	A
7	D
8	C
9	A
10	C

Section II

Question 11 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\left(\underset{\sim}{i} + 6\underset{\sim}{j}\right) + \left(2\underset{\sim}{i} - 7\underset{\sim}{j}\right) = 3\underset{\sim}{i} - \underset{\sim}{j}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Expands using the binomial expansion, or equivalent merit	1

Sample answer:

$$\begin{aligned}(2a - b)^4 &= (2a)^4 - 4(2a)^3 b + 6(2a)^2 b^2 - 4(2a)b^3 + b^4 \\ &= 16a^4 - 32a^3 b + 24a^2 b^2 - 8ab^3 + b^4\end{aligned}$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive in terms of u	2
• Obtains the integrand in terms of u, or equivalent merit	1

Sample answer:

$$\int x\sqrt{x+1}\ dx \qquad\qquad \begin{aligned} u &= x+1 \qquad \therefore x = u-1 \\ du &= dx \end{aligned}$$

$$= \int (u-1)\sqrt{u}\ du$$

$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$^{10}C_5 \times {}^{8}C_3 = 14\ 112$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Uses the chain rule $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$ OR • Obtains $\frac{dv}{dr}$ from the volume of a sphere	1

Sample answer:

$$\frac{dr}{dr} = 0.2 \text{ mm/s} \quad \text{and} \quad \frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt}$$

$$v = \frac{4}{3}\pi r^3 \qquad \therefore \frac{dv}{dr} = 4\pi^2 \text{ and } \frac{dv}{dt} = 4\pi^2 . \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi \times (0.6)^2 \times (0.2) \simeq 0.9 \text{ mm}^3\text{/s} \qquad \text{(1 decimal place)}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Integrates to obtain an arcsin function, or equivalent merit	1

Sample answer:

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\, dx = \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^{\sqrt{3}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	3
• Finds all three possible values of $\sin x$ OR • Finds at least two possible values of x, or equivalent merit	2
• Notes that $\sin x = -1$ is a solution of the polynomial, or equivalent merit OR • Takes a factor of $\sin^2 x$ OR $2\sin^2 x$ out of first two terms	1

Sample answer:

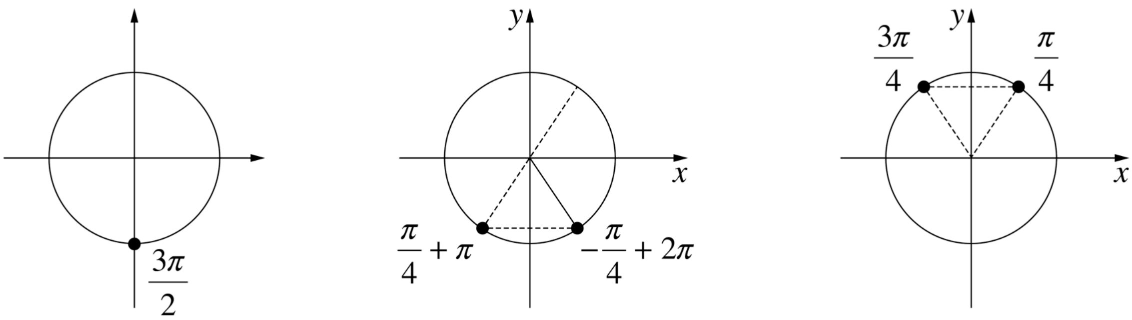
$2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$

$2\sin^2 x(\sin x + 1) - (\sin x + 1) = 0$

$(\sin x + 1)(2\sin^2 x - 1) = 0$

$\sin x = -1 \qquad \text{or} \qquad \sin^2 x = \dfrac{1}{2}$

$\sin x = -1 \qquad \text{or} \qquad \sin x = -\dfrac{1}{\sqrt{2}} \qquad \text{or} \qquad \sin x = \dfrac{1}{\sqrt{2}}$



The solutions in $[0, 2\pi]$ are: $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{3\pi}{2}, \dfrac{7\pi}{4}$.

Question 11 (h)

Criteria	Marks
• Provides correct solution	2
• Writes $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ with a common denominator OR • Applies one formula relating roots and coefficients	1

Sample answer:

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{\beta\gamma\delta+\alpha\gamma\delta+\alpha\beta\delta+\alpha\beta\gamma}{\alpha\beta\gamma\delta}$$

$x^4-3x+6=0 \qquad \therefore\ a=1,\ b=0,\ c=0,\ d=-3,\ e=6$

$$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\delta\gamma=\frac{-d}{a} \qquad \text{(coeff of } x\text{)}$$
$$=3$$

and $\alpha\beta\gamma\delta=\frac{e}{a}$ (constant term)

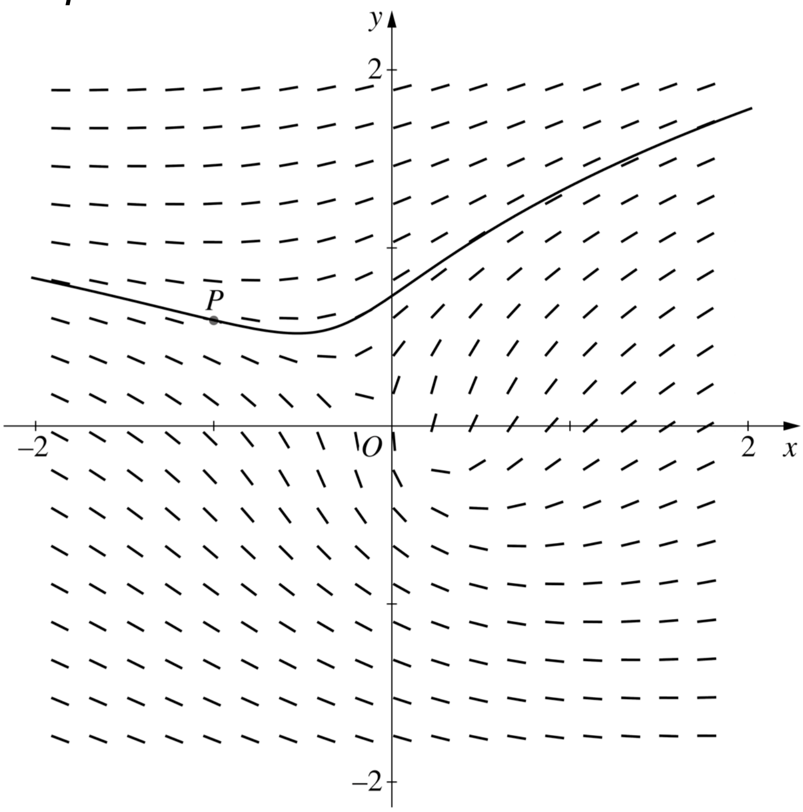
$$=6$$

Therefore $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{3}{6}=\frac{1}{2}$.

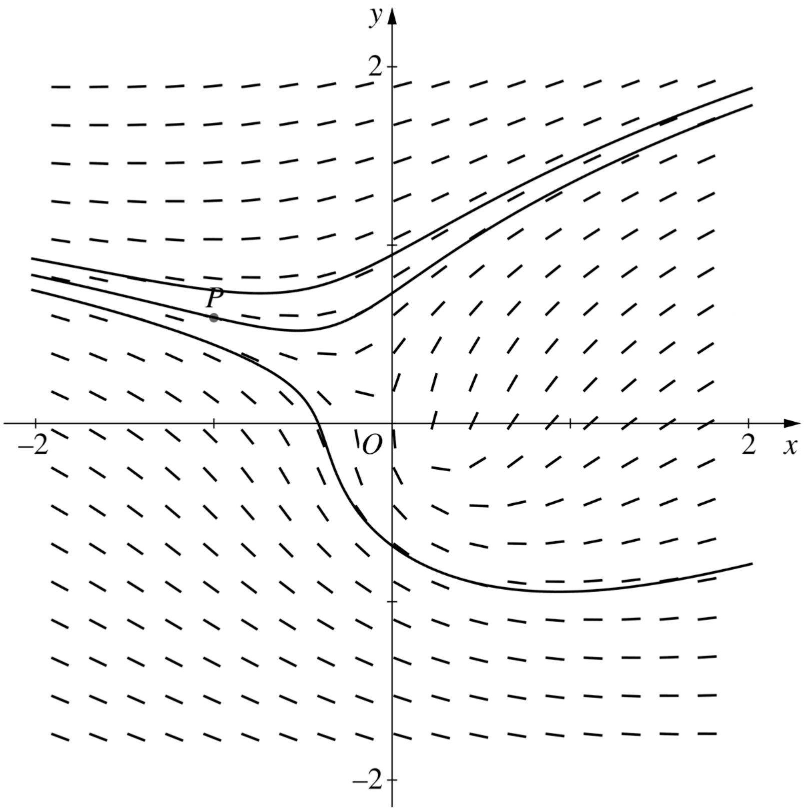
Question 12 (a)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Answers could include:



Note: An acceptable solution curve does not cross any tangent line in the direction field.

Question 12 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Finds the value of k, or equivalent merit	2
• Obtains a solution to the differential equation, that is, $T = 25 + Ae^{kt}$, or equivalent merit OR • Finds the value of A	1

Sample answer:

$$\int \frac{dT}{T-25} = \int k\ dt$$

$$\therefore kt = \ln(T-25) + c$$

$$\therefore T - 25 = Ae^{kt}$$

when $t = 0,\ T = 5$

$$\therefore -20 = A$$

$$\therefore T = 25 - 20e^{kt}$$

when $t = 8,\ T = 10$

$$\therefore 10 = 25 - 20e^{8k}$$

$$-15 = -20e^{8k}$$

$$e^{8k} = \frac{3}{4}$$

$$8k = \ln\left(\frac{3}{4}\right)$$

$$k = \frac{1}{8}\ln\left(\frac{3}{4}\right)$$

when $T = 20$

$$20 = 25 - 20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$-5 = -20e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{4} = e^{\frac{1}{8}\ln\left(\frac{3}{4}\right)t}$$

$$\frac{1}{8}\ln\left(\frac{3}{4}\right)t = \ln\left(\frac{1}{4}\right)$$

$$\therefore t = \frac{8\ln\left(\frac{1}{4}\right)}{\ln\left(\frac{3}{4}\right)}$$

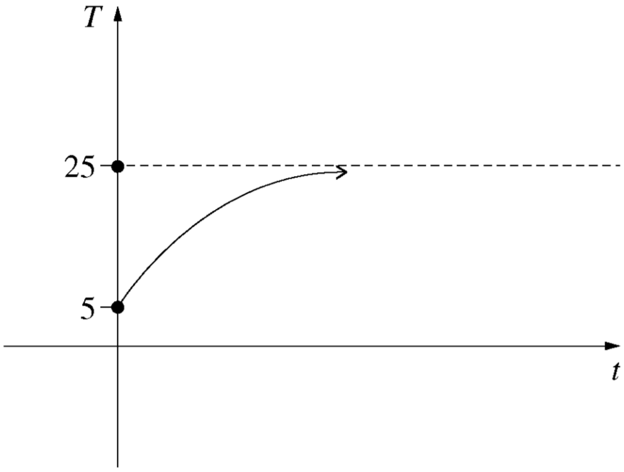
$$= 38.55\ldots$$

$$\simeq 39 \text{ minutes.}$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Proves that $p(k)$ *true* $\Rightarrow p(k+1)$ is true, or equivalent merit	2
• Verifies the initial case, or equivalent merit	1

Sample answer:

When $n = 1$

$$\text{LHS} = \frac{1}{1(1+1)(1+2)} = \frac{1}{1\times 2\times 3} = \frac{1}{6}$$

$$\text{RHS} = \frac{1}{4} - \frac{1}{2(1+1)(1+2)} = \frac{1}{4} - \frac{1}{2\times 2\times 3} = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

$\therefore$ statement is true when $n = 1$.

Assume statement is true when $n = k$, some integer $k \geq 1$, that is

$$\frac{1}{1\times 2\times 3} + \cdots + \frac{1}{k(k+1)(k+2)} = \frac{1}{4} - \frac{1}{2(k+1)(k+2)}$$

Consider $n = k + 1$:

$$\frac{1}{1\times 2\times 3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)}$$

$$= \frac{1}{4} - \frac{1}{2\big((k+1)+1\big)\big((k+1)+2\big)} = \frac{1}{4} - \frac{1}{2(k+2)(k+3)}$$

$$\begin{aligned}
\text{LHS} &= \frac{1}{1\times 2\times 3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)} \\
&= \frac{1}{4} - \frac{1}{2(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \\
&= \frac{1}{4} - \left\{\frac{1}{2(k+1)(k+2)} - \frac{1}{(k+1)(k+2)(k+3)}\right\} \\
&= \frac{1}{4} - \left\{\frac{k+3-2}{2(k+1)(k+2)(k+3)}\right\} \\
&= \frac{1}{4} - \frac{1}{2\big((k+1)+1\big)\big((k+1)+2\big)} \\
&= \frac{1}{4} - \frac{1}{2(k+2)(k+3)} \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ statement is true for all integers $n \geq 1$, by mathematical induction.

Question 12 (d) (i)

Criteria	Marks
• Provides correct sketch	2
• Sketches the left half of a concave-down parabola OR • Sketches a full concave-down parabola and provides one intercept or the vertex	1

Sample answer:

$y = 4 - \left(1 - \dfrac{x}{2}\right)^2$ is a parabola.

Vertex when $\left(1 - \dfrac{x}{2}\right)^2 = 0 \qquad \therefore\ x = 2$

$$\begin{aligned} y &= 4 - 0 \\ &= 4 \end{aligned}$$

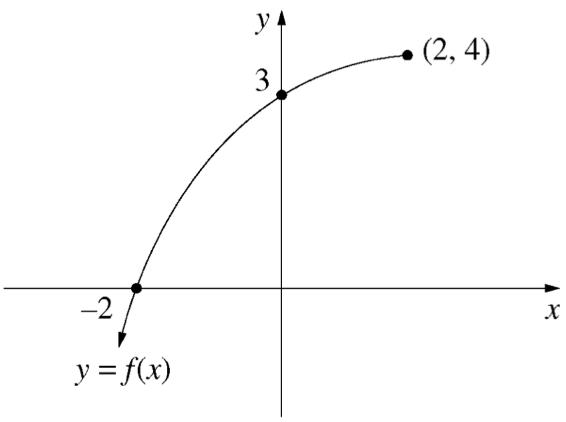
$\therefore$ vertex (2, 4)

y-intercept: $\quad \begin{aligned} y &= 4 - \left(1 - \dfrac{0}{2}\right)^2 \\ &= 3 \end{aligned}$

x-intercept:
$$4 - \left(1 - \frac{x}{2}\right)^2 = 0$$
$$\left(1 - \frac{x}{2}\right)^2 = 4$$
$$1 - \frac{x}{2} = \pm 2$$

$$\therefore\ \frac{x}{2} = 1 + 2 \quad \text{or} \quad \frac{x}{2} = 1 - 2$$
$$= 3 \qquad\qquad = -1$$
$$x = 6 \qquad\qquad x = -2$$

Coefficient of x^2 is negative, so concave down.
Domain $(-\infty, 2]$, so left half only.



Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the expression for $f^{-1}(x)$ OR • States the domain and attempts to either – make x the subject, or – find the inverse by switching variables	2
• States the domain $(\infty, 4]$ OR • Attempts to make x the subject OR • Attempts to find the inverse by switching variables	1

Sample answer:

For $f(x)$, $\quad y = 4 - \left(1 - \frac{x}{2}\right)^2$, x in the domain $(-\infty, 2]$

For inverse, $\quad x = 4 - \left(1 - \frac{y}{2}\right)^2$, y in the domain $(-\infty, 2]$

$$\left(1 - \frac{y}{2}\right)^2 = 4 - x$$

$$\left(\frac{y}{2} - 1\right)^2 = 4 - x$$

$$\frac{y}{2} - 1 = \pm\sqrt{4 - x}$$

$$y = 2 \pm 2\sqrt{4 - x}$$

Range of $f(x)$ is $(-\infty, 4]$, so domain of $f^{-1}(x)$ is $(-\infty, 4]$.

$\therefore\ f^{-1}(x) = 2 - 2\sqrt{4 - x}$ for x in the domain $(-\infty, 4]$.

Alternative solution for Q12 (d) (ii)

$$y = 4 - \left(1 - \frac{x}{2}\right)^2 \quad \text{for} \quad x \le 2$$

For $f^{-1}(x)$, make x the subject:

$$\left(1 - \frac{x}{2}\right)^2 = 4 - y$$

$$x \le 2 \quad \therefore 1 - \frac{x}{2} \ge 0$$

$$\therefore\ 1 - \frac{x}{2} = \sqrt{4 - y}$$

$$\frac{x}{2} = 1 - \sqrt{4 - y}$$

$$x = 2 - 2\sqrt{4 - y}$$

$$\therefore f^{-1}(x) = 2 - 2\sqrt{4 - x}$$

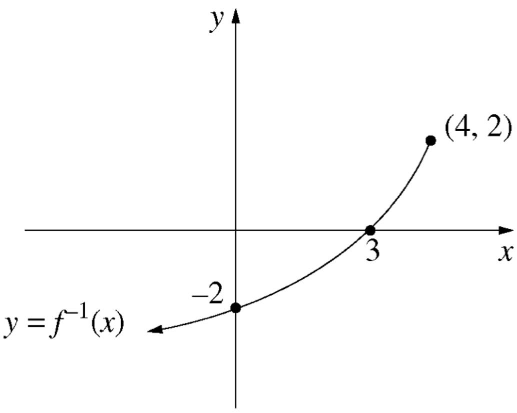
From the graph, range of $f(x)$ is $(-\infty, 4]$

$\therefore$ domain of $f^{-1}(x)$ is $(-\infty, 4]$.

Question 12 (d) (iii)

Criteria	Marks
• Provides correct sketch	1

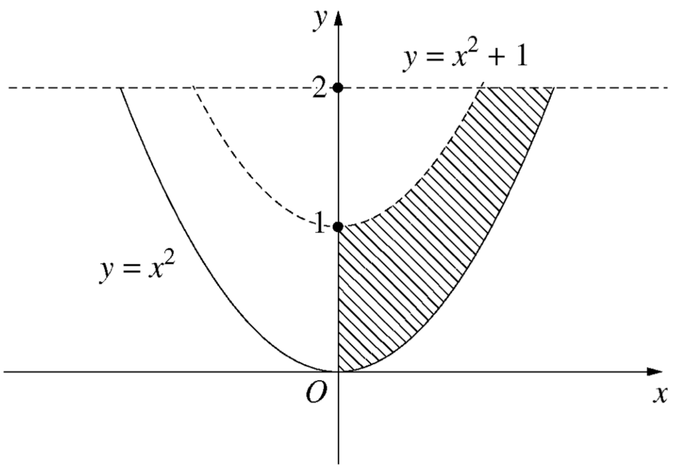
Sample answer:



Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Evaluates one volume OR • Obtains a correct expression for the volume as a difference of two integrals that are only in terms of y	2
• Obtains an integral for one relevant volume OR • Writes the required volume as the difference of two relevant volumes OR • Finds the limits of integration for both volumes	1

Sample answer:



The volume of the garden sculpture is the difference between the outer volume and the inner one.

$$\text{Outer volume} = \pi\int_0^2 x^2\,dy$$

$$= \pi\int_0^2 y\,dy \quad \text{since } y = x^2$$

$$\text{Inner volume} = \pi\int_1^2 x^2\,dy$$

$$= \pi\int_1^2 y - 1\,dy \quad \text{since } y = x^2 + 1$$

$$\therefore V = \int_0^2 \pi y\,dy - \int_1^2 \pi(y-1)\,dy$$

$$V = \pi\left[\frac{y^2}{2}\right]_0^2 - \pi\left[\frac{y^2}{2} - y\right]_1^2$$

$$= \pi\left(\frac{4}{2} - 0\right) - \pi\left(\left[\frac{4}{2} - 2\right] - \left[\frac{1}{2} - 1\right]\right)$$

$$= \frac{3\pi}{2}$$

The volume of the sculpture is $\dfrac{3\pi}{2}$ m^3.

Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Finds the maximum height and the time of flight, or equivalent merit.	3
• Finds the maximum height, or equivalent merit	2
• Finds an expression for the vertical velocity, or equivalent merit	1

Sample answer:

$\underset{\sim}{v}(t) = V\cos\theta\underset{\sim}{i} + (-10t + V\sin\theta)\underset{\sim}{j}$

The height is greatest when $\dot{y} = 0$ ie when $-10t + V\sin\theta = 0$

$$t = \frac{V\sin\theta}{10} = \frac{12\sin 30°}{10} = \frac{6}{10} = 0.6 \text{ s}$$

$$\therefore y = -5\times(0.6)^2 + 12\times(0.6)\times\sin 30° + 1$$

$$= 2.8 \text{ m}$$

The room is 3 metres high and maximum height reached by the object is 2.8 m so the object will not hit the ceiling.

To know if the object hits the far wall or not, it is enough to determine if the unrestricted horizontal range would be more or less than 10 metres.

$y = 0$ if $-5t^2 + Vt\sin\theta + h = 0$

$$-5t^2 + 12\times\frac{1}{2}t + 1 = 0$$

$$-5t^2 + 6t + 1 = 0$$

$$\triangle = 6^2 - 4(-5) = 56$$

$$\therefore\ t = \frac{-6+\sqrt{56}}{-10} < 0 \quad \text{or} \quad \frac{-6-\sqrt{56}}{-10} > 0$$

We need $t > 0$, so the object, in the absence of a far wall, would hit the floor after $t = \dfrac{6+\sqrt{56}}{10} \simeq 1.348$ s.

Substitute in $x(t)$:

$$x(t) = Vt\cos\theta = 12\times 1.348\cos 30°$$
$$\approx 14 \text{ m}$$

Since the far wall is only 10 m away, the object will hit the wall.

Alternative solution for second part

When $x = 10$ $\quad 12t\cos 30° = 10$

$$6\sqrt{3}t = 10$$

$$t = \frac{10}{6\sqrt{3}} = 0.9622 \text{ seconds}$$

When $t = 0.9622$

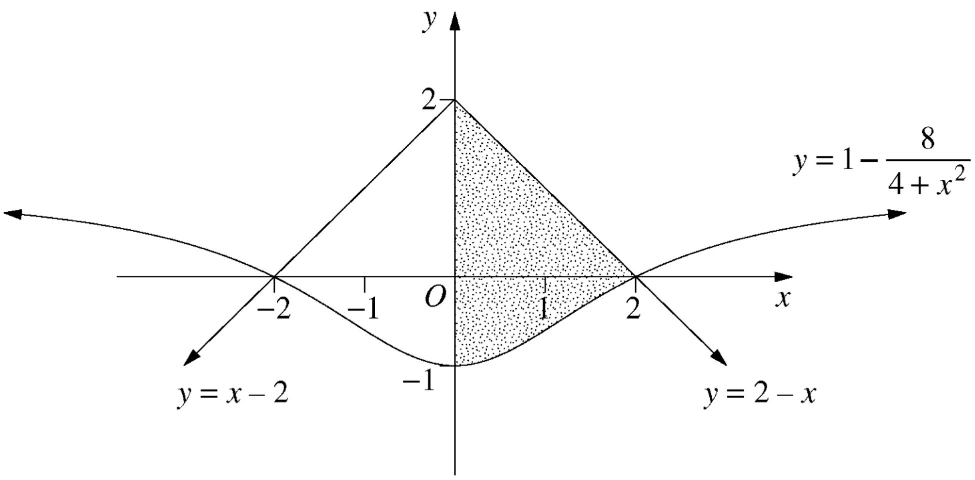
$$y = -5(0.9622\ldots)^2 + 12(0.9622\ldots)\sin 30° + 1$$
$$= 2.144 \text{ m}$$

$\therefore$ Ball is still above the floor when $x = 10$ m and so will hit the far wall without hitting the floor.

Question 13 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains the (signed) area OR the area of either side of the y-axis between the x-axis and the curve $y = 1 - \dfrac{8}{4+x^2}$, or equivalent merit	2
• Finds an integral expression for the area, or equivalent merit OR • Recognises that the total area is twice the area on one side of the y-axis OR • Finds the area of a relevant triangle	1

Sample answer:



By symmetry, area required is double the shaded region above

$$\frac{\text{Area}}{2} = \int_0^2 (2-x) - \left(1 - \frac{8}{4+x^2}\right) dx$$

$$= \int_0^2 1 - x + \frac{8}{4+x^2}\, dx$$

$$= \left[x - \frac{x^2}{2} + \frac{8}{2}\tan^{-1}\left(\frac{x}{2}\right) \right]_0^2$$

$$= \left(2 - 2 + 4\tan^{-1} 1\right) - 0$$

$$= \pi$$

So Area $= 2\pi$

Question 13 (d) (i)

Criteria	Marks
• Provides correct solution	2
• Uses $A = B - d$ and $C = B + d$ in the left hand side and attempts to use a suitable trigonometric identity, or equivalent merit	1

Sample answer:

Given $A = B - d$ and $C = B + d$

$$\therefore \frac{\sin A + \sin C}{\cos A + \cos C} = \frac{\sin(B-d) + \sin(B+d)}{\cos(B-d) + \cos(B+d)}$$

$$= \frac{2\sin B \cos d}{2\cos B \cos d}$$

$$= \tan B$$

Question 13 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Identifies the value of B in order to use as in part (i), or equivalent merit	1

Sample answer:

$$A = \frac{5\theta}{7}, \quad C = \frac{6\theta}{7} \quad \therefore B = \frac{\frac{5\theta}{7} + \frac{6\theta}{7}}{2} = \frac{11\theta}{14}$$

$$\therefore \text{ LHS} = \tan\frac{11\theta}{14}$$

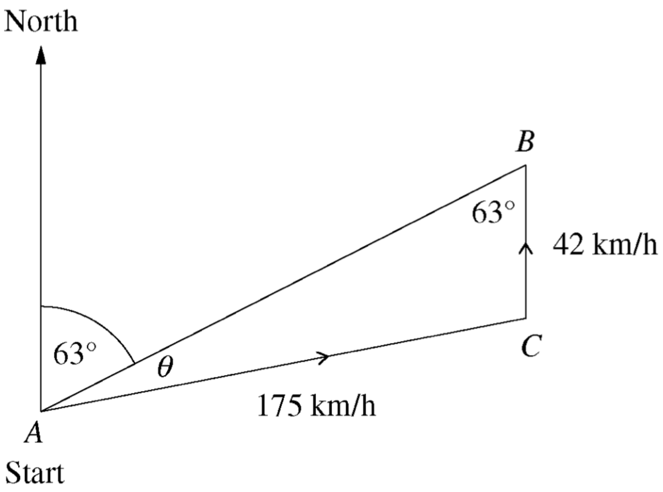
$$\tan\frac{11\theta}{14} = \sqrt{3}$$

$$\theta = \frac{14\pi}{33} \quad \text{or} \quad \frac{56\pi}{33}$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to use the sine rule in the correct triangle, or equivalent merit	2
• Sketches a suitable diagram, or equivalent merit	1

Sample answer:



$$\angle ABC = 63°$$

Let $\angle BAC = \theta$

$$\therefore \quad \frac{\sin\theta}{42} = \frac{\sin 63°}{175}$$

$$\sin\theta = 0.2138\ldots$$

$$\theta = 12.34\ldots$$

$$\approx 12°$$

$\therefore$ required bearing $= 63 + 12$
$= 075°$

Question 14 (b)

Criteria	Marks
• Provides correct solution	4
• Uses the given information to obtain two equations in A and C, or equivalent merit	3
• Integrates both sides correctly, or equivalent merit	2
• Attempts to separate the variables in the differential equation, or equivalent merit	1

Sample answer:

$$\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$$

$$\int \frac{C}{P(C-P)}\,dP = \int 0.1\,dt$$

$$\int \left(\frac{1}{P} + \frac{1}{C-P}\right)dP = \int 0.1\,dt$$

$\ln|P| - \ln|C-P| = 0.1t + k$ where k is a constant

$$\ln\left|\frac{P}{C-P}\right| = 0.1t + k$$

$\dfrac{P}{C-P} = Ae^{0.1t}$ where $A = e^k$ is a constant

When $t = 0$, $P = 150\,000$ so $\dfrac{150\,000}{C - 150\,000} = A$ ①

When $t = 20$, $P = 600\,000$ so $\dfrac{600\,000}{C - 600\,000} = Ae^2$ ②

Substituting ① into ②: $\dfrac{600\,000}{C - 600\,000} = \dfrac{150\,000}{C - 150\,000}e^2$

Taking the reciprocal of both sides

$$\frac{C - 600\,000}{600\,000} = \frac{C - 150\,000}{150\,000}e^{-2}$$

$$150\,000(C - 600\,000) = 600\,000(C - 150\,000)e^{-2}$$

$$C\left(150\,000 - 600\,000e^{-2}\right) = 150\,000 \times 600\,000\left(1 - e^{-2}\right)$$

$$C = \frac{150\,000 \times 600\,000\left(1 - e^{-2}\right)}{150\,000 - 600\,000e^{-2}} \simeq 1\,131\,121$$

$$\simeq 1\,131\,000$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Let $\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ be a vector

$$\underset{\sim}{v} \cdot \underset{\sim}{v} = x \times x + y \times y = x^2 + y^2 = |\underset{\sim}{v}|^2$$

Alternative:

$$\begin{aligned} \underset{\sim}{v} \cdot \underset{\sim}{v} &= |\underset{\sim}{v}|.|\underset{\sim}{v}|\cos\theta \\ &= |\underset{\sim}{v}|^2 \cos\theta \end{aligned}$$

where θ is the angle between and $\underset{\sim}{v}$ and $\underset{\sim}{v}$

$\therefore \quad \theta = 0 \quad$ and $\quad \cos\theta = 1$

$\therefore \quad \underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Equates $\overrightarrow{AC} \cdot \overrightarrow{AC}$ with $\overrightarrow{BD} \cdot \overrightarrow{BD}$, in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, or equivalent merit	2
• Expresses AC or BD in terms of a and b, or equivalent merit	1

Sample answer:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underset{\sim}{a} + \underset{\sim}{b}$$

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -\underset{\sim}{a} + k\underset{\sim}{b}$$

By part (i) $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$ implies

$$\overrightarrow{AC} \cdot \overrightarrow{AC} = \overrightarrow{BD} \cdot \overrightarrow{BD}$$

$$\left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} + \underset{\sim}{b}\right) = \left(-\underset{\sim}{a} + k\underset{\sim}{b}\right) \cdot \left(-\underset{\sim}{a} + k\underset{\sim}{b}\right)$$

$$\cancel{\underset{\sim}{a} \cdot \underset{\sim}{a}} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b} = \cancel{\underset{\sim}{a} \cdot \underset{\sim}{a}} - 2k\underset{\sim}{a} \cdot \underset{\sim}{b} + k^2\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$2(1+k)\underset{\sim}{a} \cdot \underset{\sim}{b} = \left(k^2 - 1\right)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$2(1+k)\underset{\sim}{a} \cdot \underset{\sim}{b} = (k+1)(k-1)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$1 + k \neq 0 \quad \text{since } k > 0$$

$$\therefore \quad 2\underset{\sim}{a} \cdot \underset{\sim}{b} = (k-1)\underset{\sim}{b} \cdot \underset{\sim}{b}$$

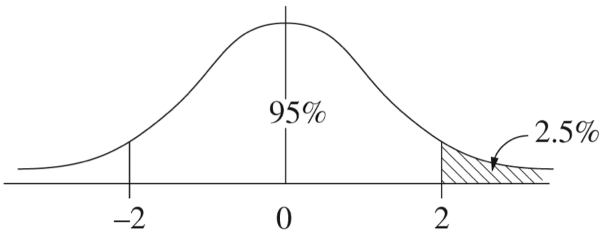
$$\therefore \quad 2\underset{\sim}{a} \cdot \underset{\sim}{b} = (k-1)\left|\underset{\sim}{b}\right|^2$$

$$2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1-k)\left|\underset{\sim}{b}\right|^2 = 0$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Recognises 2σ is important AND has σ in terms of n OR • Recognises 2σ is important AND has the value of σ OR • Obtains $2 = \dfrac{\sqrt{n}}{c}$ where c is a constant, or equivalent merit	2
• Finds σ in terms of n OR • Sketches a normal distribution and shades the correct region OR • Writes $P\left(\hat{p} \geq \dfrac{4}{500}\right) < 0.025$, or explains in words, or equivalent merit	1

Sample answer:



A tail with area 2.5% means $\hat{p} = \dfrac{4}{500}$ is two standard deviations above the mean

so $$\frac{4}{500} = \mu + 2\sigma$$

$$\frac{4}{500} = \frac{3}{500} + 2\sqrt{\frac{\frac{3}{500} \times \left(1 - \frac{3}{500}\right)}{n}}$$

$$\frac{1}{500} = 2\frac{\sqrt{\frac{3}{500}\left(\frac{497}{500}\right)}}{\sqrt{n}}$$

$$\sqrt{n} = 2 \times 500\sqrt{\frac{3}{500} \times \frac{497}{500}}$$

$$= 2 \times \cancel{500}\,\frac{\sqrt{3 \times 497}}{\cancel{500}}$$

$$n = 4 \times 3 \times 497 = 5964$$

The sample size to be chosen so that the chances of shutting down unnecessarily is less than 2.5% is $n = 5964$, ie approximately 6000.

Question 14 (e)

Criteria	Marks
• Provides correct solution	2
• Obtains an expression for the derivative of $g^{-1}(x)$ in terms of x, or equivalent merit	1

Sample answer:

$g(1) = 3 \quad \therefore \; g^{-1}(3) = 1$

Using product rule:

$$f'(x) = g^{-1}(x).1 + x.\frac{d}{dx}\left(g^{-1}(x)\right)$$

Now if $\; g^{-1}(x) = y \;$ then $\; x = g(y)$

$$\therefore \quad \frac{dx}{dy} = g'(y)$$

and $\dfrac{dy}{dx} = \dfrac{1}{g'(y)}$

$$\therefore \quad \frac{d}{dx}\left(g^{-1}(x)\right) = \frac{1}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(x) = g^{-1}(x) + \frac{x}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(3) = g^{-1}(3) + \frac{3}{g'\left(g^{-1}(3)\right)}$$

$$= 1 + \frac{3}{g'(1)}$$

Now, $g(x) = x^3 + 4x - 2$

$$\therefore \quad g'(x) = 3x^2 + 4$$

$$\therefore \quad f'(3) = 1 + \frac{3}{3(1)^2 + 4}$$

$\therefore$ gradient of tangent $= \dfrac{10}{7}$.

2021 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME V1 Introduction to vectors	ME 12–2
2	1	ME C2 Further calculus skills	ME 12–1
3	1	ME F2 Polynomials	ME 11–2
4	1	ME C3 Applications of calculus	ME 12–4
5	1	ME V1 Introduction to vectors	ME 12–2
6	1	ME S1 The binomial distribution	ME 12–5
7	1	ME T3 Trigonometric equations	ME 12–3
8	1	ME F1 Further work with functions	ME 11–2
9	1	ME T1 Inverse trigonometric functions	ME 11–3
10	1	ME A1 Working with combinations	ME 11–5

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	ME V1 Introduction to vectors	ME 12–2
11 (b)	2	ME A1 Working with combinations	ME 11–5
11 (c)	3	ME C2 Further calculus skills	ME 12–4
11 (d)	1	ME C1 Applications of calculus	ME 11–4
11 (e)	2	ME A1 Working with combinations	ME 11–5
11 (f)	2	ME C2 Further calculus skills	ME 12–4
11 (g)	3	ME T3 Trigonometric equations	ME 12–3
11 (h)	2	ME F2 Polynomials	ME 11–2
12 (a)	1	ME C3 Applications of calculus	ME 12–4
12 (b) (i)	3	ME C1 Rates of change	ME 11-4, ME 12–4
12 (b) (ii)	1	ME C1 Rates of change	ME 11–4
12 (c)	3	ME P1 Proof by mathematical induction	ME 12–1
12 (d) (i)	2	ME F1 Further work with functions	ME 11–2
12 (d) (ii)	3	ME F1 Further work with functions	ME 11–1
12 (d) (iii)	1	ME F1 Further work with functions	ME 11–1

Question	Marks	Content	Syllabus outcomes
13 (a)	3	ME C3 Applications of calculus	ME 12–4
13 (b)	4	ME V1 Introduction to vectors	ME 12–2
13 (c)	3	ME C2 Further calculus skills ME C3 Applications of calculus	ME 12–1
13 (d) (i)	2	ME T2 Further trigonometric identities	ME 11–3
13 (d) (ii)	2	ME T3 Trigonometric equations	ME 12–3
14 (a)	3	ME V1 Introduction to vectors	ME 12–2
14 (b)	4	ME C3 Applications of calculus	ME 12–4
14 (c) (i)	1	ME V1 Introduction to vectors	ME 12–2
14 (c) (ii)	3	ME V1 Introduction to vectors	ME 12–2
14 (d)	3	ME S1 The binomial distribution	ME 12–5
14 (e)	2	ME C2 Further calculus skills	ME 12–1