Centre Number

Student Number

NSW Education Standards Authority

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page and on the Question 13 Writing Booklet attached

Total marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–13)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The polynomial $x^3 + 2x^2 - 5x - 6$ has zeros -1, -3 and α.

What is the value of α?

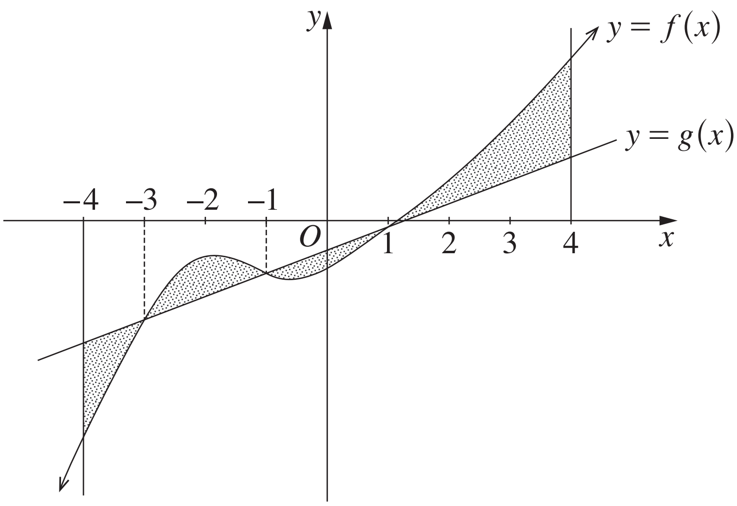
A. -2

B. 2

C. 3

D. 6

2 Consider the functions $y = f(x)$ and $y = g(x)$, and the regions shaded in the diagram below.



Which of the following gives the total area of the shaded regions?

A. $\displaystyle\int_{-4}^{4} f(x) - g(x)\,dx$

B. $\displaystyle\left|\int_{-4}^{4} f(x) - g(x)\,dx\right|$

C. $\displaystyle\int_{-4}^{-3} f(x) - g(x)\,dx + \int_{-3}^{-1} f(x) - g(x)\,dx + \int_{-1}^{1} f(x) - g(x)\,dx + \int_{1}^{4} f(x) - g(x)\,dx$

D. $\displaystyle-\int_{-4}^{-3} f(x) - g(x)\,dx + \int_{-3}^{-1} f(x) - g(x)\,dx - \int_{-1}^{1} f(x) - g(x)\,dx + \int_{1}^{4} f(x) - g(x)\,dx$

3 Students from 4 different schools come together to form a choir.

What is the minimum size of the choir to know that there must be at least 20 students in the choir from one of the schools?

A. 76

B. 77

C. 80

D. 81

4 What are the domain and range of the function $y = 2\cos^{-1}(2x) + 2\sin^{-1}(2x)$?

A. Domain: $[-0.5, 0.5]$ and Range: $\{\pi\}$

B. Domain: $[-0.5, 0.5]$ and Range: $[-\pi, 3\pi]$

C. Domain: $[-2, 2]$ and Range: $\{\pi\}$

D. Domain: $[-2, 2]$ and Range: $[-\pi, 3\pi]$

5 Consider the function $g(x) = 2\sin^{-1}(3x)$.

Which transformations have been applied to $f(x) = \sin^{-1}(x)$ to obtain $g(x)$?

A. Vertical dilation by a factor of $\frac{1}{2}$ and a horizontal dilation by a factor of $\frac{1}{3}$

B. Vertical dilation by a factor of $\frac{1}{2}$ and a horizontal dilation by a factor of 3

C. Vertical dilation by a factor of 2 and a horizontal dilation by a factor of $\frac{1}{3}$

D. Vertical dilation by a factor of 2 and a horizontal dilation by a factor of 3

6 How many real value(s) of x satisfy the equation

$$|b| = |b\sin(4x)|,$$

where $x \in [0, 2\pi]$ and b is not zero?

A. 1

B. 2

C. 4

D. 8

7 A driver's knowledge test contains 30 multiple-choice questions, each with 4 options. An applicant must get at least 29 correct to pass.

If an applicant correctly answers the first 25 questions and randomly guesses the last 5 questions, what is the probability that the applicant will pass the test?

A. $\dfrac{1}{256}$

B. $\dfrac{15}{1024}$

C. $\dfrac{1}{64}$

D. $\dfrac{21}{256}$

8 A local council is proposing to ban dog-walking on the beach. It is known that the proportion of households that have a dog is $\dfrac{7}{12}$.

The local council wishes to poll n households about this proposal.

Let $\hat{p}$ be the random variable representing the proportion of households polled that have a dog.

What is the smallest sample size, n, for which the standard deviation of $\hat{p}$ is less than 0.06?

A. 67

B. 68

C. 94

D. 95

9 A bag contains n metal coins, $n \geq 3$, that are made from either silver or bronze.

There are k silver coins in the bag and the rest are bronze.

Two coins are to be drawn at random from the bag, with the first coin drawn not being replaced before the second coin is drawn.

Which of the following expressions will give the probability that the two coins drawn are made of the same metal?

A. $\dfrac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$

B. $\dbinom{n}{2}\dbinom{n}{k}\left(1-\dfrac{k}{n}\right)^{n-2}$

C. $\dfrac{\dbinom{k}{2}+\dbinom{n-k}{2}}{n(n-1)}$

D. $\dfrac{k^2+(n-k)^2}{n^2}$

10 For real numbers a and b, where $a \neq 0$ and $b \neq 0$, we can find numbers $\alpha, \beta, \gamma, \delta$ and R such that $a\cos x + b\sin x$ can be written in the following 4 forms:

$$R\sin(x+\alpha)$$
$$R\sin(x-\beta)$$
$$R\cos(x+\gamma)$$
$$R\cos(x-\delta)$$

where $R > 0$ and $0 < \alpha, \beta, \gamma, \delta < 2\pi$.

What is the value of $\alpha + \beta + \gamma + \delta$?

A. 0

B. π

C. 2π

D. 4π

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Consider the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 4\underset{\sim}{j}$.

(i) Find $2\underset{\sim}{a} - \underset{\sim}{b}$. **1**

(ii) Find $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **1**

(b) Solve $x^2 - 8x - 9 \leq 0$. **2**

(c) Using the substitution $u = x - 1$, find $\int x\sqrt{x-1}\, dx$. **3**

(d) Solve the differential equation $\frac{dy}{dx} = xy$, given $y > 0$. Express your answer in the form $y = e^{f(x)}$. **2**

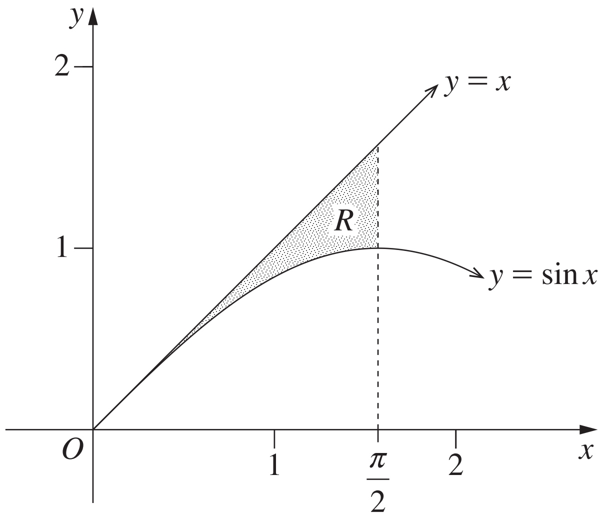
(e) Differentiate the function $f(x) = \arcsin\left(x^5\right)$. **1**

Question 11 continues on page 7

(f) The volume of a sphere of radius r cm, is given by $V = \frac{4}{3}\pi r^3$, and the volume of the sphere is increasing at a rate of $10 \text{ cm}^3 \text{ s}^{-1}$. **2**

Show that the rate of increase of the radius is given by $\frac{dr}{dt} = \frac{5}{2\pi r^2} \text{ cm s}^{-1}$.

(g) The region, R, is bounded by the curves $y = \sin x$, $y = x$ and the line $x = \frac{\pi}{2}$ as shown in the diagram. **3**



Find the area of the region R.

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet

(a) The vectors $\begin{pmatrix} a^2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} a+5 \\ a-4 \end{pmatrix}$ are perpendicular. **3**

Find the possible values of a.

(b) The region, R, is bounded by the function, $y = x^3$, the x-axis and the lines $x = 1$ and $x = 2$. **3**

What is the volume of the solid of revolution obtained when the region R is rotated about the x-axis?

(c) A charity employs a worker to collect donations. There is a 0.31 chance that when the charity worker talks to someone a donation is made to the charity. **3**

Each day the charity worker must talk to exactly 100 people.

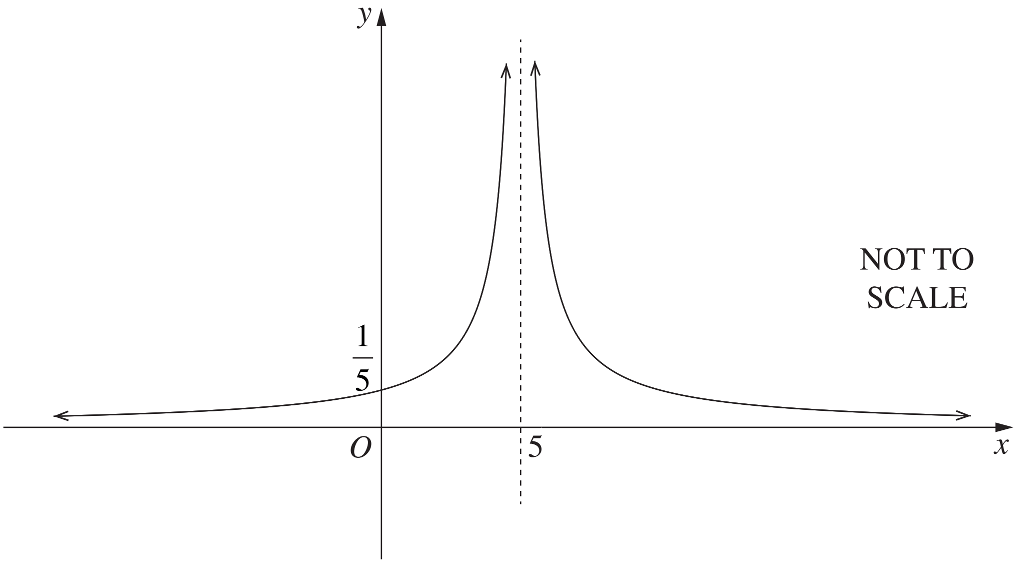
Use the standard normal distribution and the information on page 13 to approximate the probability that, on a particular day, at least 35% of the people talked to made a donation.

(d) Use mathematical induction to prove that $2^{3n} + 13$ is divisible by 7 for all integers $n \geq 1$. **3**

Question 12 continues on page 9

Question 12 (continued)

(e) The diagram shows the graph of $y = \dfrac{1}{|x-5|}$. **3**



For what values of x is $\dfrac{x}{6} \geq \dfrac{1}{|x-5|}$?

End of Question 12

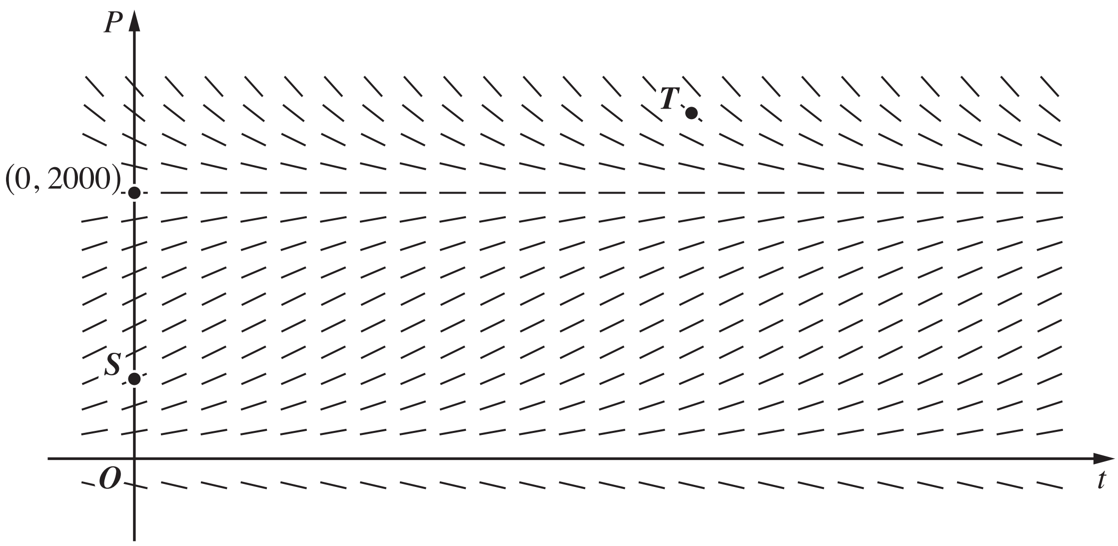
Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) In an experiment, the population of insects, $P(t)$, was modelled by the logistic differential equation

$$\frac{dP}{dt} = P(2000 - P)$$

where t is the time in days after the beginning of the experiment.

The diagram shows a direction field for this differential equation, with the point S representing the initial population.



(i) Explain why the graph of the solution that passes through the point S cannot also pass through the point T. **1**

(ii) On the diagram provided on page 1 of the Question 13 Writing Booklet, clearly sketch the graph of the solution that passes through the point S. **1**

(iii) Find the predicted value of the population, $P(t)$, at which the rate of growth of the population is largest. **2**

Question 13 continues on page 11

(b) (i) Show that $\cos^4 x + \sin^4 x = \dfrac{1 + \cos^2 2x}{2}$. **2**

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \left(\cos^4 x + \sin^4 x\right) dx$. **3**

(c) The vector $\underset{\sim}{a}$ is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and the vector $\underset{\sim}{b}$ is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. **4**

The projection of a vector $\underset{\sim}{x}$ onto the vector $\underset{\sim}{a}$ is $k\underset{\sim}{a}$, where k is a real number.

The projection of the vector $\underset{\sim}{x}$ onto the vector $\underset{\sim}{b}$ is $p\underset{\sim}{b}$, where p is a real number.

Find the vector $\underset{\sim}{x}$ in terms of k and p.

(d) Using the substitution $u = e^x + 2e^{-x}$, and considering u^2, find **3**

$$\int \frac{e^{3x} - 2e^x}{4 + 8e^{2x} + e^{4x}}\, dx.$$

End of Question 13

Question 14 (14 marks) Use the Question 14 Writing Booklet

(a) Find the domain and range of the function that is the solution to the differential equation **4**

$$\frac{dy}{dx} = e^{x+y}$$

and whose graph passes through the origin.

(b) For what values of the constant k would the function $f(x) = \frac{kx}{1+x^2} + \arctan x$ have an inverse? **3**

(c) (i) Explain why the equation $\tan^{-1}(3x) + \tan^{-1}(10x) = \theta$, where $-\pi < \theta < \pi$, has exactly one solution. **1**

(ii) Solve $\tan^{-1}(3x) + \tan^{-1}(10x) = \frac{3\pi}{4}$. **2**

(d) A particle is projected from the origin, with initial speed V at an angle of θ to the horizontal. The position vector of the particle, $\underset{\sim}{r}(t)$, where t is the time after projection and g is the acceleration due to gravity, is given by **4**

$$\underset{\sim}{r}(t) = \begin{pmatrix} Vt\cos\theta \\ Vt\sin\theta - \frac{gt^2}{2} \end{pmatrix}. \qquad \text{(Do NOT prove this.)}$$

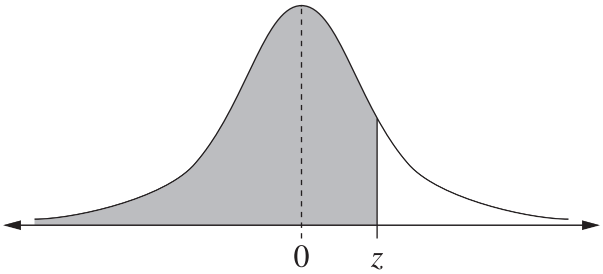
Let $D(t)$ be the distance of the particle from the origin at time t, so $D(t) = \left|\underset{\sim}{r}(t)\right|$.

Show that for $\theta < \sin^{-1}\left(\sqrt{\frac{8}{9}}\right)$ the distance, $D(t)$, is increasing for all $t > 0$.

End of paper

Use the information below to answer Question 12 (c).

Table of values $P(Z \leq z)$ for the normal distribution $N(0, 1)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

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2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

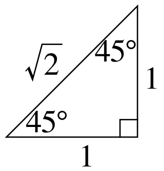
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

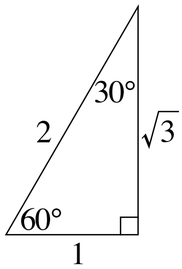


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

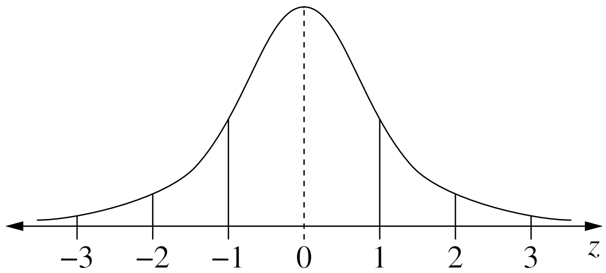
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\, dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\, dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\, dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x)\, dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left| \underset{\sim}{u} \right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left| \underset{\sim}{u} \right| \left| \underset{\sim}{v} \right| \cos\theta = x_1 x_2 + y_1 y_2,$$

where $\underset{\sim}{u} = x_1 \underset{\sim}{i} + y_1 \underset{\sim}{j}$

and $\underset{\sim}{v} = x_2 \underset{\sim}{i} + y_2 \underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta) \right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

2024 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	D
3	B
4	A
5	C
6	D
7	C
8	B
9	A
10	D

Section II

Question 11 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}, \qquad \underset{\sim}{b} = -\underset{\sim}{i} + 4\underset{\sim}{j}$

$$\begin{aligned} 2\underset{\sim}{a} - \underset{\sim}{b} &= 2\left(3\underset{\sim}{i} + 2\underset{\sim}{j}\right) - \left(-\underset{\sim}{i} + 4\underset{\sim}{j}\right) \\ &= 6\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{i} - 4\underset{\sim}{j} \\ &= 7\underset{\sim}{i} \end{aligned}$$

Question 11 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} \underset{\sim}{a} \cdot \underset{\sim}{b} &= 3 \times -1 + 2 \times 4 \\ &= -3 + 8 \\ &= 5 \end{aligned}$$

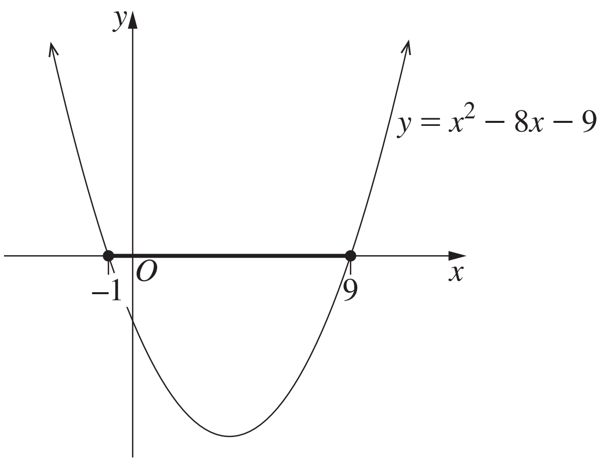
Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to factor the LHS, or equivalent merit	1

Sample answer:

$$x^2 - 8x - 9 \le 0$$

$$\begin{aligned} y &= x^2 - 8x - 9 \\ &= (x-9)(x+1) \end{aligned}$$



$$-1 \le x \le 9$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral in terms of u, or equivalent merit	2
• Correctly substitutes for one element of the integrand, or equivalent merit	1

Sample answer:

$$u = x - 1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\begin{aligned} \int x\sqrt{x-1} \cdot dx &= \int (u+1)u^{\frac{1}{2}} \cdot du \\ &= \int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du \\ &= \frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} + C \\ &= \frac{2\left(\sqrt{x-1}\right)^5}{5} + \frac{2\left(\sqrt{x-1}\right)^3}{3} + C \end{aligned}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Separates the variables, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = xy$$

$$\int \frac{dy}{y} = \int x\,dx$$

$$\log_e y = \frac{x^2}{2} + C \qquad \text{as } y > 0$$

$$y = e^{\frac{x^2}{2} + C}$$

Answers could include:

$$Ae^{\frac{x^2}{2}}$$

Question 11 (e)

Criteria	Marks
• Provides correct derivative	1

Sample answer:

$$y = \arcsin\left(x^5\right)$$

Let $\quad f(x) = x^5$

$\therefore \quad f'(x) = 5x^4$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$$

$$= \frac{5x^4}{\sqrt{1 - x^{10}}}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Correctly applies the chain rule to find an expression for $\frac{dr}{dt}$, or equivalent merit	1

Sample answer:

$$V = \frac{4}{3}\pi r^3, \qquad \frac{dV}{dt} = 10\ \text{cm}^3\,\text{s}^{-1}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\begin{aligned}\frac{dr}{dt} &= \frac{dr}{dV} \cdot \frac{dV}{dt} \\ &= \frac{1}{4\pi r^2} \cdot 10 \\ &= \frac{5}{2\pi r^2}\ \text{cm}\,\text{s}^{-1}\end{aligned}$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	3
• Finds the primitive of $x - \sin x$ OR • Finds the two relevant areas, or equivalent merit	2
• Considers the function $x - \sin x$ OR • Calculates the area of the triangle OR • Considers the difference of two areas, or equivalent merit	1

Sample answer:

$$\text{Area} = \int_0^{\frac{\pi}{2}} (x - \sin x)\, dx$$

$$= \left[\frac{x^2}{2} + \cos x \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi^2}{8} + 0 \right) - (0 + 1)$$

$$= \frac{\pi^2}{8} - 1$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Obtains the solution $a = 1$, or equivalent merit	2
• Obtains the equation $a^2(a+5) + 2(a-4) = 0$, or equivalent merit	1

Sample answer:

$$\begin{pmatrix} a^2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} a+5 \\ a-4 \end{pmatrix} = 0 \qquad \text{as vectors are } \perp$$

$$\begin{aligned} a^2(a+5) + 2(a-4) &= 0 \\ a^3 + 5a^2 + 2a - 8 &= 0 \end{aligned}$$

Let $a = 1$

$$\begin{aligned} 1^3 + 5(1)^2 + 2(1) - 8 &= 1 + 5 + 2 - 8 \\ &= 0 \end{aligned}$$

$\therefore\ a - 1$ is a factor.

Using sum and product of roots:

$$\begin{aligned} \alpha + \beta + \gamma &= -5 \\ 1 + \beta + \gamma &= -5 \\ \beta + \gamma &= -6 \end{aligned}$$

$$\beta\gamma = 8$$

$$\therefore \quad \beta = -4, \quad \gamma = -2$$

Hence, roots are $1, -2, -4$

Answers could include:

- Inspection and equating of coefficients
- Division of polynomials
- Other methods may also be used

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive, or equivalent merit	2
• Obtains a correct expression for the volume, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
V &= \pi \int_1^2 \left(x^3\right)^2 dx \\
&= \pi \left[\frac{x^7}{7} \right]_1^2 \\
&= \frac{\pi}{7}(128 - 1) \\
&= \frac{127\pi}{7}
\end{aligned}
$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Finds the *z*-score, or equivalent merit	2
• Obtains relevant standard deviation, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
\sigma &= \sqrt{np(1-p)} \\
&= \sqrt{100 \times 0.31 \times 0.69} \\
&= \sqrt{21.39}
\end{aligned}
$$

$$
\begin{aligned}
z &= \frac{x - \mu}{\sigma} \\
&= \frac{35 - 31}{\sqrt{21.39}} \\
&\doteqdot 0.86
\end{aligned}
$$

From normal distribution graph

$$
\begin{aligned}
P(\geq 35\% \text{ make a donation}) &= 1 - 0.8051 \\
&= 0.1949 \\
&= 19.49\%
\end{aligned}
$$

Question 12 (d)

Criteria	Marks
• Provides correct proof	3
• Establishes that $p(n) \Rightarrow p(n+1)$ is true, or equivalent merit	2
• Establishes base case, or equivalent merit	1

Sample answer:

Base case $n = 1$

$$\begin{aligned} 2^{3\times 1} + 13 &= 2^3 + 13 \\ &= 8 + 13 \\ &= 21 \quad \text{(which is divisible by 7)} \end{aligned}$$

Therefore true for $n = 1$

Assume true for $n = k$ ie $2^{3k} + 13 = 7M$ where M is a positive integer

Prove true for $n = k + 1$ ie $2^{3(k+1)} + 13 = 7N$ where N is a positive integer

$$\begin{aligned} \text{LHS} &= 2^{3k+3} + 13 \\ &= 2^{3k} \cdot 2^3 + 13 \\ &= (7M - 13)8 + 13 \quad \text{using assumption} \\ &= 7M \times 8 - 104 + 13 \\ &= 7M \times 8 - 91 \\ &= 7(8M - 13) \\ &= 7N \quad \text{where} \quad N = 8M - 13 \quad \text{which is an integer} \end{aligned}$$

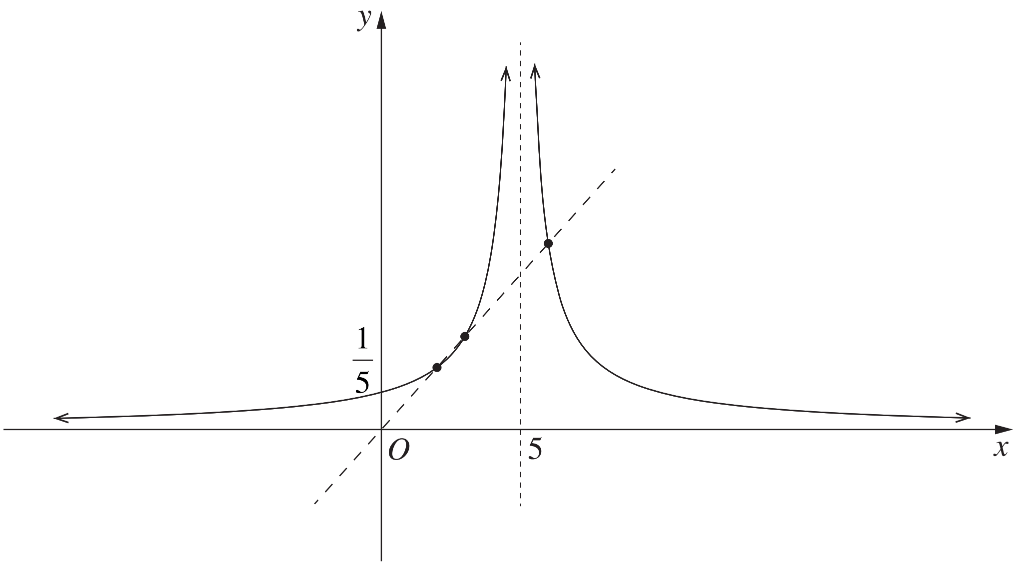
$7N$ is divisible by 7, as required.

Hence, by mathematical induction, $2^{3n} + 13$ is divisible by 7, for all integers $n \geq 1$.

Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Obtains the three critical points 2, 3 and 6, or equivalent merit	2
• Recognises the two cases OR • Observes that $x = 6$ is a critical point OR • Graphs the two curves on the same axis, or equivalent merit	1

Sample answer:



$x > 5$

$$\frac{x}{6} = \frac{1}{x-5}$$
$$x^2 - 5x = 6$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x = 6 \qquad \text{since } x > 5$$
$$\therefore\ [6, \infty)$$

$x < 5$

$$\frac{x}{6} = \frac{-1}{x-5}$$
$$x^2 - 5x = -6$$
$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2,\ 3 \qquad \text{since } x < 5$$
$$\therefore\ [2, 3]$$

$\therefore$ Solution is $[2, 3] \cup [6, \infty)$

Question 13 (a) (i)

Criteria	Marks
• Provides correct explanation	1

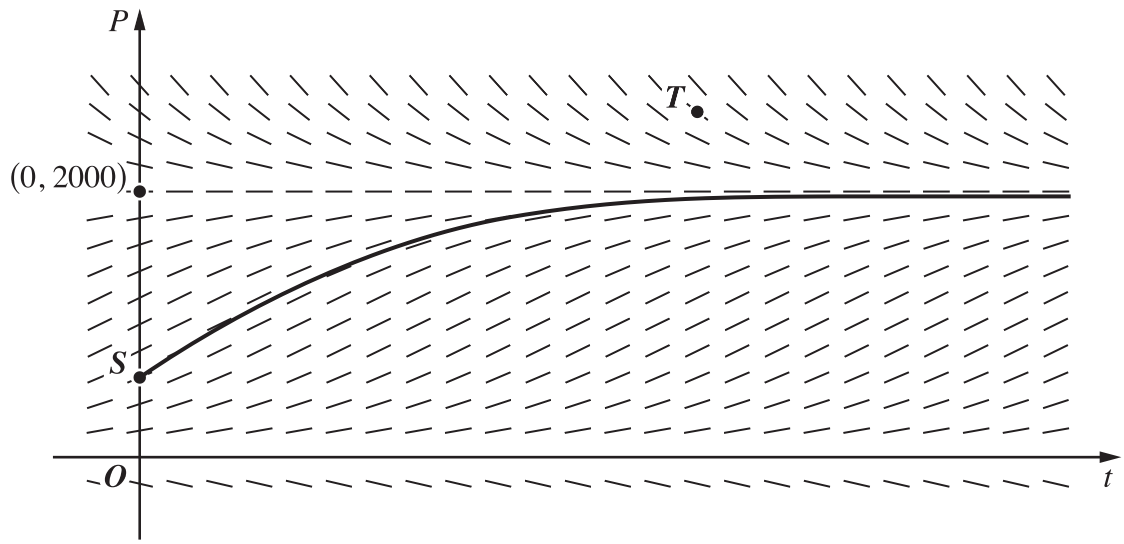
Sample answer:

All graphs passing through T must be decreasing to the left of T. So the y-intercept of such a curve, if any y-intercept, must be above T. The point S isn't above T so S can't be on any such curve.

Question 13 (a) (ii)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Question 13 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains the equation $\frac{d^2P}{dt^2} = 0$, or equivalent merit	1

Sample answer:

Rate of growth is a maximum when $\frac{dP}{dt}$ is a maximum.

ie when $\frac{d^2P}{dt^2} = 0$

$$0 = 1 \times (2000 - P) - P$$
$$2P = 2000$$
$$P = 1000$$

From the direction field it can be seen that the slope at $P = 1000$ is the maximum slope. (Rather than a minimum slope or other type of stationary point.)

Question 13 (b) (i)

Criteria	Marks
• Provides correct proof	2
• Expands $\left(\cos^2 x + \sin^2 x\right)^2$, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
\cos^4 x + \sin^4 x &= \left(\cos^2 x + \sin^2 x\right)^2 - 2\cos^2 x \sin^2 x \\
&= 1^2 - \frac{1}{2}(2\cos x \sin x)^2 \\
&= 1 - \frac{1}{2}\sin^2 2x \\
&= 1 - \frac{1}{2}\left(1 - \cos^2 2x\right) \\
&= 1 - \frac{1}{2} + \frac{1}{2}\cos^2 2x \\
&= \frac{1}{2} + \frac{1}{2}\cos^2 2x \\
&= \frac{1 + \cos^2 2x}{2}
\end{aligned}
$$

Answers could include:

• Use of double angle formula

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive, or equivalent merit	2
• Obtains correct integral involving $\cos 4x$, or equivalent merit	1

Sample answer:

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} \left(\cos^4 x + \sin^4 x\right) dx &= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 + \cos^2 2x\right) dx, \qquad \text{using part (i)} \\
&= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 + \frac{1}{2}(1 + \cos 4x)\right) dx \\
&= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\frac{3}{2} + \frac{1}{2}\cos 4x\right) dx \\
&= \frac{1}{2}\left[\frac{3x}{2} + \frac{1}{2}\cdot\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{4}} \\
&= \left[\frac{3x}{4} + \frac{1}{16}\sin 4x\right]_0^{\frac{\pi}{4}} \\
&= \left(\frac{3}{4}\left(\frac{\pi}{4}\right) + \frac{1}{16}\sin \pi\right) - (0 + 0) \\
&= \frac{3\pi}{16} + 0 \\
&= \frac{3\pi}{16}
\end{aligned}$$

Question 13 (c)

Criteria	Marks
• Provides correct solution	4
• Obtains both k and p in terms of the components of $\underset{\sim}{x}$, or equivalent merit	3
• Obtains k or p in terms of the components of $\underset{\sim}{x}$, or equivalent merit	2
• Writes $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{x}}{\underset{\sim}{a} \cdot \underset{\sim}{a}} \underset{\sim}{a}$ or $\dfrac{\underset{\sim}{b} \cdot \underset{\sim}{x}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$, or equivalent merit	1

Sample answer:

Let $\quad \underset{\sim}{x} = \begin{pmatrix} u \\ v \end{pmatrix}$

Then $\quad k\underset{\sim}{a} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{x}}{\underset{\sim}{a} \cdot \underset{\sim}{a}} \underset{\sim}{a}$

So $\quad k = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{x}}{\underset{\sim}{a} \cdot \underset{\sim}{a}} = \dfrac{u + 3v}{10}$

$$u + 3v = 10k$$

Also $\quad p\underset{\sim}{b} = \dfrac{\underset{\sim}{b} \cdot \underset{\sim}{x}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$

So $\quad p = \dfrac{\underset{\sim}{b} \cdot \underset{\sim}{x}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} = \dfrac{2u - v}{5}$

$$2u - v = 5p$$

Solving

$$\begin{aligned}
& & u + 3v &= 10k & \quad ① \\
& & 2u - v &= 5p & \quad ② \\
3 \times ② & & 6u - 3v &= 15p & \\
① + 3 \times ② & & 7u &= 10k + 15p & \\
& & u &= \frac{10k + 15p}{7} & \\
& & v &= -5p + 2u & \\
& & &= -5p + \frac{20k + 30p}{7} & \\
& & &= \frac{20k - 5p}{7} &
\end{aligned}$$

$$\underset{\sim}{x} = \begin{pmatrix} \dfrac{10k + 15p}{7} \\[2mm] \dfrac{20k - 5p}{7} \end{pmatrix} = \frac{5}{7}\begin{pmatrix} 2k + 3p \\ 4k - p \end{pmatrix}$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	3
• Changes the integrand into an appropriate form using algebra, or equivalent merit	2
• Obtains $\frac{du}{dx} = e^x - 2e^{-x}$, or equivalent merit	1

Sample answer:

$$\int \frac{e^{3x} - 2e^x}{4 + 8e^{2x} + e^{4x}}\,dx$$

Let $u = e^x + 2e^{-x}$

$$u^2 = e^{2x} + 4 + 4e^{-2x}$$

$$\frac{du}{dx} = e^x - 2e^{-x}$$

$$du = \left(e^x - 2e^{-x}\right)dx$$

$$\begin{aligned} 4 + 8e^{2x} + e^{4x} &= e^{2x}\left(4e^{-2x} + 8 + e^{2x}\right) \\ &= e^{2x}\left(4e^{-2x} + 4 + e^{2x}\right) + 4e^{2x} \end{aligned}$$

$$\begin{aligned} \int \frac{e^{3x} - 2e^x}{4 + 8e^{2x} + e^{4x}}\,dx &= \int \frac{e^{2x}\left(e^x - 2e^{-x}\right)}{e^{2x}\left(4e^{-2x} + 4 + e^{2x}\right) + 4e^{2x}}\,dx \\ &= \int \frac{\left(e^x - 2e^{-x}\right)dx}{\left(4e^{-2x} + 4 + e^{2x}\right) + 4} \\ &= \int \frac{du}{u^2 + 4} \\ &= \frac{1}{2}\tan^{-1}\frac{u}{2} + C \\ &= \frac{1}{2}\tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right) + C \end{aligned}$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	4
• Obtains correct domain or range, or equivalent merit	3
• Integrates to obtain relation with constant evaluated, or equivalent merit	2
• Separates the variables, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = e^{x+y} = e^x e^y$$

$$\int e^{-y}\,dy = \int e^x\,dx$$

$$-e^{-y} = e^x + k$$

$y = 0,\ x = 0$

$$-1 = 1 + k$$
$$k = -2$$

$$-e^{-y} = e^x - 2$$
$$e^{-y} = 2 - e^x$$
$$-y = \ln\left(2 - e^x\right)$$
$$y = -\ln\left(2 - e^x\right)$$
$$e^x < 2$$
$$x < \ln 2$$

Domain is $(-\infty, \ln 2)$

To find the range:

$$x \longrightarrow -\infty \qquad x \longrightarrow \ln 2$$
$$e^x \longrightarrow 0 \qquad e^x \longrightarrow 2$$
$$y \longrightarrow -\ln 2 \qquad \ln\left(2 - e^x\right) \longrightarrow -\infty$$
$$y \longrightarrow \infty$$

and $\dfrac{dy}{dx} = e^{x+y} > 0$ so graph is increasing

Range is $(-\ln 2, \infty)$

Question 14 (b)

Criteria	Marks
• Provides correct solution	3
• Correctly completes one of two cases, or equivalent merit	2
• Recognises that $f(x)$ needs to be monotonically increasing or monotonically decreasing, or equivalent merit	1

Sample answer:

As $f(x)$ is defined and differentiable for all reals, it will have an inverse if $f'(x) \geq 0$ always or $f'(x) \leq 0$ always.

$$f'(x) = \frac{k\left(1+x^2\right) - kx(2x)}{\left(1+x^2\right)^2} + \frac{1}{1+x^2}$$

$$= \frac{(k+1) - kx^2 + x^2}{\left(1+x^2\right)^2}$$

$$= \frac{(k+1) + (1-k)x^2}{\left(1+x^2\right)^2} \quad \left(\text{and } \left(1+x^2\right)^2 \text{ is positive}\right)$$

Case 1: $f'(x) \geq 0$

$(k+1) + (1-k)x^2$ will always be greater or equal to zero
if and only if $(1-k) \geq 0$ and $(k+1) \geq 0$.

$\therefore \quad k \leq 1$ and $k \geq -1$

ie $-1 \leq k \leq 1$

Case 2: $f'(x) \leq 0$

$(k+1) + (1-k)x^2$ will always be less than or equal to zero
if and only if $(1-k) \leq 0$ and $(k+1) \leq 0$.

$\therefore \quad k \geq 1$ and $k \leq -1$

ie no solution

Hence, if $-1 \leq k \leq 1$, then $f'(x)$ is increasing and so has an inverse.

Question 14 (c) (i)

Criteria	Marks
• Provides correct explanation	1

Sample answer:

The graphs of $\tan^{-1}(3x)$ and $\tan^{-1}(10x)$ are $y = \tan^{-1}(x)$ shrunk horizontally.

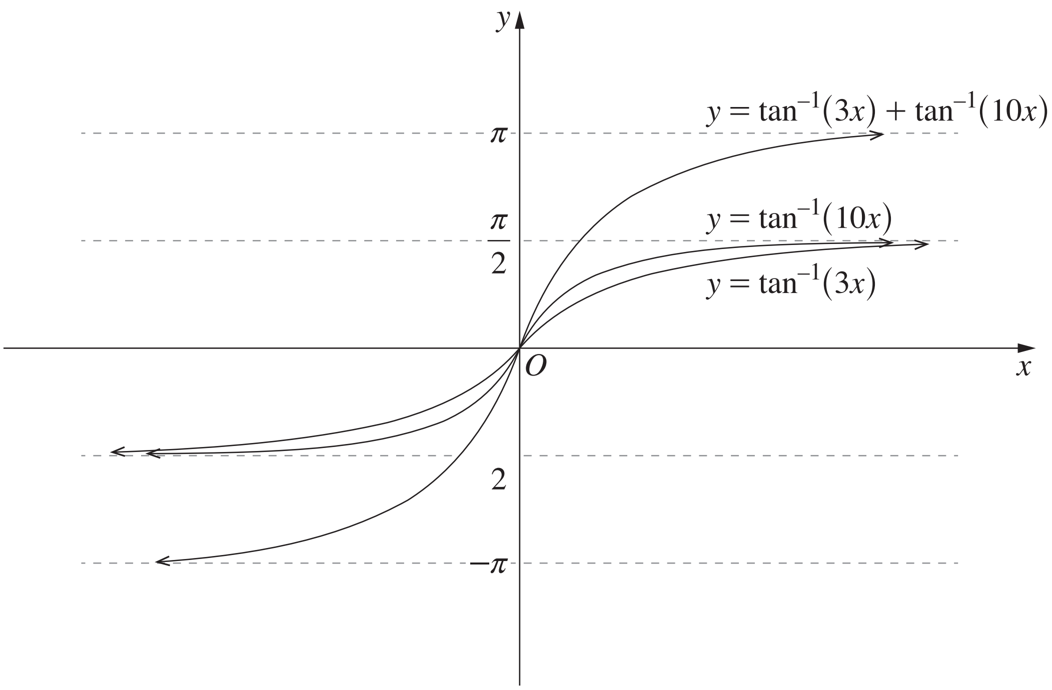
Both are increasing functions with a range $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

Adding gives an increasing function, considering large values of x shows the range of $\tan^{-1}(3x) + \tan^{-1}(10x)$ is $(-\pi, \pi)$.

For $-\pi < \theta < \pi$ the line $y = \theta$ will cross the graph of $y = \tan^{-1}(3x) + \tan^{-1}(10x)$ at exactly one point.

So, $\tan^{-1}(3x) + \tan^{-1}(10x) = \theta$ has exactly one solution.

Answers could include:



Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains correct quadratic equation for x, or equivalent merit	1

Sample answer:

$$\tan^{-1}(3x) + \tan^{-1}(10x) = \frac{3\pi}{4}$$

$$\tan\left(\tan^{-1}(3x) + \tan^{-1}(10x)\right) = \tan\frac{3\pi}{4}$$

$$\frac{\tan\left(\tan^{-1}(3x)\right) + \tan\left(\tan^{-1}(10x)\right)}{1 - \tan\left(\tan^{-1}(3x)\right)\tan\left(\tan^{-1}(10x)\right)} = -1$$

$$\frac{3x + 10x}{1 - (3x)(10x)} = -1$$

$$13x = -1 + 30x^2$$

$$30x^2 - 13x - 1 = 0$$

$$(2x - 1)(15x + 1) = 0$$

$$x = \frac{1}{2}, \quad -\frac{1}{15}$$

When $x = \frac{1}{2}$, $\quad \tan^{-1}\left(\frac{3}{2}\right) + \tan^{-1}(5) > 0$

When $x = -\frac{1}{15}$, $\quad \tan^{-1}\left(-\frac{3}{15}\right) + \tan^{-1}\left(-\frac{10}{15}\right) < 0$

The only solution is $x = \frac{1}{2}$.

Question 14 (d)

Criteria	Marks
• Provides correct proof	4
• Explains why $\Delta < 0$ for their correct quadratic, or equivalent merit	3
• Obtains correct derivative of $\left[D(t)\right]^2$ or $D(t)$, or equivalent merit	2
• Obtains $\lvert D(t) \rvert$ or $\lvert D(t) \rvert^2$, or equivalent merit	1

Sample answer:

$$D^2 = V^2t^2\cos^2\theta + V^2t^2\sin^2\theta - Vgt^3\sin\theta + \frac{g^2t^4}{4}$$

$$= V^2t^2 - Vgt^3\sin\theta + \frac{g^2t^4}{4}$$

$$\frac{d\left(D^2\right)}{dt} = 2V^2t - 3Vgt^2\sin\theta + g^2t^3$$

$$= t\left(2V^2 - 3Vgt\sin\theta + g^2t^2\right)$$

$D(t)$ is increasing for all $t > 0$ when $\frac{d\left(D^2\right)}{dt} > 0$.

When $\Delta < 0$

$$9V^2g^2\sin^2\theta - 4 \times g^2 \times 2V^2 < 0$$

$$9V^2g^2\sin^2\theta - 8V^2g^2 < 0$$

As $V^2, g^2 \neq 0$ $\quad 9\sin^2\theta - 8 < 0$

$$\sin^2\theta < \frac{8}{9}$$

For $0 \leq \theta \leq \frac{\pi}{2}, \sin\theta > 0$ $\quad \therefore\ \sin\theta < \sqrt{\frac{8}{9}}$

$$\therefore\ \theta < \sin^{-1}\left(\sqrt{\frac{8}{9}}\right)$$

2024 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-F2 Polynomials	ME11-2
2	1	ME-C2 Further Calculus Skills	ME12-4
3	1	ME-A1 Working with Combinatorics	ME11-5
4	1	ME-T1 Inverse Trigonometric Functions	ME11-1
5	1	ME-T1 Inverse Trigonometric Functions	ME11-1
6	1	ME-F1 Further Work with Functions	ME11-2
7	1	ME-A1 Working with Combinatorics	ME11-5
8	1	ME-S1 The Binomial Distribution	ME12-5
9	1	ME-A1 Working with Combinatorics	ME11-5
10	1	ME-T3 Trigonometric Equations	ME12-3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a) (i)	1	ME-V1 Introduction to Vectors	ME12-2
11 (a) (ii)	1	ME-V1 Introduction to Vectors	ME12-2
11 (b)	2	ME-F1 Further Work with Functions	ME11-2
11 (c)	3	ME-C2 Further Calculus Skills	ME12-1
11 (d)	2	ME-C2 Further Calculus Skills	ME12-1
11 (e)	1	ME-C2 Further Calculus Skills	ME12-1
11 (f)	2	ME-C1 Rates of Change	ME11-4
11 (g)	3	ME-C3 Applications of Calculus	ME12-4
12 (a)	3	ME-V1 Introduction to Vectors	ME12-2
12 (b)	3	ME-C3 Applications of Calculus	ME12-4
12 (c)	3	ME-S1 The Binomial Distribution	ME12-5
12 (d)	3	ME-P1 Proof by Mathematical induction	ME12-1
12 (e)	3	ME-F1 Further Work with Function	ME11-2
13 (a) (i)	1	ME-C3 Applications of Calculus	ME12-4
13 (a) (ii)	1	ME-C3 Applications of Calculus	ME12-4
13 (a) (iii)	2	ME-C3 Applications of Calculus	ME12-4
13 (b) (i)	2	ME-T2 Further Trigonometric Identities	ME11-3
13 (b) (ii)	3	ME-C2 Further Calculus Skills	ME12-4
13 (c)	4	ME-V1 Introduction to Vectors	ME12-2
13 (d)	3	ME-C2 Further Calculus Skills	ME12-7
14 (a)	4	ME-C3 Applications of Calculus	ME12-4
14 (b)	3	ME-C2 Further Calculus Skills	ME12-7

Question	Marks	Content	Syllabus outcomes
14 (c) (i)	1	ME-T1 Inverse Trigonometric Functions ME-T3 Trigonometric Equations	ME11-1, ME12-3
14 (c) (ii)	2	ME-T3 Trigonometric Equations	ME12-3
14 (d)	4	ME-C3 Applications of Calculus ME-V1 Introduction to Vectors	ME12-2, ME12-4