



NSW Education Standards Authority

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Centre Number

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Student Number

**2025** HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Extension 1

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## General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

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## Total marks: 70

### Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

### Section II – 60 marks (pages 7–13)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

## Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the solution to  $|2x + 3| < 5$ ?

- A.  $-4 < x < 1$
- B.  $x < -4$  or  $x > 1$
- C.  $-1 < x < 4$
- D.  $x < -1$  or  $x > 4$

2 The projection of  $\underline{u}$  onto  $\underline{v}$  is given by  $\left( \frac{\underline{u} \cdot \underline{v}}{|\underline{v}|^2} \right) \underline{v}$ .

What is the projection of  $\underline{u} = \underline{i} + 2\underline{j}$  onto  $\underline{v} = 2\underline{i} - 3\underline{j}$ ?

- A.  $-\frac{4}{5}(\underline{i} + 2\underline{j})$
- B.  $-\frac{4}{13}(2\underline{i} - 3\underline{j})$
- C.  $-\frac{4}{\sqrt{5}}(\underline{i} + 2\underline{j})$
- D.  $-\frac{4}{\sqrt{13}}(2\underline{i} - 3\underline{j})$

- 3 Consider the integral  $\int_{-\frac{5}{2}}^{\frac{5}{2}} \left( \frac{1}{25 - x^2} \right) dx$ .

The substitution  $x = 5 \sin \theta$  is applied.

Which of the following is obtained?

- A.  $\frac{1}{5} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cosec} \theta d\theta$
- B.  $\frac{1}{5} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec \theta d\theta$
- C.  $\frac{1}{25} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cosec}^2 \theta d\theta$
- D.  $\frac{1}{25} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^2 \theta d\theta$

- 4 A Bernoulli random variable  $X$  has probability distribution

$$P(x) = \frac{x+1}{3} \text{ for } x = 0, 1.$$

What are the mean and variance of  $X$ ?

- A.  $E(X) = \frac{1}{3}, \quad \operatorname{Var}(X) = \frac{2}{9}$
- B.  $E(X) = \frac{1}{3}, \quad \operatorname{Var}(X) = \frac{2}{3}$
- C.  $E(X) = \frac{2}{3}, \quad \operatorname{Var}(X) = \frac{2}{9}$
- D.  $E(X) = \frac{2}{3}, \quad \operatorname{Var}(X) = \frac{2}{3}$

- 5 How many distinct solutions are there to the equation  $\cos 5x + \sin x = 0$  for  $0 \leq x \leq 2\pi$ ?
- A. 5  
B. 6  
C. 9  
D. 10
- 6 Given that  $a$  is a non-zero constant, which of the following integrals is equal to zero?

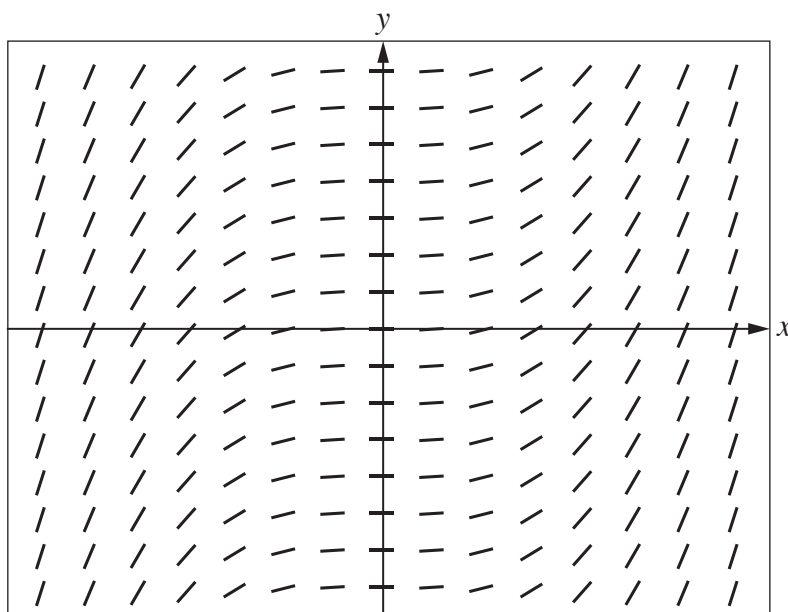
A.  $\int_{-a}^a x \cos^{-1}(x) dx$

B.  $\int_{-a}^a x^2 \cos^{-1}(x) dx$

C.  $\int_{-a}^a x \tan^{-1}(x) dx$

D.  $\int_{-a}^a x^2 \tan^{-1}(x) dx$

- 7 A slope field is shown.



Which of the following could be the differential equation represented by the slope field?

- A.  $\frac{dy}{dx} = x^2$
  - B.  $\frac{dy}{dx} = x^2 + C, C \neq 0$
  - C.  $\frac{dy}{dx} = x^3$
  - D.  $\frac{dy}{dx} = x^3 + C, C \neq 0$
- 8 Points  $A$  and  $B$  have non-zero, non-parallel position vectors  $\underline{a}$  and  $\underline{b}$  respectively.

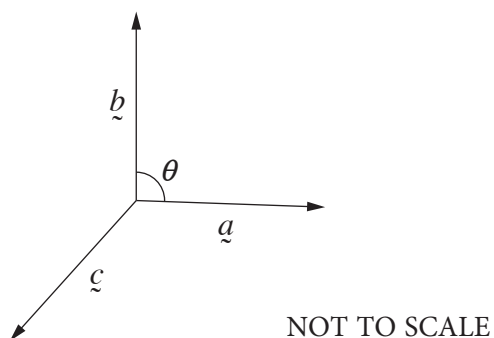
Point  $C$  has position vector  $\underline{c} = 3\underline{a} - 2\underline{b}$ .

The points  $A, B$  and  $C$  lie on the same line.

Which of the following must be true?

- A. Point  $A$  always lies between Points  $B$  and  $C$ .
- B. Point  $B$  always lies between Points  $A$  and  $C$ .
- C. Point  $C$  always lies between Points  $A$  and  $B$ .
- D. The order of the points cannot be determined.

- 9 The vectors  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  have magnitudes 3, 5 and 7 respectively.



Given that  $\underline{a} + \underline{b} + \underline{c} = \underline{0}$ , what is the size of angle  $\theta$  between  $\underline{a}$  and  $\underline{b}$ ?

- A.  $\frac{\pi}{6}$
- B.  $\frac{\pi}{3}$
- C.  $\frac{2\pi}{3}$
- D.  $\frac{5\pi}{6}$
- 10 For the function  $f(x)$ , it is known that  $f(3) = 1$ ,  $f'(3) = 2$  and  $f''(3) = 4$ .

Let  $g(x) = f^{-1}(x)$ .

What is the value of  $g''(1)$ ?

- A.  $\frac{1}{4}$
- B.  $-\frac{1}{4}$
- C.  $-\frac{1}{2}$
- D.  $-1$

## Section II

**60 marks**

**Attempt Questions 11–14**

**Allow about 1 hour and 45 minutes for this section**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use the Question 11 Writing Booklet

(a) Find the inverse function,  $f^{-1}(x)$ , of the function  $f(x) = 1 - \frac{1}{x-2}$ . **2**

(b) Solve  $\sin 2\theta - \sin \theta = 0$  for  $0 \leq \theta \leq \pi$ . **3**

(c) Find  $\int \sin 3x \cos x \, dx$ . **2**

(d) Sketch the graph of  $y = \frac{1}{3} \cos^{-1}(2x)$ . **2**

(e) For what value of  $m$  is the vector  $\begin{pmatrix} 1 \\ m \end{pmatrix}$  parallel to the vector  $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ ? **1**

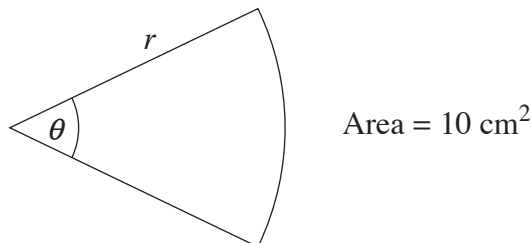
(f) The roots of  $2x^3 + 6x^2 + x - 1 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ . **2**

What is the value of  $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$ ?

(g) Evaluate  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2(3x) \, dx$ . **3**

**Question 12** (14 marks) Use the Question 12 Writing Booklet

- (a) The radius,  $r$  cm, and angle,  $\theta$  radians, of a sector vary in such a way that its area remains a constant  $10 \text{ cm}^2$ . **3**



The angle  $\theta$  is increasing at a constant rate of 2 radians per second.

Find the rate at which the radius is changing when the radius is 4 cm.

- (b) Consider the region bounded by the hyperbola  $y = \frac{1}{x}$ , the  $y$ -axis and the lines  $y = 1$  and  $y = a$  for  $a > 1$ . **2**

Find the volume of the solid of revolution formed when the region is rotated about the  $y$ -axis.

- (c) Prove by mathematical induction that **3**

$$1 \times (1!) + 2 \times (2!) + \cdots + n \times (n!) = (n+1)! - 1$$

for integers  $n \geq 1$ .

- (d) Find the solution of  $\frac{dy}{dx} = \sqrt{(2-y)(2+y)}$ , given that  $y = 1$  when  $x = 0$ . **3**

- (e) (i) Express  $\sqrt{3} \sin x - \cos x$  in the form  $2 \sin(x - \alpha)$ , where  $0 < \alpha < \frac{\pi}{2}$ . **1**
- (ii) Hence, or otherwise, solve  $\sqrt{3} \sin x = \cos x + 1$ , where  $0 \leq x \leq 2\pi$ . **2**

**Question 13** (16 marks) Use the Question 13 Writing Booklet

- (a) It is given that  $\frac{dy}{dx} = \frac{5}{y}$  and  $y = -4$  when  $x = 0$ . **3**

Find  $y$  as a function of  $x$ .

- (b) Eight guests are to be seated at a round table. If two of these guests refuse to sit next to each other, how many seating arrangements are possible? **2**

- (c) At time  $t$ , a particle has position vector  $\underline{r}(t) = t\mathbf{i} + \frac{t^2}{9}\mathbf{j}$ , velocity vector  $\underline{v}(t)$  and acceleration vector  $\underline{a}(t)$ . **4**

Find the time when the angle between  $\underline{v}(t)$  and  $\underline{a}(t)$  is  $\frac{\pi}{4}$ .

- (d) A bag contains counters, some of which are green. **4**

One hundred trials of an experiment are run. In each trial, one counter is selected from the bag at random and its colour noted. The counter is returned to the bag after each trial.

Let  $X$  be the random variable representing the number of times that a green counter is selected.

Given that  $E(X) = 20$  and  $P(X \geq k) = 0.0668$ , find the value of  $k$ . You may use the standard normal approximation and the information on page 13.

**Question 13 continues on page 10**

Question 13 (continued)

- (e) (i) The Pascal's triangle relation can be expressed as **1**

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}. \quad (\text{Do NOT prove this.})$$

Show that  $\binom{m}{R} = \binom{m+1}{R+1} - \binom{m}{R+1}.$

- (ii) Hence, or otherwise, prove that **2**

$$\binom{2000}{2000} + \binom{2001}{2000} + \binom{2002}{2000} + \cdots + \binom{2050}{2000} = \binom{2051}{2001}.$$

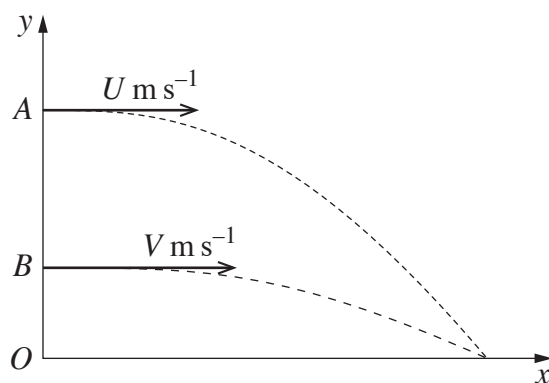
**End of Question 13**

**Question 14** (15 marks) Use the Question 14 Writing Booklet

- (a) Prove that the product of any seven distinct factors of 60 must be a multiple of 60. **2**

- (b) Points  $A$  and  $B$  lie vertically above the origin. Point  $A$  is higher than point  $B$  such that  $\frac{OA}{OB} = k$ , where  $k > 1$ . **4**

A particle is projected horizontally from point  $A$  with velocity  $U \text{ m s}^{-1}$ . After  $T$  seconds, another particle is projected horizontally from point  $B$  with velocity  $V \text{ m s}^{-1}$ . The two particles land on the ground in the same place.



Show that the ratio  $\frac{V}{U}$  depends only on  $k$ .

- (c) The hands of an analogue clock are  $OA$  and  $OB$ , **3**  
where  $A$  is  $\left(\sin\left(\frac{\pi t}{360}\right), \cos\left(\frac{\pi t}{360}\right)\right)$ ,  $B$  is  $\left(2\sin\left(\frac{\pi t}{30}\right), 2\cos\left(\frac{\pi t}{30}\right)\right)$ ,  
 $O$  is the origin, and  $t \geq 0$  is the number of minutes past midnight.

Find the values of  $t$  when the hands are perpendicular for the first and second time after midnight. Give your answers to 3 decimal places.

**Question 14 continues on page 12**

Question 14 (continued)

- (d) The function  $f(x)$  is defined by  $f(x) = \cos^{-1}(\sin x)$  in the domain  $(0, \pi)$ . **3**

Find  $f'(x)$  for those values of  $x$  where it is defined.

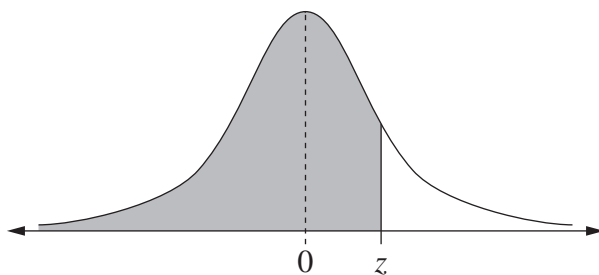
- (e) It is given that  $\tan \alpha$ ,  $\tan \beta$  and  $\tan \gamma$  are the three real roots of the polynomial equation  $x^3 + bx^2 + cx - 1 + b + c = 0$ , where  $b$  and  $c$  are real numbers and  $c \neq 1$ . **3**

Find the smallest positive value of  $\alpha + \beta + \gamma$ .

**End of paper**

Use the information below to answer Question 13 (d).

**Table of values  $P(Z \leq z)$  for the normal distribution  $N(0, 1)$**



| $z$ | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |

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Mathematics Advanced  
Mathematics Extension 1  
Mathematics Extension 2

**REFERENCE SHEET**

**Measurement**

**Length**

$$l = \frac{\theta}{360} \times 2\pi r$$

**Area**

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

**Surface area**

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

**Volume**

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

**Functions**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For  $ax^3 + bx^2 + cx + d = 0$ :

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

**Relations**

$$(x - h)^2 + (y - k)^2 = r^2$$

**Financial Mathematics**

$$A = P(1 + r)^n$$

**Sequences and series**

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

**Logarithmic and Exponential Functions**

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

## Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

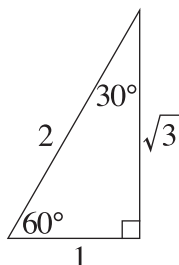
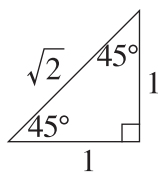
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



## Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

## Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

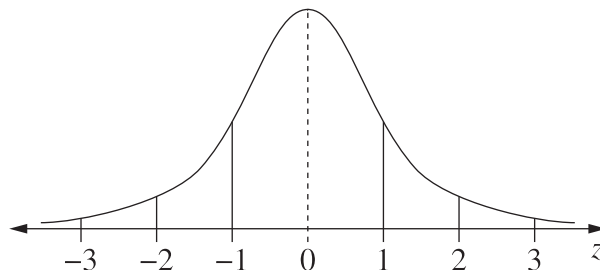
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

## Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score  
less than  $Q_1 - 1.5 \times IQR$   
or  
more than  $Q_3 + 1.5 \times IQR$

## Normal distribution



- approximately 68% of scores have  $z$ -scores between  $-1$  and  $1$
- approximately 95% of scores have  $z$ -scores between  $-2$  and  $2$
- approximately 99.7% of scores have  $z$ -scores between  $-3$  and  $3$

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

## Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

## Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

## Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

## Differential Calculus

### Function

### Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

## Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where  $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where  $a = x_0$  and  $b = x_n$

## Combinatorics

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

---

## Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1 x_2 + y_1 y_2,$$

$$\text{where } \underline{u} = x_1 \underline{i} + y_1 \underline{j}$$

$$\text{and } \underline{v} = x_2 \underline{i} + y_2 \underline{j}$$

$$\underline{r} = a + \lambda \underline{b}$$

---

## Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= r e^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n (\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

---

## Mechanics

$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

# 2025 HSC Mathematics Extension 1 Marking Guidelines

## Section I

### Multiple-choice Answer Key

| Question | Answer |
|----------|--------|
| 1        | A      |
| 2        | B      |
| 3        | B      |
| 4        | C      |
| 5        | D      |
| 6        | D      |
| 7        | A      |
| 8        | A      |
| 9        | B      |
| 10       | C      |

## Section II

### Question 11 (a)

| Criteria                                                  | Marks |
|-----------------------------------------------------------|-------|
| • Provides correct solution                               | 2     |
| • Rearranges to make $x$ the subject, or equivalent merit | 1     |

**Sample answer:**

Let  $y = f(x)$ . Then

$$y = 1 - \frac{1}{x-2}$$

$$y - 1 = -\frac{1}{x-2}$$

$$x - 2 = -\frac{1}{y-1}$$

$$x = 2 - \frac{1}{y-1}$$

Swap  $y$  and  $x$  for the inverse function,  $y = 2 - \frac{1}{x-1}$ .

$$\therefore f^{-1}(x) = 2 - \frac{1}{x-1}.$$

**Question 11 (b)**

| Criteria                              | Marks |
|---------------------------------------|-------|
| • Provides correct solution           | 3     |
| • Finds one correct value of $\theta$ | 2     |
| • Applies double angle formula        | 1     |

**Sample answer:**

$$\sin 2\theta - \sin \theta = 0$$

$$2 \sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (2 \cos \theta - 1) = 0$$

$$\text{either } \sin \theta = 0 \quad \text{or} \quad 2 \cos \theta - 1 = 0$$

$$\theta = 0, \pi \quad \cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$$\theta = 0, \frac{\pi}{3}, \pi$$

**Question 11 (c)**

| Criteria                                                    | Marks |
|-------------------------------------------------------------|-------|
| • Provides correct solution                                 | 2     |
| • Applies trigonometric product as sum, or equivalent merit | 1     |

**Sample answer:**

$$\int \sin 3x \cos x \, dx$$

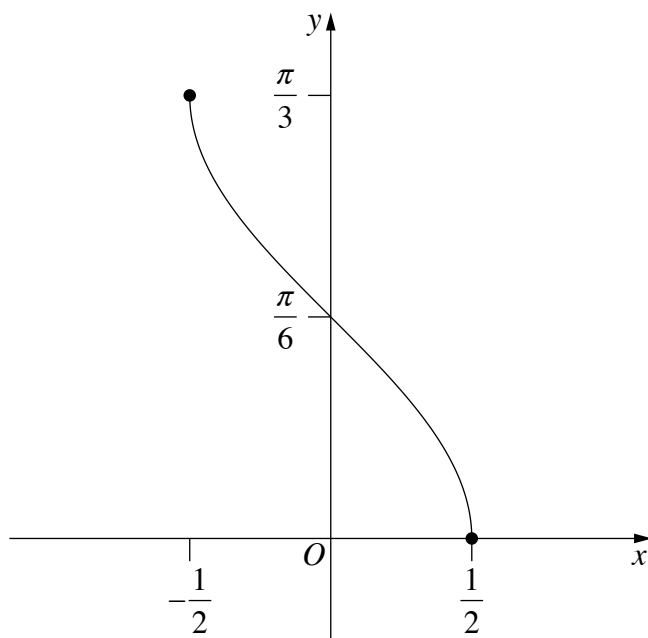
$$= \frac{1}{2} \int (\sin(4x) + \sin(2x)) \, dx$$

$$= -\frac{\cos(4x)}{8} - \frac{\cos(2x)}{4} + C$$

### Question 11 (d)

| Criteria                                                  | Marks |
|-----------------------------------------------------------|-------|
| • Provides correct graph                                  | 2     |
| • Provides correct domain and range, or equivalent method | 1     |

**Sample answer:**



### Question 11 (e)

| Criteria                  | Marks |
|---------------------------|-------|
| • Provides correct answer | 1     |

**Sample answer:**

$$\begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

So  $m = 3$  makes the vectors parallel.

### Question 11 (f)

| Criteria                                                                                                            | Marks |
|---------------------------------------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                                                         | 2     |
| • Arranges over a common denominator<br>OR<br>• Applies formulas for sums or products of roots, or equivalent merit | 1     |

**Sample answer:**

$$\begin{aligned}
 & \frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} \\
 &= \frac{\gamma}{\alpha\beta\gamma} + \frac{\beta}{\alpha\beta\gamma} + \frac{\alpha}{\alpha\beta\gamma} \\
 &= \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} \\
 &= \frac{-6}{\frac{1}{2}} \\
 &= -6
 \end{aligned}$$

### Question 11 (g)

| Criteria                      | Marks |
|-------------------------------|-------|
| • Provides correct solution   | 3     |
| • Integrates expression       | 2     |
| • Uses compound angle formula | 1     |

**Sample answer:**

$$\begin{aligned}
 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2(3x) dx &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1 + \cos(6x)}{2} dx \\
 &= \frac{1}{2} \left[ x + \frac{\sin(6x)}{6} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\
 &= \frac{1}{2} \left( \frac{\pi}{3} + \frac{\sin(2\pi)}{6} \right) - \frac{1}{2} \left( \frac{\pi}{6} + \frac{\sin(\pi)}{6} \right) \\
 &= \frac{\pi}{6} + 0 - \frac{\pi}{12} - 0 \\
 &= \frac{\pi}{12}
 \end{aligned}$$

## Question 12 (a)

| Criteria                                                                                | Marks |
|-----------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                             | 3     |
| • Finds $\frac{dr}{dt}$ in terms of $r$ , or equivalent merit                           | 2     |
| • Finds $\frac{d\theta}{dr}$<br>OR<br>• Attempts to use chain rule, or equivalent merit | 1     |

**Sample answer:**

$$\text{Area} = \frac{1}{2}r^2\theta = 10$$

$$\therefore \theta = \frac{20}{r^2}$$

$$\therefore \frac{d\theta}{dr} = -\frac{40}{r^3}$$

$$\frac{d\theta}{dt} = 2 \text{ (given)}$$

$$\begin{aligned} \therefore \frac{dr}{dt} &= \frac{dr}{d\theta} \times \frac{d\theta}{dt} \\ &= -\frac{r^3}{40} \times 2 \\ &= -\frac{r^3}{20} \end{aligned}$$

When  $r = 4$ ,

$$\frac{dr}{dt} = -\frac{4^3}{20}$$

$$= -\frac{16}{5}$$

$$= -3.2 \text{ cm s}^{-1}$$

(That is radius is *decreasing* at a rate of  $3.2 \text{ cm s}^{-1}$ .)

## Question 12 (b)

| Criteria                                         | Marks |
|--------------------------------------------------|-------|
| • Provides correct solution                      | 2     |
| • Provides correct integral, or equivalent merit | 1     |

### Sample answer:

The volume is

$$V = \pi \int_1^a \left(\frac{1}{y}\right)^2 dy = \pi \left[-\frac{1}{y}\right]_1^a = \pi - \frac{\pi}{a}$$

## Question 12 (c)

| Criteria                                             | Marks |
|------------------------------------------------------|-------|
| • Provides correct proof                             | 3     |
| • Uses inductive assumption, or equivalent merit     | 2     |
| • Establishes initial statement, or equivalent merit | 1     |

### Sample answer:

When  $n = 1$

$$\text{LHS} = 1 \times (1!) = 1$$

$$\text{RHS} = (1 + 1)! - 1 = 2 - 1 = 1$$

$\therefore$  True when  $n = 1$

Assume true for  $n = k$ , that is

$$1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!) = (k + 1)! - 1$$

Now prove true for  $n = k + 1$ , that is prove

$$1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!) + (k + 1) \times ((k + 1)!) = (k + 2)! - 1$$

$$\text{LHS} = [1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!)] + (k + 1) \times ((k + 1)!)$$

$$= [(k + 1)! - 1] + (k + 1) \times ((k + 1)!) \text{ by assumption}$$

$$= [1 + (k + 1)](k + 1)! - 1$$

$$= (k + 2)! - 1 = \text{RHS.}$$

Hence the statement is also true for  $n = k + 1$ . So the statement is true for all positive integers by mathematical induction.

## Question 12 (d)

| Criteria                                                                                       | Marks |
|------------------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                                    | 3     |
| • Finds the general solution $x = \sin^{-1}\left(\frac{y}{2}\right) + C$ , or equivalent merit | 2     |
| • Separates variables, or equivalent merit                                                     | 1     |

**Sample answer:**

$$\begin{aligned}\frac{dy}{dx} &= \sqrt{(2-y)(2+y)} \\ &= \sqrt{4-y^2}\end{aligned}$$

$$\therefore \int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$x = \sin^{-1}\left(\frac{y}{2}\right) + C$$

When  $x = 0$ ,  $y = 1$

$$\therefore 0 = \sin^{-1}\left(\frac{1}{2}\right) + C$$

$$= \frac{\pi}{6} + C$$

$$\therefore C = -\frac{\pi}{6}$$

$$\therefore x = \sin^{-1}\left(\frac{y}{2}\right) - \frac{\pi}{6} \quad \text{or} \quad y = 2 \sin\left(x + \frac{\pi}{6}\right)$$

**Question 12 (e) (i)**

| Criteria                  | Marks |
|---------------------------|-------|
| • Provides correct answer | 1     |

**Sample answer:**

$$2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right) = 2\sin\left(x - \frac{\pi}{6}\right)$$

**Question 12 (e) (ii)**

| Criteria                                                      | Marks |
|---------------------------------------------------------------|-------|
| • Provides correct answer                                     | 2     |
| • Uses part (i) to simplify the equation, or equivalent merit | 1     |

**Sample answer:**

$$\sqrt{3}\sin x = \cos x + 1$$

$$\sqrt{3}\sin x - \cos x = 1$$

$$2\sin\left(x - \frac{\pi}{6}\right) = 1$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$x - \frac{\pi}{6} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}.$$

$$\text{So } x = \frac{2\pi}{6} = \frac{\pi}{3} \text{ or } x = \frac{6\pi}{6} = \pi.$$

**Question 13 (a)**

| Criteria                                                              | Marks |
|-----------------------------------------------------------------------|-------|
| • Provides correct solution                                           | 3     |
| • Obtains $y^2 = 10x + 16$ , or equivalent merit                      | 2     |
| • Integrates to obtain $\frac{y^2}{2} = 5x + C$ , or equivalent merit | 1     |

**Sample answer:**

Since  $\frac{dy}{dx} = \frac{5}{y}$ ,

$$\therefore \int y \, dy = \int 5 \, dx$$

$$\frac{y^2}{2} = 5x + C$$

When  $x = 0$ ,  $y = -4$ .

$$\therefore \frac{16}{2} = 0 + C$$

$$C = 8$$

ie  $\frac{y^2}{2} = 5x + 8$

$$y^2 = 10x + 16$$

$$y = \pm\sqrt{10x + 16}$$

When  $x = 0$ ,  $y = -4$

$$\therefore y = -\sqrt{10x + 16}$$

**Question 13 (b)**

| Criteria                                                                             | Marks |
|--------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                          | 2     |
| • Obtains 7! unrestricted seating arrangements at a round table, or equivalent merit | 1     |

**Sample answer:**

There are 7! possible seating arrangements at a round table without restriction.

There are  $6! \times 2$  possible seating arrangements where the two people sit together.

Hence, there are  $7! - 6! \times 2 = 3600$  seating arrangements where they do not sit together.

### Question 13 (c)

| Criteria                                                                                                   | Marks |
|------------------------------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                                                | 4     |
| • Equates the two expressions of $\underline{v} \cdot \underline{a}$ in terms of $t$ , or equivalent merit | 3     |
| • Obtains expression $\underline{v} \cdot \underline{a} = \frac{4t}{81}$ , or equivalent merit             | 2     |
| • Obtains expression for $\underline{v}$                                                                   | 1     |

**Sample answer:**

Given  $\underline{r}(t) = t\underline{i} + \frac{t^2}{9}\underline{j}$

then  $\underline{v}(t) = \underline{i} + \frac{2t}{9}\underline{j}$

and  $\underline{a}(t) = 0\underline{i} + \frac{2}{9}\underline{j}$

Using scalar product,

$$\underline{v} \cdot \underline{a} = \frac{4t}{81}$$

and  $\underline{v} \cdot \underline{a} = |\underline{v}| \cdot |\underline{a}| \cos \frac{\pi}{4}$

$$= \sqrt{1 + \frac{4t^2}{81}} \cdot \sqrt{\frac{4}{81}} \cdot \frac{1}{\sqrt{2}}$$

Equating,

$$\frac{4t}{81} = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{4t^2}{81}} \cdot \frac{2}{9}$$

$$\frac{2t}{9} = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{4t^2}{81}}$$

Squaring,

$$\frac{4t^2}{81} = \frac{1}{2} \left( 1 + \frac{4t^2}{81} \right)$$

$$\frac{8t^2}{81} = 1 + \frac{4t^2}{81}$$

$$\frac{4t^2}{81} = 1$$

$$t^2 = \frac{81}{4}$$

$$t = \pm \frac{9}{2}$$

$$t = \frac{9}{2}, \text{ since } t > 0.$$

### Question 13 (d)

| Criteria                                                                                                      | Marks |
|---------------------------------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                                                   | 4     |
| • Uses $Z = \frac{X - 20}{4}$ as the normal approximation and finds $P(X < k) = 0.9332$ , or equivalent merit | 3     |
| • Attempts to use $Z = \frac{X - 20}{4}$ as the normal approximation, or equivalent merit                     | 2     |
| • Obtains $p = \frac{1}{5}$ or finds $P(Z \leq 1.5) = 0.9332$ , or equivalent merit                           | 1     |

**Sample answer:**

If  $p$  is the probability that a counter is green then  $E(X) = np = 100p = 20$  so  $p = \frac{1}{5}$ .

The variance is  $\sigma^2 = 100 \times p \times (1 - p)$   

$$= 100 \times \frac{1}{5} \times \frac{4}{5} = 16.$$

The probability of selecting at least  $k$  green counters is

$$P(X \geq k) = P\left(\frac{X - 20}{4} \geq \frac{k - 20}{4}\right) = P\left(Z \geq \frac{k - 20}{4}\right)$$

where  $Z = \frac{X - 20}{4} \sim N(0, 1)$ .

Since  $P(X \geq k) = 0.0668$  then  $P(X < k) = 1 - 0.0668 = 0.9332$  and so

$$P\left(Z \leq \frac{k - 20}{4}\right) = 0.9332 = P(Z \leq 1.5) \text{ by table.}$$

So  $1.5 = \frac{k - 20}{4}$ , hence  $k = 26$ .

### Question 13 (e) (i)

| Criteria                    | Marks |
|-----------------------------|-------|
| • Provides correct solution | 1     |

**Sample answer:**

Re-arranging

$$\binom{n-1}{r-1} = \binom{n}{r} - \binom{n-1}{r}$$

Let  $r = R + 1$ ,  $n = m + 1$ . Therefore

$$\binom{m}{R} = \binom{m+1}{R+1} - \binom{m}{R+1}$$

### Question 13 (e) (ii)

| Criteria                                | Marks |
|-----------------------------------------|-------|
| • Provides correct solution             | 2     |
| • Applies part (i), or equivalent merit | 1     |

**Sample answer:**

Therefore,

$$\begin{aligned}
 \text{LHS} &= \binom{2000}{2000} + \left[ \binom{2002}{2001} - \binom{2001}{2001} \right] && \text{Using part (i)} \\
 &+ \left[ \binom{2003}{2001} - \binom{2002}{2001} \right] \\
 &+ \cdots + \left[ \binom{2051}{2001} - \binom{2050}{2001} \right] \\
 &= \binom{2000}{2000} - \binom{2001}{2001} + \binom{2051}{2001} \\
 &= 1 - 1 + \binom{2051}{2001} \\
 &= \binom{2051}{2001} \\
 &= \text{RHS}
 \end{aligned}$$

**Question 14 (a)**

| Criteria                                             | Marks |
|------------------------------------------------------|-------|
| • Provides correct proof                             | 2     |
| • Creates 6 appropriate buckets, or equivalent merit | 1     |

**Sample answer:**

Create 6 buckets  $[1, 60]$ ,  $[2, 30]$ ,  $[3, 20]$ ,  $[4, 15]$ ,  $[5, 12]$ ,  $[6, 10]$ .

If we choose 7 numbers, then by the pigeonhole principle two numbers must be chosen from the same bucket. These two numbers multiply to 60. Hence, the product of all 7 numbers is a multiple of 60.

### Question 14 (b)

| Criteria                                                                              | Marks |
|---------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                           | 4     |
| • Equates $x$ -components, and equates $y$ -components with zero, or equivalent merit | 3     |
| • Equates $x$ -components, or equates $y$ -components with zero, or equivalent merit  | 2     |
| • Establishes displacement in either direction for either particle                    | 1     |

**Sample answer:**

Let  $t$  be the time after the first particle is projected, let the time difference be  $T$  seconds and let  $OB = h$  metres so  $OA = kh$  metres.

$\therefore$  For the particle projected from  $A$ , the equations of motion are  $x = Ut$  and  $y = -\frac{1}{2}gt^2 + kh$  and for the particle projected from  $B$ , the equations of motion are  $x = V(t - T)$  and  $y = -\frac{1}{2}g(t - T)^2 + h$ .

The particles land in the same place, so

$$x = Ut = V(t - T)$$

$$\therefore \frac{V}{U} = \frac{t}{t - T}$$

Also, they land on the ground, so

$$y = -\frac{1}{2}gt^2 + kh = 0 \quad \text{and} \quad y = -\frac{1}{2}g(t - T)^2 + h = 0$$

$$\therefore kh = \frac{1}{2}gt^2 \quad \therefore h = \frac{1}{2}g(t - T)^2$$

$$\frac{2kh}{g} = t^2 \quad \frac{2h}{g} = (t - T)^2$$

$$\therefore t^2 = k(t - T)^2$$

$$\frac{t^2}{(t - T)^2} = k$$

$$\therefore \frac{V}{U} = \frac{t}{t - T}$$

$$= \sqrt{k} \quad \text{since } t > T > 0.$$

### Question 14 (c)

| Criteria                                                      | Marks |
|---------------------------------------------------------------|-------|
| • Provides correct solution                                   | 3     |
| • Attempts to use compound angle formula, or equivalent merit | 2     |
| • Uses the scalar product, or equivalent merit                | 1     |

**Sample answer:**

The hands are perpendicular when the scalar product is zero, which leads to

$$\sin\left(\frac{\pi t}{360}\right)\sin\left(\frac{\pi t}{30}\right) + \cos\left(\frac{\pi t}{360}\right)\cos\left(\frac{\pi t}{30}\right) = 0,$$

which simplifies to  $\cos\left(\frac{\pi t}{30} - \frac{\pi t}{360}\right) = \cos\left(\frac{11\pi t}{360}\right) = 0$ .

This means  $\frac{11\pi t}{360} = \dots - \frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$ .

The first time that the hands are perpendicular is when  $\frac{11\pi t}{360} = \frac{\pi}{2}$ ,

which gives  $t = \frac{360 \times \pi}{2 \times 11 \times \pi} \approx 16.364$  minutes.

The second time that the hands are perpendicular is when  $\frac{11\pi t}{360} = \frac{3\pi}{2}$ ,

which gives  $t = \frac{3 \times 360 \times \pi}{2 \times 11 \times \pi} \approx 49.091$  minutes.

### Question 14 (d)

| Criteria                                                                       | Marks |
|--------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                    | 3     |
| • Recognises $f'(x)$ is undefined at $x = \frac{\pi}{2}$ , or equivalent merit | 2     |
| • States $f'(x) = 1$ or $f'(x) = -1$ or $f'(x) = \pm 1$ , or equivalent merit  | 1     |

**Sample answer:**

$$\begin{aligned}
 f(x) &= \cos^{-1}(\sin x) \\
 \therefore f'(x) &= \frac{-\cos x}{\sqrt{1 - \sin^2 x}} \\
 &= \frac{-\cos x}{|\cos x|} \\
 &= \pm 1
 \end{aligned}$$

$$\text{For } 0 < x < \frac{\pi}{2}, \quad \cos x > 0 \quad \therefore f'(x) = -1$$

$$\text{For } \frac{\pi}{2} < x < \pi, \quad \cos x < 0 \quad \therefore f'(x) = 1$$

When  $x = \frac{\pi}{2}$ ,  $f'(x)$  is undefined.

$$\therefore f'(x) = -1 \quad \text{for } x \in \left(0, \frac{\pi}{2}\right)$$

$$\text{and } f'(x) = 1 \quad \text{for } x \in \left(\frac{\pi}{2}, \pi\right)$$

### Question 14 (e)

| Criteria                                                                           | Marks |
|------------------------------------------------------------------------------------|-------|
| • Provides correct solution                                                        | 3     |
| • Applies $\tan(A + B)$ formula, or equivalent merit                               | 2     |
| • Uses relation between roots and coefficients of polynomials, or equivalent merit | 1     |

**Sample answer:**

$$\tan \alpha + \tan \beta + \tan \gamma = -b$$

$$\tan \alpha \tan \beta + \tan \alpha \tan \gamma + \tan \beta \tan \gamma = c$$

$$\tan \alpha \tan \beta \tan \gamma = 1 - b - c$$

$$\tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan(\beta + \gamma)}{1 - \tan \alpha \tan(\beta + \gamma)}$$

$$= \frac{\tan \alpha + \frac{\tan \beta + \tan \gamma}{1 - \tan \beta \tan \gamma}}{1 - \tan \alpha \left( \frac{\tan \beta + \tan \gamma}{1 - \tan \beta \tan \gamma} \right)}$$

$$= \frac{\tan \alpha - \tan \alpha \tan \beta \tan \gamma + \tan \beta + \tan \gamma}{1 - \tan \beta \tan \gamma - \tan \alpha \tan \beta - \tan \alpha \tan \gamma}$$

$$= \frac{(\tan \alpha + \tan \beta + \tan \gamma) - \tan \alpha \tan \beta \tan \gamma}{1 - (\tan \alpha \tan \beta + \tan \alpha \tan \gamma + \tan \beta \tan \gamma)}$$

$$= \frac{-b - (1 - b - c)}{1 - c}$$

$$= -\frac{(1 - c)}{1 - c}$$

$$= -1$$

$$\therefore \alpha + \beta + \gamma = \dots, -\frac{\pi}{4}, \frac{3\pi}{4}, \dots$$

Smallest positive value is  $\frac{3\pi}{4}$ .

# 2025 HSC Mathematics Extension 1 Mapping Grid

## Section I

| Question | Marks | Content                               | Syllabus outcomes |
|----------|-------|---------------------------------------|-------------------|
| 1        | 1     | ME-F1 Further Work with Functions     | ME11-2            |
| 2        | 1     | ME-V1 Introduction to Vectors         | ME12-2            |
| 3        | 1     | ME-C2 Further Calculus Skills         | ME12-1            |
| 4        | 1     | ME-S1 The Binomial Distribution       | ME12-5            |
| 5        | 1     | ME-T3 Trigonometric Equations         | ME12-3            |
| 6        | 1     | ME-T1 Inverse Trigonometric Functions | ME11-3            |
| 7        | 1     | ME-C3 Applications of Calculus        | ME12-4            |
| 8        | 1     | ME-V1 Introduction to Vectors         | ME12-2            |
| 9        | 1     | ME-V1 Introduction to Vectors         | ME12-2            |
| 10       | 1     | ME-C2 Further Calculus Skills         | ME12-4            |

## Section II

| Question    | Marks | Content                                | Syllabus outcomes |
|-------------|-------|----------------------------------------|-------------------|
| 11 (a)      | 2     | ME-F1 Further Work with Functions      | ME11-1            |
| 11 (b)      | 3     | ME-T3 Trigonometric Equations          | ME12-3            |
| 11 (c)      | 2     | ME-T2 Further Trigonometric Identities | ME11-3            |
| 11 (d)      | 2     | ME-T1 Inverse Trigonometric Functions  | ME11-3            |
| 11 (e)      | 1     | ME-V1 Introduction to Vectors          | ME12-2            |
| 11 (f)      | 2     | ME-F2 Polynomials                      | ME11-2            |
| 11 (g)      | 3     | ME-C2 Further Calculus Skills          | ME12-1            |
| 12 (a)      | 3     | ME-C1 Rates of Change                  | ME11-4            |
| 12 (b)      | 2     | ME-C3 Applications of Calculus         | ME12-4            |
| 12 (c)      | 3     | ME-P1 Proof by Mathematical Induction  | ME12-1            |
| 12 (d)      | 3     | ME-C3 Applications of Calculus         | ME12-4            |
| 12 (e) (i)  | 1     | ME-T3 Trigonometric Equations          | ME12-3            |
| 12 (e) (ii) | 2     | ME-T3 Trigonometric Equations          | ME12-3            |
| 13 (a)      | 3     | ME-C3 Applications of Calculus         | ME12-4            |
| 13 (b)      | 2     | ME-A1 Working with Combinatorics       | ME11-5            |
| 13 (c)      | 4     | ME-V1 Introduction to Vectors          | ME12-2            |
| 13 (d)      | 4     | ME-S1 The Binomial Distribution        | ME12-5            |
| 13 (e) (i)  | 1     | ME-A1 Working with Combinatorics       | ME11-5            |
| 13 (e) (ii) | 2     | ME-A1 Working with Combinatorics       | ME11-5            |
| 14 (a)      | 2     | ME-A1 Working with Combinatorics       | ME11-5            |
| 14 (b)      | 4     | ME-V1 Introduction to Vectors          | ME12-2            |

| Question | Marks | Content                                                        | Syllabus outcomes |
|----------|-------|----------------------------------------------------------------|-------------------|
| 14 (c)   | 3     | ME-V1 Introduction to Vectors<br>ME-T3 Trigonometric Equations | ME12-2, ME12-3    |
| 14 (d)   | 3     | ME-C2 Further Calculus Skills                                  | ME12-1            |
| 14 (e)   | 3     | ME-F2 Polynomials<br>ME-T3 Trigonometric Equations             | ME12-3            |