Student Number

Barker College

2022 YEAR 12 ASSESSMENT BLOCK

Mathematics Extension 1

Staff Involved:
- ESP ● AXD ● KJL ● PDJ*
- JZT ● GPF ● GDH

Thursday 7th April
8:30 am
110 copies

TOPICS COVERED:		
Combinatorics	Binomial Theorem	Further Rates
Trigonometry	Further Graphs	Polynomials
	Induction	

General Instructions:
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- Write your student number at the top of the page

Total marks: 80

Section I - 8 marks (pages 2 - 5)
- Attempt Multiple Choice Questions 1 - 8
- Allow about 15 minutes for this section

Section II - 72 marks (pages 6 - 13)
- Attempt Questions 9 - 14
- Show all necessary working
- Allow about 1 hour and 45 minutes for this section

Section I

8 Marks
Attempt Questions 1-8
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-8.

---

1 Consider the polynomial equation $x^3 - 3x^2 - 1 = 0$, with roots α, β and γ.

Which of the following is the value of $\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$?

(A) -3

(B) 3

(C) -2

(D) 2

2 In a group of 20 people, 7 of them can drive a manual car. If people are chosen at random, what is the least number of people you would need to choose to ensure you have picked at least two people who can drive a manual car?

(A) 9

(B) 13

(C) 15

(D) 17

3 Consider the function defined parametrically by: $x = \sqrt{t-1}$, $y = t$
What is the range of the function?

(A) $y \in (-\infty, \infty)$

(B) $y \in [0, \infty)$

(C) $y \in [1, \infty)$

(D) $y \in [-1, \infty)$

4 What is the value of $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$?

(A) $\frac{4\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $-\frac{\pi}{3}$

5 The angle θ satisfies $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$.
What is the value of $\sin 2\theta$?

(A) $\frac{10}{13}$

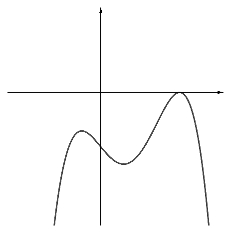
(B) $-\frac{10}{13}$

(C) $\frac{120}{169}$

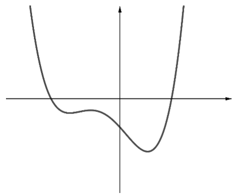
(D) $-\frac{120}{169}$

6 A polynomial $P(x)$ of degree 4 has one zero of multiplicity 2 and is divisible by $x^2 + x + 4$. Which could be the graph of $P(x)$?

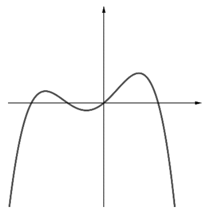
(A)



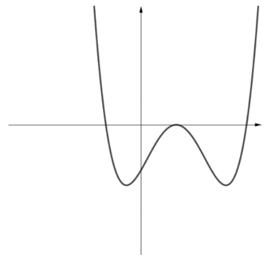
(B)



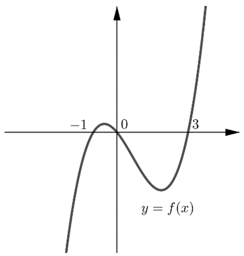
(C)

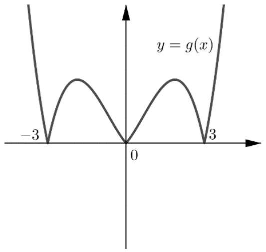


(D)



7 Consider the graphs of $y = f(x)$ and $y = g(x)$





Which of the following describes the transformation relating $f(x)$ and $g(x)$?

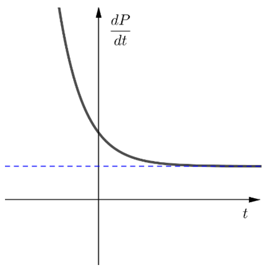
(A) $g(x) = f(|x|)$

(B) $|g(x)| = f(x)$

(C) $g(x) = |f(|x|)|$

(D) $|g(x)| = |f(x)|$

8 The graph of $\dfrac{dP}{dt}$ is shown.



It is also known that $\dfrac{dP}{dr} = (kr)^2$.

Which of the following is a true statement?

(A) $\dfrac{dr}{dt} > 0$

(B) $\dfrac{dr}{dt} \geq 0$

(C) $\dfrac{dr}{dt} \leq 0$

(D) $\dfrac{dr}{dt} < 0$

Section II

72 Marks
Attempt Questions 9-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 9-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (12 Marks) Use the Question 9 Writing Booklet

(a) (i) Sketch the graph of $y = x(x-3)(x+1)$ without using calculus. **1**

(ii) Hence or otherwise, solve the inequality $\dfrac{x(x+1)}{x-3} \geq 0$ **2**

(b) For an upcoming tennis match, 5 ball boys and 5 ball girls will be chosen from a group of 10 boys and 10 girls.

(i) In how many ways can the selection be made? **1**

(ii) Brother and sister Aidan and Amber are among those who may be chosen. In how many ways can the selection be made so that one of them is chosen, but not both? **2**

(c) Given that $A = 4t$ and $r = \sqrt{A}$, find $\dfrac{dr}{dt}$ when $A = 4$. **2**

(d) Consider the function $f(x) = \sin^{-1}\left(\dfrac{x}{2}\right) - \pi$.

(i) State the domain and range of $f(x)$. **2**

(ii) Sketch the graph of $y = f(x)$, clearly indicating any intercept(s). **2**

Question 10 (12 Marks) Use the Question 10 Writing Booklet

(a) (i) Prove the identity: $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$ **2**

(ii) Hence or otherwise, find the value of $\tan\left(\dfrac{\pi}{8}\right)$ in simplest exact form. **2**

(b) Consider the function $y = 3\cos x + \sqrt{3}\sin x$, for $0 \le x \le 2\pi$.

(i) Express the function in the form $R\cos(x-\alpha)$. **2**

(ii) Hence or otherwise, determine the number of solutions for the equation $3\cos x + \sqrt{3}\sin x = 2\sqrt{3}$ in the domain $0 \le x \le 2\pi$. **1**

Answer Question 10 (c) on the additional answer page at the end of the booklet. Detach and hand it in with your booklet for Question 10.

Question 11 (12 Marks) Use the Question 11 Writing Booklet

(a) Use mathematical induction to prove that $2^n + (-1)^{n+1}$ is divisible by 3 for integers $n \ge 1$. **3**

(b) Consider the function $f(x) = (x-1)^2 - 5$.

(i) State the largest positive domain for which $f(x)$ is a 1-1 function. **1**

(ii) Let $g(x) = f(x)$ with the domain specified in (i). State the domain and range of $y = g^{-1}(x)$. **1**

(iii) Find the inverse function $y = g^{-1}(x)$. **2**

(iv) Graph $y = g^{-1}(x)$, clearly showing the domain and range obtained in part (ii). **1**

(v) Determine the point(s) of intersection of $y = g^{-1}(x)$ and $y = f(x)$. **2**

(c) In a course of 300 students, there are 10 different tutorial classes available each week. Each student must choose 2 of these tutorial classes to enrol in. **2**

Show that there are at least 7 students who will choose exactly the same 2 tutorial classes.

Question 12 (12 Marks) Use the Question 12 Writing Booklet

(a) A representative debating team is to be formed by choosing a total of three debaters from two schools.
The three debaters are chosen from among 5 students in school A, and 7 students in school B.

(i) How many different debating teams can be formed if at least one student is chosen from each school? **2**

(ii) Instead, the three students are randomly chosen. **2**
Given that all three students chosen are from the same school, what is the probability that they are all from school B?

(b) Consider the expansion of $(3-x)^n$.

(i) Show that the coefficient of x^4 is given by $\binom{n}{4}3^{n-4}$. **1**

(ii) The coefficients of x^4 and x^5 in the expansion of $(3-x)^n$ are equal in magnitude and opposite in sign. **3**

Solve for n.

(c) Use the principle of mathematical induction to show that for all positive integers n: **4**

$$\frac{1}{4\times1^2-1}+\frac{1}{4\times2^2-1}+\ldots+\frac{1}{4n^2-1}=\frac{n}{2n+1}.$$

Question 13 (12 Marks) Use the Question 13 Writing Booklet

(a) A monic polynomial $P(x)$ of degree 3 is divisible by (x^2-2x-3) and leaves a remainder of 10 when divided by $(x-4)$. **3**

Define $P(x)$ as a product of linear factors.

(b) Solve the equation: **4**

$$\tan 2x - 2\sin x = 0 \qquad \text{for} \quad 0 \le x \le 2\pi$$

(c) (i) Show that for all positive integers n: **2**

$$x\left[1+(1+x)+(1+x)^2+\ldots+(1+x)^{n-2}+(1+x)^{n-1}\right]=(1+x)^n-1$$

(ii) Hence show that for $1 \le k \le n$: **2**

$$\binom{k-1}{k-1}+\binom{k}{k-1}+\cdots+\binom{n-3}{k-1}+\binom{n-2}{k-1}+\binom{n-1}{k-1}=\binom{n}{k}$$

(iii) Hence find the value of $\binom{3}{3}+\binom{4}{3}+\cdots+\binom{48}{3}+\binom{49}{3}+\binom{50}{3}$ **1**

Question 14 (12 Marks) Use the Question 14 Writing Booklet

(a) The population P of animals in a certain species t years after being introduced in a new area satisfies the equation:

$$\frac{dP}{dt} = kP(200 - P), \text{ where } k \text{ is a constant.}$$

(i) Verify using differentiation that $P = \dfrac{200}{1 + 49e^{-200kt}}$ is a solution of the above equation. **3**

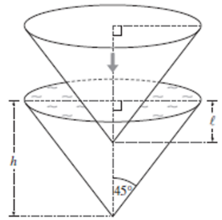
(ii) Given that after 2 years, the population grew to 50, solve for the exact value of k. **2**

(iii) Determine the initial rate of growth of the population, to 2 decimal places. **2**

Question 14 continues on the next page.

Question 14 (continued)

(b) The diagram shows two identical circular cones with a common vertical axis. Each cone has height h cm and semi-vertical angle 45°.



NOT TO SCALE

The lower cone is completely filled with water. The upper cone is lowered vertically into the water as shown in the diagram. The rate at which it is lowered is given by

$$\frac{dl}{dt} = 10$$

where l cm is the distance the upper cone has descended into the water after t seconds.

As the upper cone is lowered, water spills from the lower cone. The volume of water remaining in the lower cone at time h is V cm^3.

(i) Show that $V = \dfrac{\pi}{3}(h^3 - l^3)$ **2**

(ii) Find the rate at which V is changing with respect to time when the lower cone has lost $\dfrac{1}{8}$ of its water. Give your answer in terms of h. **3**

END OF PAPER

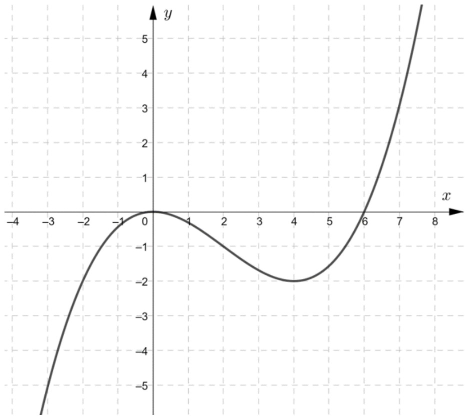
Additional Answer page for Question 10 (c)

Detach and hand in with your Question 10 booklet.

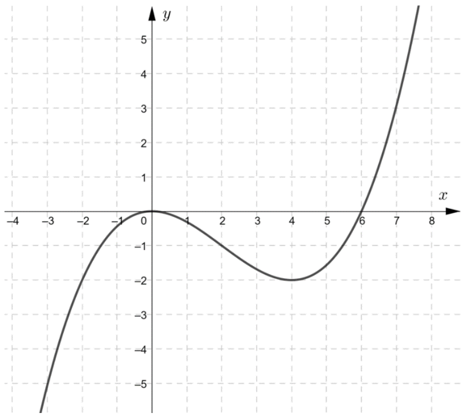
Student Number

(c) The diagram below shows the graph of $y = f(x)$.

(i) On the same set of axes, sketch $y = \frac{1}{f(x)}$ clearly showing **all** asymptotes and important features. **3**



(ii) Hence or otherwise, sketch $y = \frac{1}{f(|x|)}$ clearly showing **all** asymptotes and important features. **2**



Yr 12 Ext 1 7/4/22

1. B 2. C 3. C 4. D

5. D 6. A 7. C 8. A

1. $\alpha+\beta+\gamma = -\frac{b}{a} = -\frac{-3}{1}$

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-1}{1}$

$\therefore \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{3}{1} = 3$ $\therefore$ B

2. worst case - first 13 drive auto then 2 manual $\therefore$ 15 $\therefore$ C

3. $x = \sqrt{t-1}$, $y = t$

$\therefore x = \sqrt{y-1}$

$\therefore y - 1 \geq 0$

$y \geq 1$ $\therefore y \in [1, \infty)$ $\therefore$ C

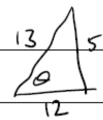
4. $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (as even function)

$= -\frac{\pi}{3}$

$\therefore$ D

5. $\sin\theta = \frac{5}{13}$



$\sin\theta = \frac{5}{13}$

$\cos\theta = -\frac{12}{13}$ $\left(\frac{\pi}{2} < \theta < \pi\right)$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{5}{13} \times \frac{-12}{13}$

$= -\frac{120}{169}$ $\therefore$ D

6. $P(x) = (x^2+x+4)(x+a)^2$

multiplicity of 2 $\therefore (x+a)^2$

$\therefore$ curve bounces off x axis

$\therefore$ only A or D

$y = x^2+x+4 \Rightarrow \Delta < 0$ $\therefore$ no roots

$\therefore$ does not cross x axis again

$\therefore$ A

7. $f(x) \to g(x)$

positive x reflected across y axis

$g(x) = f(|x|)$

negative y reflected across x axis

$g(x) = |f(x)|$

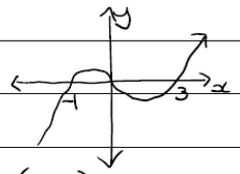
$\therefore g(x) = |f(|x|)|$ $\therefore$ C

8. $\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$

$\frac{dP}{dt} > 0$, $\frac{dP}{dr} = (kr)^2 \geq 0$

$\therefore$ for $\frac{dP}{dt} > 0$ $\frac{dr}{dt} > 0$ $\therefore$ A

9(a)(i)



(ii) $\frac{x(x+1)}{x-3} \geqslant 0$ NB: $x \neq 3$

$x(x+1)(x-3) \geqslant 0$

$\therefore -1 \leq x \leq 0,\ x > 3$

(b) i, $\binom{10}{5} \times \binom{10}{5} = 63504$

ii, $\binom{9}{4}\binom{9}{5} \times 2 = 31752$

(c) $\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$= \frac{1}{2\sqrt{A}} \times 4$

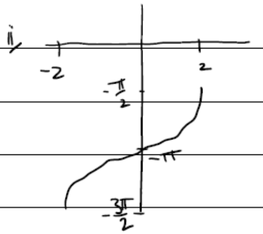
$= \frac{2}{\sqrt{A}}$

when $A = 4$ $\frac{dr}{dt} = 1$

(d) i, D: $-2 \leq x \leq 2$

R: $-\frac{3\pi}{2} \leq y \leq -\frac{\pi}{2}$

ii,



10 (a) i, LHS $= \frac{\sin 2\theta}{1+\cos 2\theta}$

$= \frac{2\sin\theta\cos\theta}{1+\cos^2\theta-\sin^2\theta}$

$= \frac{2\sin\theta\cos\theta}{1+\cos^2\theta-(1-\cos^2\theta)}$

$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ RHS

(ii) $\tan\frac{\pi}{8} = \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$

$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}+1}$

$= \sqrt{2}-1$

(b) i, $R^2 = (\sqrt{3})^2 + 3^2$

$= \sqrt{12}$ or $2\sqrt{3}$

$R\cos\alpha = 3$

$\cos\alpha = \frac{3}{\sqrt{12}}$

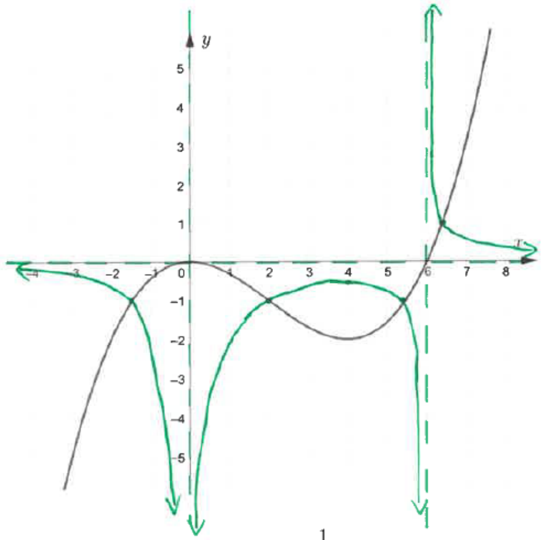
$\alpha = \frac{\pi}{6}$

$\therefore 2\sqrt{3}\cos\left(x - \frac{\pi}{6}\right)$

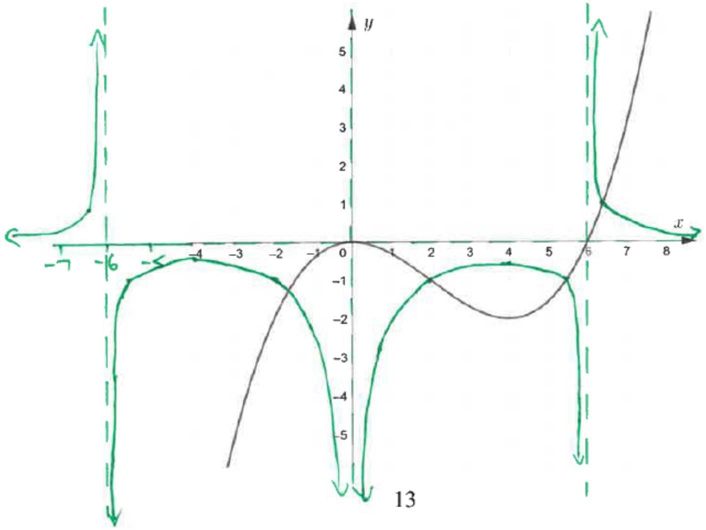
ii, one

(c) The diagram below shows the graph of $y = f(x)$.

(i) On the same set of axes, sketch $y = \frac{1}{f(x)}$ clearly showing **all** asymptotes and important features. **3**



(ii) Hence or otherwise, sketch $y = \frac{1}{f(|x|)}$ clearly showing **all** asymptotes and important features. **2**



11(a) S1 - Prove true for $n=1$

$2^1 + (-1)^2 = 2+1$

$= 3$ ∴ true

S2: Assume true for $n=k$

$2^k + (-1)^{k+1} = 3M$ where M is an integer

∴ $2^k = 3M - (-1)^{k+1}$

S3 Prove true for $n=k+1$

RTP: $2^{k+1} + (-1)^{k+2} = 3N$ where N is an integer

LHS $= 2\cdot 2^k + (-1)(-1)^{k+1}$

$= 2[3M-(-1)^{k+1}] - (-1)^{k+1}$ from S2

$= 6M - 2(-1)^{k+1} - (-1)^{k+1}$

$= 6M - 3(-1)^{k+1}$

$= 3(2M - (-1)^{k+1})$

$= 3N$

= RHS

∴ By PMI the statement is true for $n \geq 1$

(b) i, $x \geq 1$

ii, D: $x \geq -5$ R: $y \geq 1$

iii, $x = (y-1)^2 - 5$

$x+5 = (y-1)^2$

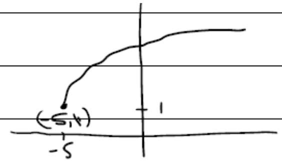
$\pm\sqrt{x+5} = y-1$

$y = 1 \pm \sqrt{x+5}$

but $y \geq 1$

∴ $y = 1+\sqrt{x+5}$

iv,



v, $x = 1+\sqrt{x+5}$

$x-1 = \sqrt{x+5}$

$(x-1)^2 = x+5$

$x^2-2x+1 = x+5$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

∴ $x = 4$ or -1

but $x \geq 1$

∴ $x = 4$ $(4,4)$

(c) $\binom{10}{2} = 45$

$300 \div 45 = 6\frac{2}{3}$

there are 45 combinations, we can have at most 270 students without exceeding 6 students per combination. But we have 300 students, so at least one combination has 7 or more students

12(a) i, $\binom{5}{1}\binom{7}{2} + \binom{5}{2}\binom{7}{1} = 105 + 70$

$= 175$

ii, $\dfrac{\binom{7}{3}}{\binom{5}{3}+\binom{7}{3}} = \dfrac{7}{9}$

(b) i, $\binom{n}{4}(3)^{n-4}(-x)^4 = \binom{n}{4}3^{n-4}x^4$

∴ co-eff of x^4 is $\binom{n}{4}3^{n-4}$

ii, $\binom{n}{5}3^{n-5}(-x)^5 = -\binom{n}{5}3^{n-5}x^5$

∴ $\binom{n}{4}3^{n-4} = \binom{n}{5}3^{n-5}$

$\dfrac{n!}{4!(n-4)!}3^{n-4} = \dfrac{n!}{5!(n-5)!}3^{n-5}$

$\dfrac{\cancel{n!}\,5!\,\cancel{(n-5)!}\,\cancel{3^{n-4}}}{\cancel{n!}\,\cancel{4!}\,(n-4)!\,\cancel{3^{n-5}}} = 1$

$\dfrac{15}{n-4} = 1$

$15 = n-4$

$n = 19$

(c) S1 Prove true for $n=1$

LHS $= \frac{1}{4-1}$ RHS $= \frac{1}{2\times1+1}$

$= \frac{1}{3}$ $= \frac{1}{3}$

∴ true for $n=1$

S2 Assume true for $n=k$

$\frac{1}{4\times1^2-1}+\frac{1}{4\times2^2-1}+\dots+\frac{1}{4k^2-1}=\frac{k}{2k+1}$

S3 Prove true for $n=k+1$

RTP: $\frac{1}{4\times1^2-1}+\dots+\frac{1}{4k^2-1}+\frac{1}{4(k+1)^2-1}=\frac{k+1}{2(k+1)+1}$

$=\frac{k+1}{2k+3}$

LHS $= \frac{k}{2k+1}+\frac{1}{4(k^2+2k+1)-1}$

$=\frac{k}{2k+1}+\frac{1}{4k^2+8k-3}$

$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$

$=\frac{(2k+3)k+1}{(2k+1)(2k+3)}$

$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$=\frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$

$=\frac{k+1}{2k+3}$

= RHS

∴ By PMI the statement is true for all positive integers n.

13(a) $P(4)=10$

$P(x)=(x+k)(x^2-2x-3)$

$\therefore P(4)=(4+k)(4^2-2\times4-3)$

$10=(4+k)\times5$

$2=4+k$

$k=-2$

$\therefore P(x)=(x-2)(x-3)(x+1)$

(b) $\tan 2x-2\sin x=0$

$\dfrac{2\tan x}{1-\tan^2 x}-2\sin x=0$

$\dfrac{\frac{2\sin x}{\cos x}}{1-\tan^2 x}-2\sin x=0$

$\dfrac{2\sin x}{\cos x(1-\tan^2 x)}-2\sin x=0$

$2\sin x\left(\dfrac{1}{\cos x(1-\tan^2 x)}-1\right)=0$

$\therefore 2\sin x=0$ and $\dfrac{1}{\cos x(1-\tan^2 x)}-1=0$

$\sin x=0$

$\therefore x=0,\pi,2\pi$

$\dfrac{1}{\cos x(1-\tan^2 x)}=1$

$1=\cos x(1-\tan^2 x)$

$1=\cos x(1-(\sec^2 x-1))$

$1=\cos x(2-\sec^2 x)$

$1=\cos x\left(2-\dfrac{1}{\cos^2 x}\right)$

$1=2\cos x-\dfrac{1}{\cos x}$

$\cos x=2\cos^2 x-1$

$2\cos^2 x-\cos x-1=0$

$(2\cos x+1)(\cos x-1)=0$

$\therefore \cos x=-\frac{1}{2},\ 1$

$\therefore x=\frac{2\pi}{3},\frac{4\pi}{3},\ 0,2\pi$

$\therefore x=0,\frac{2\pi}{3},\pi,\frac{4\pi}{3},2\pi$

(c) i) $LHS=x\left[\dfrac{1((1+x)^n-1)}{(1+x)-1}\right]$

$=x\left[\dfrac{(1+x)^n-1}{x}\right]$

$=(1+x)^n-1$

ii) $(1+x)^n-1=\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{k}x^k+\ldots+\binom{n}{n}x^n-1$

$\therefore \binom{n}{k}$ is the coeff of x^k

$\therefore$ looking for coeff of x^k on LHS

$LHS=x\left[1+(1+x)+(1+x)^2+\ldots+(1+x)^{k-1}+(1+x)^k+(1+x)^{k+1}+\ldots+(1+x)^{n-2}+(1+x)^{n-1}\right]$

due to the x out the front

looking for x^{k-1} in brackets are

from $(1+x)^{k-1}=\binom{k-1}{0}+\binom{k-1}{1}x+\ldots+\binom{k-1}{k-1}x^{k-1}$

from $(1+x)^k=\binom{k}{0}+\binom{k}{1}x+\ldots+\binom{k}{k-1}x^{k-1}+\binom{k}{k}x^k$

from $(1+x)^{k+1}=\binom{k+1}{0}+\binom{k+1}{1}x+\ldots+\binom{k+1}{k-1}x^{k-1}+\ldots$

$\vdots$

from $(1+x)^{n-2}=\binom{n-2}{0}+\binom{n-2}{1}x+\ldots+\binom{n-2}{k-1}x^{k-1}+\ldots$

from $(1+x)^{n-1}=\binom{n-1}{0}+\binom{n-1}{1}x+\ldots+\binom{n-1}{k-1}x^{k-1}+\ldots$

$\therefore$ coeff of x^k will be

$\binom{k-1}{k-1}+\binom{k}{k-1}+\ldots+\binom{n-3}{k-1}+\binom{n-2}{k-1}+\binom{n-1}{k-1}$

iii) $\binom{51}{4}=249900$

14.(a) i) $P=\dfrac{200}{1+49e^{-200kt}}$

$\dfrac{dP}{dt}=\dfrac{-200(-200k\cdot49e^{-200kt})}{(1+49e^{-200kt})^2}$

$=P^2k\times49e^{-200kt}$

$=P^2k\times\left(\frac{200}{P}-1\right)$

$=kP(200-P)$

ii) $t=2$ $P=50$

$50=\dfrac{200}{1+49e^{-400k}}$

$1+49e^{-400k}=\dfrac{200}{50}$

$49e^{-400k}=3$

$e^{-400k}=\dfrac{3}{49}$

$-400k=\ln\frac{3}{49}$

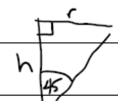
$k=\dfrac{\ln\frac{3}{49}}{-400}$

iii) when $t=0$ $P=\frac{200}{50}=4$

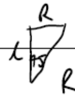
$\dfrac{dP}{dt}=\dfrac{\ln\frac{3}{49}}{-400}\times4(200-4)$

$=5.47$

(b) i) $V=\frac{1}{3}\pi r^2h-\frac{1}{3}\pi R^2 l$



$\therefore r=h$



$R=l$

$V=\frac{1}{3}\pi h^3-\frac{1}{3}\pi l^3$

$=\frac{\pi}{3}(h^3-l^3)$

ii) Find l when $\frac{1}{8}$ water lost

$\dfrac{\pi l^3}{3}=\dfrac{1}{8}\times\dfrac{\pi h^3}{3}$

$l^3=\dfrac{h^3}{8}$

$l=\dfrac{h}{2}$

$\dfrac{dV}{dt}=\dfrac{dl}{dt}\times\dfrac{dV}{dl}$

$V=\frac{\pi}{3}h^3-\frac{\pi}{3}l^3$

$\dfrac{dV}{dl}=-\pi l^2$

$\therefore \dfrac{dV}{dt}=10\times-\pi l^2$

$=-10\pi l^2$

when $l=\frac{h}{2}$

$\dfrac{dV}{dl}=-10\pi\left(\frac{h}{2}\right)^2$

$=-\frac{5\pi}{2}h^2$