
Student Number

Barker College

2023 YEAR 12 ASSESSMENT BLOCK

Mathematics Extension 1

Staff Involved:
- DZP • BHC • PDJ • ARP
- ALY • RJW • ESP*

Wednesday 5th April
1:30 pm
100 copies

TOPICS COVERED:		
Combinatorics	Binomial Theorem	Further Rates
Further Trigonometry	Further Graphs	Polynomials
Trigonometric Equations	Induction	

General Instructions:
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- Write your student number at the top of the page

Total marks: 80

Section I - 8 marks (pages 2 - 5)
- Attempt Multiple Choice Questions 1 - 8
- Allow about 15 minutes for this section

Section II – 72 marks (pages 6 - 13)
- Attempt Questions 9 - 14
- Show all necessary working
- Allow about 1 hour and 45 minutes for this section

Section I

8 Marks
Attempt Questions 1-8
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-8.

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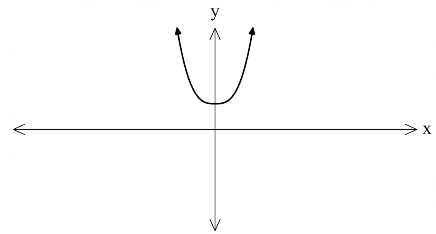
1 The function $f(x) = x^3$ was transformed to produce the graph below.



Which function describes this transformation?

(A) $y = |f(x-1)|$

(B) $y = |f(x)| + 1$

(C) $y = |f(x) + 1|$

(D) $y = |f(x) - 1|$

2 The polynomial $P(x) = x^3 + x^2 - 5x + 3$ has a double root at $x = \alpha$. What is the value of α ?

(A) $\frac{-5}{3}$

(B) 1

(C) $\frac{-5}{3}$ and 1

(D) $\frac{5}{3}$ and -1

3 How many arrangements of the letters in the word FIBONACCI are possible?

(A) $9!$

(B) $\frac{9!}{2!\,2!}$

(C) $\frac{9!}{4!}$

(D) $\binom{9}{2}$

4 What is the range of $y = \sin(\sin^{-1} x)$?

(A) $[-1,1]$

(B) $(-1,1)$

(C) All real values

(D) Undefined everywhere

5 What is the constant term in the binomial expansion of $\left(3x - \frac{5}{x^2}\right)^{27}$?

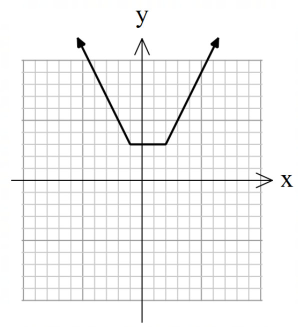
(A) ${}^{27}C_{18}\, 3^{18}\, 5^{9}$

(B) ${}^{27}C_{9}\, 3^{9}\, 5^{18}$

(C) $-{}^{27}C_{18}\, 3^{18}\, 5^{9}$

(D) $-{}^{27}C_{9}\, 3^{9}\, 5^{18}$

6 Given $f(x) = |x+1|$ and $g(x) = |x-2|$, $h(x)$ is shown below. Which of the following is the equation of $h(x)$?



(A) $h(x) = f(x) + g(x)$

(B) $h(x) = f(x) \times g(x)$

(C) $h(x) = f(x) - g(x)$

(D) $h(x) = f(x) \div g(x)$

7 What is the smallest group of people that can be assembled where it is guaranteed that at least two people are born in the same month and share the same first letter of their first name?

(A) 38

(B) 39

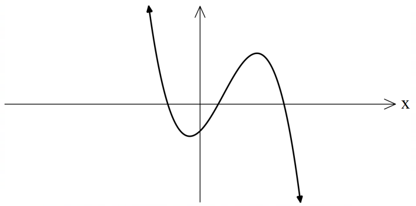
(C) 312

(D) 313

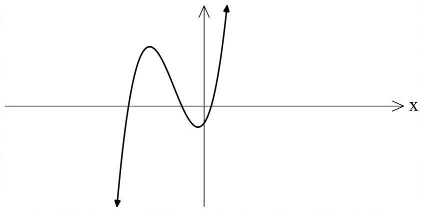
8 Consider the polynomial $g(x) = ax^3 + bx^2 - 11x - 6$ with a and b both negative integers.

Which graph could represent $g(x)$?

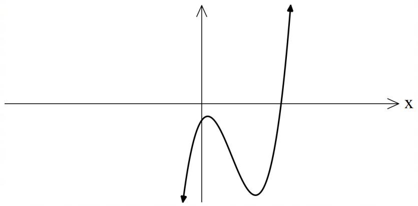
(A)



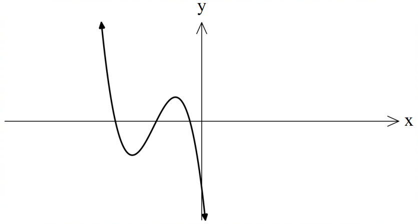
(B)



(C)



(D)



Section II

72 Marks
Attempt Questions 9-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 9-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (12 Marks) Use the Question 9 Writing Booklet

(a) Find the exact value of $\sin 15°$. Express your answer as a single fraction. **2**

(b) Consider the function $\sqrt{3}\sin\theta + \cos\theta$, for $0 \le \theta \le 2\pi$.

(i) Express the function in the form $Rsin(\theta + \alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $\sqrt{3}\sin\theta + \cos\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

(c) Sketch $y = 2\arctan x$ showing all important features. **2**

(d) Solve $\frac{x^2-2}{x} + 1 \ge 0$ **3**

Question 10 (12 Marks) Use the Question 10 Writing Booklet

(a) Use mathematical induction to prove

$$log_2(1) + log_2(2) + log_2(3) + \ldots + log_2(n) = log_2(n!) \quad \text{for } n \geq 1$$ **3**

(b) Prove $\sin^2 A = \frac{1}{2}\sin(2A)\tan A$ **2**

(c) In a game of Guessy Number, a barrel contains 48 discs, each with a different number from 1 to 48. Players must guess the first 3 numbers to be drawn, without replacement and in the correct order, to win first prize .

(i) In how many ways can 3 discs be randomly drawn from the barrel in order? **1**

(ii) Otto has 6 entries in the same draw of *Guessy Number*. What is the probability that he will win first prize? **1**

(d) Consider the function $f(x) = \dfrac{x}{9-x^2}$.

(i) Show that $f'(x) > 0$ for all x in the domain of $f(x)$ **2**

(ii) Is $f(x)$ odd, even or neither? Justify your answer with working. **1**

(iii) Sketch the graph of $y = f(x)$ showing horizontal and vertical asymptotes. **2**

Question 11 (12 Marks) Use the Question 11 Writing Booklet

(a) Find the values of a, b, and c for which $3x^2 - 2x - 7 \equiv a(x+2)^2 + b(x+2) + c$ **3**

(b) Consider the function $f(x) = x^3 - 3x$.

(i) Find the stationary points and determine their nature. **2**

(ii) Sketch $f(x)$ showing the stationary points and intercepts. **1**

(iii) What is the largest restricted domain containing $x = 0$ such that $f(x)$ has an inverse function? **1**

(iv) Let $f^{-1}(x)$ be the inverse function of $f(x)$ with the restricted domain found in part (iii). State the domain and range of $f^{-1}(x)$. **2**

(c) Use the principle of mathematical induction to show that

$$3^{2n+4} - 2^{2n}$$

is a multiple of 5 for integers $n \geq 1$. **3**

Question 12 (12 Marks) Use the Question 12 Writing Booklet

(a) (i) Prove that $tan^2\theta = \frac{1-cos2\theta}{1+cos2\theta}$ provided that $\cos 2\theta \neq -1$ **2**

(ii) Hence show the exact value of $tan\frac{\pi}{12}$ is $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$ **2**

(b) A block of chocolate is in the shape of a rectangular prism with a square base.

It has a height double the side length of the base.

As it melts, the volume of the block of chocolate is decreasing at a rate of $12\ cm^3/s$.

Find the rate at which the surface area will be decreasing when the side of the base is 3.1 cm. Give your answer to 1 decimal place. **4**

(c) Find the x^3 term in the expansion of $(1+x)^2\left(\frac{x}{2}-1\right)^{10}$ **4**

Question 13 (12 Marks) Use the Question 13 Writing Booklet

(a) **Answer Question 13 (a) on the separate answer page supplied.** **5**

Hand it in with your booklet for Question 13.

(b) A kettle of water is filled at a variable rate so that the volume of liquid, V, in the kettle at any time t minutes is given by the formula:

$$V = M(1-e^{-kt}), \quad \text{where } k > 0$$

Eventually, the kettle is filled with water.

(i) Show $\frac{dV}{dt} = k(M-V)$ **2**

(ii) Show that as t approaches ∞, V approaches M, and explain the significance of this result in the context of this question. **2**

(iii) If a quarter of the kettle is filled in the first 5 minutes, what exact fraction is filled in the next 5 minutes? **3**

Question 14 (12 Marks) Use the Question 14 Writing Booklet

(a) The height h, in metres, of two people in different seats on a Ferris Wheel are modelled by the equations $h_1 = 12\cos 3t$ and $h_2 = 12\cos(3t - \frac{\pi}{4})$, where t is the time in minutes.

When is the first time that the two people are at the same height? **3**

(b) Solve $32x^3 - 48x^2 + 22x - 3 = 0$,

given that the roots are consecutive terms of an arithmetic sequence. **3**

(c) A game is played by throwing a number of darts at a bulls-eye on a chosen dartboard.

Players can choose to play Dartboard 1 or Dartboard 2.

Dartboard 1: Players throw 2 darts and win if the bulls-eye is hit both times. The probability that a player hits the bulls-eye for Dartboard 1 is $0.9p$.

Dartboard 2: Players throw 3 darts and win if the bulls-eye is hit at least once. The probability that a player hits the bulls-eye for Dartboard 2 is $0.2p$.

(i) Find the probability of winning when choosing Dartboard 1. **1**

(ii) Show that the probability of winning when choosing Dartboard 2 is $0.6p - 0.12p^2 + 0.008p^3$. **2**

(iii) Find the range of values of p for which it is more likely to win a game on Dartboard 2 than Dartboard 1. **3**

END OF PAPER

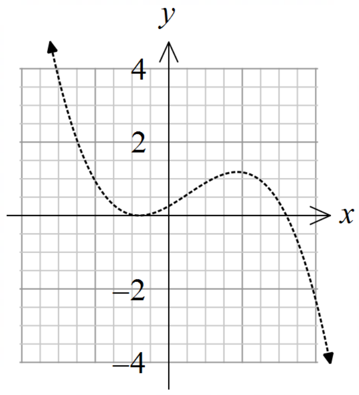
Additional Answer page for Question 13 (a)

Detach and hand in with your Question 13 booklet.

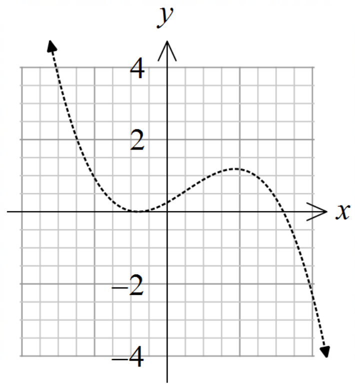
Student Number

(a) The diagram below shows the graph of $y = f(x)$.

(i) On the axes below, sketch $y = \dfrac{1}{f(x)}$ showing **all** asymptotes and important features. **3**



(ii) On the axes below, sketch $y = \sqrt{f(x)}$ clearly showing important features. **2**



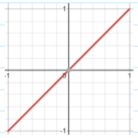
Yr 12 ext 1 Ass #2 Solutions

1. $|f(x)|$ makes all $f(x) > 0$
   $+1$ shifts up 1 $\therefore$ Ans : B

2. Double root if factor of $P(x)$ and $P'(x)$
   $P'(x) = 3x^2 + 2x - 5$
   $= (3x+5)(x-1) = 0$ when $x = -\frac{5}{3}$ or 1
   $P(1) = 1 + 1 - 5 + 3 = 0$ ✓
   $P(-\frac{5}{3}) = 9.48\ldots$ ✗ $\therefore$ Ans = B
   $\therefore$ Only 1 is a factor of both.

3. FIBONACCI → 9 letters, 2 I, 2 C
   $\therefore$ Arr $= \dfrac{9!}{2!2!}$ $\therefore$ Ans = B

4. $y = \sin(\sin^{-1} x)$

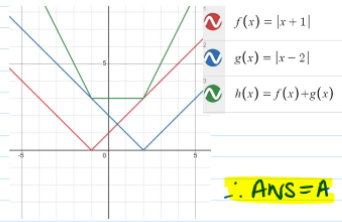


Dom restricted to $[-1, 1]$
then sin 'undoes' $\sin^{-1} x$, leaving $y = x$, range $[-1, 1]$

Ans = A

5. $(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$
   using ref sheet to find r
   $\binom{27}{r}(3x)^{27-r}\left(\frac{-5}{x^2}\right)^r$
   $= \binom{27}{r}3^{27-r}(-5)^r x^{27-r} x^{-2r}$
   $\therefore 27 - r - 2r = 0$
   $27 = 3r$
   $r = 9$
   $\therefore$ Constant term
   $-\binom{27}{9}3^{18}5^9$ which by symm in Pascals Δ $= -\binom{27}{18}3^{18}5^9$
   $\therefore$ ANS = C

6.



$\therefore$ ANS = A

7. $12 \times 26 + 1 = 313$
   $\therefore$ ANS = D

8. $ax^3 + bx^2 - 11x - 6$
   $a < 0$ so shape ↷ so not B or C
   sum of roots $= \dfrac{-b}{a} < 0$ (a, b negative)
   For sum of roots they must have greater magnitude on LHS of y-axis.
   $\therefore$ ANS = D

Question 9

a) $\sin 15^\circ = \sin(45 - 30^\circ)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

b) $\sqrt{3}\sin\theta + \cos\theta, \quad 0 \le \theta \le 2\pi$

(i) $R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R = \sqrt{3+1} = 2. \quad (R > 0)$

$2\cos\alpha = \sqrt{3}$

$2\sin\alpha = 1$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$ $\qquad \therefore \sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$

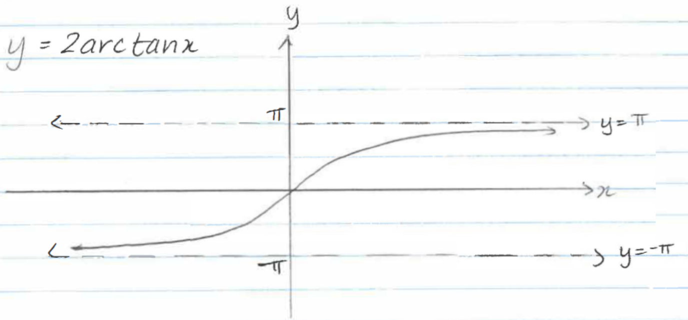
ii) $2\sin\left(\theta + \frac{\pi}{6}\right) = 1$ $\qquad \frac{\pi}{6} \le \theta + \frac{\pi}{6} \le \frac{13\pi}{6}$

$\sin\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}$

$\theta + \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$

$\theta = 0, \frac{2\pi}{3}$ or 2π

c) $y = 2\arctan x$



d) $\frac{x^2-2}{x} + 1 \ge 0 \qquad (x \ne 0)$

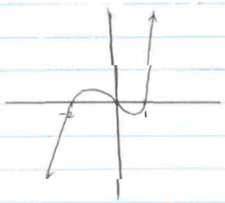
$\frac{x^2-2}{x} \ge -1$

* $x(x^2-2) \ge -x^2$

$x(x^2-2) + x^2 \ge 0$

$x\left[x^2 - 2 + x\right] \ge 0$

$x(x-1)(x+2) \ge 0$



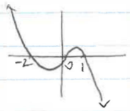
Soln : $-2 \le x < 0$ or $x \ge 1$

* note alternate option

$-x^2 - x(x^2-2) \le 0$

$-x\left[x + x^2 - 2\right] \le 0$

$-x(x+2)(x-1) \le 0$



$-2 \le x < 0$ or $x \ge 1$

Question 10

a) $\log_2 1 + \log_2 2 + \log_2 3 + \ldots + \log_2 n = \log_2(n!)$

When $n=1$ $\quad LHS = \log_2 1 = 0$

$RHS = \log_2(1!)$

$= \log_2 1$

$= 0 \qquad \therefore$ true for $n=1$

Assume for $n=k$ $\quad (k \in \mathbb{Z}^+)$

Let $\log_2 1 + \log_2 2 + \log_2 3 + \ldots + \log_2 k = \log_2(k!)$

Prove for $n=k+1$

RTP $\log_2 1 + \log_2 2 + \log_2 3 + \ldots + \log_2 k + \log_2(k+1) = \log_2(k+1)!$

$LHS = \log_2 1 + \log_2 2 + \log_2 3 + \ldots + \log_2 k + \log_2(k+1)$

$= \log_2(k!) + \log_2(k+1)$ by assumption

$= \log_2(k! \times (k+1))$ by log laws

$= \log_2(k+1)!$

$= RHS.$

$\therefore$ If true for $n=1$, and proven for $n=k+1$ when assumed true for $n=k$, true by MI.

b) RTP $\sin^2 A = \frac{1}{2}\sin 2A \tan A$

$RHS = \frac{1}{2}\sin 2A \tan A$

$= \frac{1}{2}(2\sin A\cos A) \times \frac{\sin A}{\cos A}$

$= \sin A \cancel{\cos A} \times \frac{\sin A}{\cancel{\cos} A}$

$= \sin^2 A = LHS$ Proven.

Question 10 (cont'd)

(c) (i) $48 \times 47 \times 46 = 103\,776$

OR $^{48}P_3 = 103776$

(ii) $P(\text{win}) = \frac{6}{103776}$

$= \frac{1}{17296}$

(d)(i) $f(x) = \frac{x}{9-x^2}$

$f'(x) = \frac{1.(9-x^2) - x(-2x)}{(9-x^2)^2}$

$= \frac{9-x^2+2x^2}{(9-x^2)^2}$

$= \frac{9+x^2}{(9-x^2)^2}$

$(9-x^2)^2 > 0 \quad \forall x$

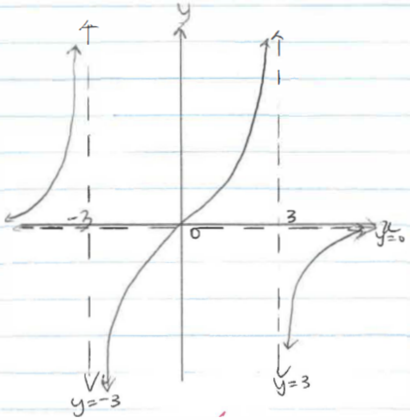
$x^2+9 > 0 \quad \forall x \qquad \therefore f'(x) > 0$ for all x in the domain

(ii) $f(-x) = \frac{-x}{9-(-x)^2}$

$= \frac{-x}{9-x^2}$

$= -f(x)$

$\therefore$ odd



Question 11

(a) $3x^2 - 2x - 7 \equiv a(x+2)^2 + b(x+2) + c$

$RHS = a(x^2 + 4x + 4) + bx + 2b + c$

$= ax^2 + 4ax + 4a + bx + 2b + c$

$= ax^2 + (4a+b)x + (4a+2b+c)$

$\therefore$ By equating coefficients

$a = 3$

$4a + b = -2$

$b = -14$

$4a + 2b + c = -7$

$12 - 28 + c = -7$

$c = 9$

$\therefore \boxed{a = 3 \quad b = -14 \quad c = 9}$

(b) $f(x) = x^3 - 3x$

(i) $f'(x) = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x-1)(x+1)$

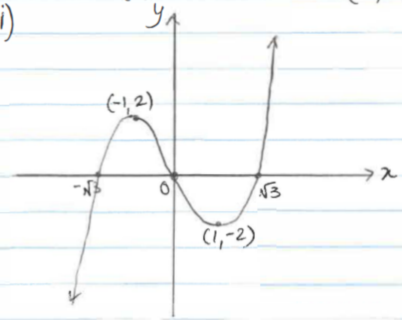
$\therefore$ Stat pts at $(1,-2)$ and $(-1,2)$

$f''(x) = 6x$

$f''(1) = 6 > 0 \quad \therefore (1,-2)$ a local min

$f''(-1) = -6 < 0 \quad \therefore (-1,2)$ a local max

(ii)



(iii) $-1 \le x \le 1$

or $[-1, 1]$

(iv) Dom: $[-2, 2]$

Range: $[-1, 1]$

Question 11 (cont'd)

(c) Prove $3^{2n+4} - 2^{2n}$ is a multiple of 5, integers $n \ge 1$

Prove for $n = 1$

$LHS = 3^{2(1)+4} - 2^{2(1)}$

$= 3^6 - 2^2$

$= 725$ which is a multiple of 5.

$= 5 \times 145$

Assume true for $n = k$

$3^{2k+4} - 2^{2k} = 5M \qquad (k, M \in \mathbb{Z}^+)$

$5M + 2^{2k} = 3^{2k+4}$

Prove true for $n = k+1$

RTP $3^{2(k+1)+4} - 2^{2(k+1)} = 5N \qquad (N \in \mathbb{Z}^+)$

$LHS = 3^{2k+6} - 2^{2k+2}$

$= 3^{2k+4} \cdot 3^2 - 2^{2k} \cdot 2^2$

$= 9(5M + 2^{2k}) - 2^2 \cdot 2^{2k}$ by assumption

$= 9 \times 5M + 9 \cdot 2^{2k} - 2^2 \cdot 2^{2k}$

$= 5 \times 9M + 5 \cdot 2^{2k}$

$= 5(9M + 2^{2k})$

$= 5N$ for $M, K \in \mathbb{Z}^+$

$\therefore$ Proven true by MI

Question 12

(a)(i) Prove $\tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$ provided $\cos\theta \ne -1$

$RHS = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$

$= \dfrac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)}$

$= \dfrac{1 - 1 + 2\sin^2\theta}{1 + 2\cos^2\theta - 1}$

$= \dfrac{2\sin^2\theta}{2\cos^2\theta}$

$= \tan^2\theta = RHS$ proven.

(ii) $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

$\tan^2\dfrac{\pi}{12} = \dfrac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}}$

$= \dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}$

$= \dfrac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

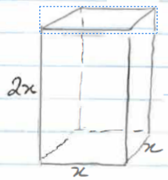
$\therefore \tan\dfrac{\pi}{12} = \pm\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}$

but $\tan\left(\frac{\pi}{12}\right) > 0$

$\therefore \tan\dfrac{\pi}{12} = \sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}$

Question 12 (cont'd)

(b)



$\dfrac{dV}{dt} = -12\ cm^3/sec$

$\dfrac{dA}{dt} = ?$ when $x = 3.1\ cm$

$V = 2x \times x \times x$
$= 2x^3$
$\dfrac{dV}{dx} = 6x^2$

$A = 4\times 2x^2 + 2\times x^2$
$= 10x^2$
$\dfrac{dA}{dx} = 20x$

$\dfrac{dV}{dt} = \dfrac{dV}{dx}\cdot\dfrac{dx}{dt}$

$-12 = 6x^2 \cdot \dfrac{dx}{dt}$

$\dfrac{dx}{dt} = \dfrac{-2}{x^2}$

$\dfrac{dt}{dx} = \dfrac{-x^2}{2}$

$\dfrac{dA}{dt} = \dfrac{dA}{dx}\cdot\dfrac{dx}{dt}$

$= 20x \cdot \dfrac{-2}{x^2}$

$= \dfrac{-40}{x}$

at $x = 3.1$

$\dfrac{dA}{dt} = \dfrac{-40}{3.1} = -12.9\ cm^2/sec$ (1 dp)

(c) $(1+x)^2\left(\frac{x}{2}-1\right)^{10}$: find x^3 term

$= (x^2+2x+1)\left(\frac{x}{2}-1\right)^{10}$

$= (x^2+2x+1)\left(\binom{10}{0}\left(\frac{x}{2}\right)^{10}(-1)^0 + \dots + \binom{10}{7}\left(\frac{x}{2}\right)^3(-1) + \binom{10}{8}\left(\frac{x}{2}\right)(-1)^0 + \binom{10}{9}\left(\frac{x}{2}\right)^1(-1)^9 + \binom{10}{10}\left(\frac{x}{2}\right)^0(-1)^{10}\right)$

For x^3 term we use $x^2\times x$, $x\times x^2$, $1\times x^3$ terms

$\therefore x^2\times\binom{10}{9}\left(\frac{x}{2}\right)(-1)^9 + 2x\times\binom{10}{8}\left(\frac{x^2}{4}\right)(-1)^8 + 1\times\binom{10}{7}\left(\frac{x^3}{8}\right)(-1)$

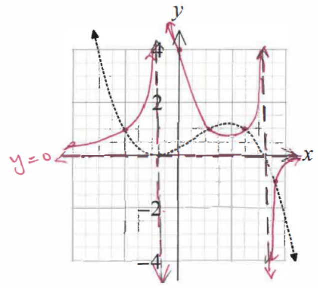
$= -5x^3 + \frac{45}{2}x^3 - 15x^3$

$= \frac{5}{2}x^3$

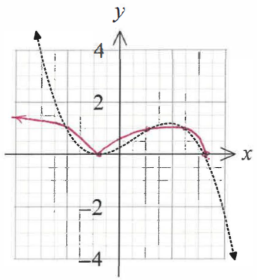
Additional Answer page for Question 13 (a)

(a) The diagram below shows the graph of $y = f(x)$.

(i) On the axes below, sketch $y = \dfrac{1}{f(x)}$ showing **all** asymptotes and important features. **3**



(ii) On the axes below, sketch $y = \sqrt{f(x)}$ clearly showing important features. **2**



Question 13 (cont'd)

(b) $V = M(1 - e^{-kt}) = M - Me^{-kt}$ (M a constant)

(i) Show $\frac{dV}{dt} = k(M - V)$

$$\frac{dV}{dt} = -k(-Me^{-kt})$$
$$= kMe^{-kt}$$

If $V = M - Me^{-kt}$

$Me^{-kt} = M - V$

$$\therefore \frac{dV}{dt} = kMe^{-kt}$$
$$= k(M - V)$$ shown.

(ii) as $t \to \infty$ $e^{-kt} \to 0$

$\therefore$ as $t \to \infty$ $1 - e^{-kt} \to 1$

$\therefore V \to M$ $\therefore$ M is full volume of kettle.

(iii) at $t = 5$ $V = \frac{1}{4}$ of M

$$\therefore \frac{1}{4} = 1 - e^{-5k}$$
$$e^{-5k} = \frac{3}{4}$$
$$-5k = \ln\frac{3}{4}$$
$$k = \frac{-\ln\frac{3}{4}}{5}$$

at $t = 10$ $V = M\left(1 - e^{\frac{\ln\frac{3}{4}}{5}}\right)$
$$= M\left(1 - e^{2\ln\frac{3}{4}}\right)$$
$$= M\left(1 - e^{\ln\frac{9}{16}}\right)$$
$$= M\left(1 - \frac{9}{16}\right)$$
$$= \frac{7}{16}M$$

$\therefore$ In the next 5 mins

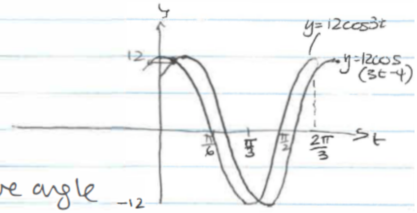
$\frac{7}{16} - \frac{1}{4} = \frac{3}{16}$

is filled.

Question 14

(a) $h_1 = 12\cos 3t$ t (mins)

$h_2 = 12\cos\left(3t - \frac{\pi}{4}\right)$



$$12\cos 3t = 12\cos\left(3t - \frac{\pi}{4}\right)$$

$3t > 0$

By (even) symmetry

$\left(3t - \frac{\pi}{4}\right)$ is the equivalent -ve angle

$$\therefore 3t = -\left(3t - \frac{\pi}{4}\right)$$
$$6t = \frac{\pi}{4}$$
$$t = \frac{\pi}{24} = 0.130899 \text{ mins}$$
$$= 7.85398 \text{ sec.}$$

OR

$$\cos 3t = \cos\left(3t - \frac{\pi}{4}\right)$$
$$= \cos 3t \cos\frac{\pi}{4} + \sin 3t \sin\frac{\pi}{4}$$
$$\cos 3t = \frac{\cos 3t}{\sqrt{2}} + \frac{\sin 3t}{\sqrt{2}}$$
$$\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)\cos 3t = \frac{\sin 3t}{\sqrt{2}}$$
$$\tan 3t = \sqrt{2} - 1$$

$3t = 0.392699$ (not 22.5 - calc needs to be RAD)

$t = 0.1308996\ldots$ or 0.13 min (2 dp)

Question 14

(b) $32x^3 - 48x^2 + 22x - 3 = 0$

Let roots be $\alpha - d$, α, $\alpha + d$

Sum roots $= 3\alpha = \frac{48}{32}$

$\therefore \boxed{\alpha = \frac{16}{32} = \frac{1}{2}}$

$P(\tfrac{1}{2}) = 4 - 12 + 11 - 3 = 0$ ✓

Product roots $= \alpha(\alpha^2 - d^2) = \frac{3}{32}$

$\frac{1}{2}\left(\frac{1}{4} - d^2\right) = \frac{3}{32}$

$\frac{1}{4} - d^2 = \frac{3}{16}$

$d^2 = \frac{1}{16}$

$\boxed{d = \pm \frac{1}{4}}$

$\therefore$ Roots are $\frac{1}{2} - \frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{2} + \frac{1}{4}$

$= \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

Question 14 (cont'd)

(c) (i) P(Hit bullseye) $= 0.9p$

$P(\text{win}) = P(h) \times P(h)$

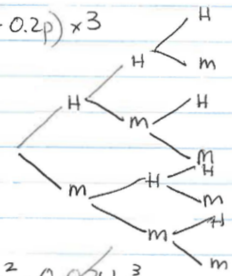
$= 0.9p \times 0.9p$

$= 0.81p^2$

(ii) P(win on DB 2) = P(Hit once) + P(Hit twice) + P(Hit 3x)

$P(H) = 0.2p$

$P(\text{win}) = 0.2p \times (1 - .2p)^2 \times 3 + (0.2p)^2 \times (1 - 0.2p) \times 3 + (0.2p)^3$



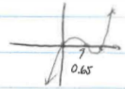
$= 0.6p(1 - 0.4p + 0.04p^2) + (3 - 0.6p) \times 0.04p^2 + 0.008p^3$

$= 0.6p - 0.24p^2 + \cancel{0.024p^3} + 0.12p^2 - \cancel{0.024p^3} + 0.008p^3$

$= 0.008p^3 - 0.12p^2 + 0.6p$

shown

iii) $0.008p^3 - 0.12p^2 + 0.6p > 0.81p^2$



$0.008p^3 - 0.93p^2 + 0.6p > 0$

$p(0.008p^2 - 0.93p + 0.6) > 0$

$p \neq 0$

$\therefore 0.008p^2 - 0.93p + 0.6 > 0 \quad (\times 1000)$

$8p^2 - 930p + 600 > 0$

$p = \frac{930 \pm \sqrt{(-930)^2 - 4(8)(600)}}{16}$

$p = 0.65$ or 115.6 but $0 < p \leq 1$

$\therefore$ Range of values is $0 < p \leq 0.65$