Barker College

Student Number

# Mathematics Extension 1

# 2020
## YEAR 11 ASSESSMENT TASK 2

**Staff Involved:**

**DZP*** **BHC**

**ARP** **PDJ**

**PJR** **ARM**

**FRIDAY 26TH JUNE**

**PERIOD 2 OR 5**

**110 copies**

**General Instructions**

- Working time – 50 minutes
- Write using blue or black pen
- Write your student number in the space provided at the top of this page
- NESA approved calculators may be used
- A reference sheet is provided
- Show ALL your working
- Marks may not be awarded for careless or badly arranged working
- Answer ALL questions in the spaces provided
- Marks are as indicated on the paper

**Total Marks: 56**

**Topics covered:**

- Functions and graphs
- Trigonometry
- Logarithms and exponentials
- Further graphs
- Differentiation

**Marks**

**Question 1 (4 marks)**

Given that $\log_a 3 = 1.7$ and $\log_a 5 = 2.5$ , find the value of:

(a) $\log_a(15a)$ **2**

(b) $\log_a\left(\frac{9}{5}\right)$ **2**

**Question 2 (5 marks)**

Given that $f(x) = 3x^2 - x$:

(i) find $f(x+h)$ in expanded simplified form. **2**

(ii) use $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ to find $f'(x)$ from first principles. **3**

**Question 3 (5 marks)**

(i) Show that $\frac{\sin^2\theta}{1-cos\theta} - \frac{\sin^2\theta}{1+\cos\theta} = 2\cos\theta$ **3**

(ii) Hence, solve $\frac{\sin^2\theta}{1-cos\,\theta} - \frac{\sin^2\theta}{1+cos\theta} = 1$ , $-90° \leq \theta \leq 90°$ **2**



**Question 4 (6 marks)**

Differentiate the following:

(a) $y = 1 - 3x^2$ **1**

(b) $f(x) = \frac{4}{\sqrt{x}} - \sqrt[3]{x}$ **2**

(c) $y = x^2\sqrt{1-x}$ **3**

**Question 5 (8 marks)**

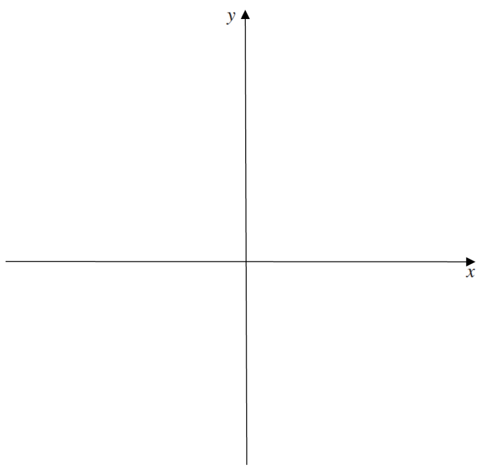
Consider the function $f(x) = \frac{2x}{x^2-1}$ .

(i) Show, with necessary working, whether it is an odd function, even function or neither. **2**

(ii) Find the equation(s) of any vertical asymptotes. **1**

(iii) By first finding $\frac{d}{dx}\frac{2x}{x^2-1}$ show that the derivative is always negative. **3**

(iv) Sketch the function, showing all asymptotes and intercepts. **2**






**Question 6 (6 marks)**

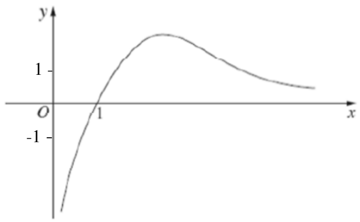
Solve for $x$:

(a) $\log_{10}(x+4) - \log_{10}(x-5) = 1$ **3**

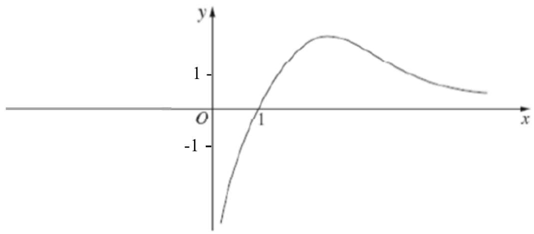
(b) $2(4^x) - 17(2^x) + 8 = 0$ **3**

**Question 7 (6 marks)**

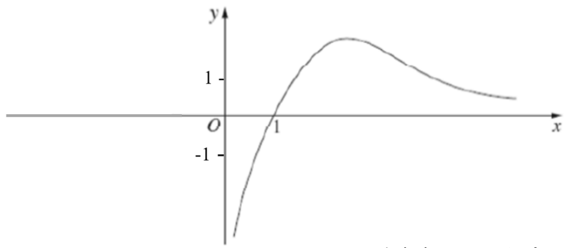
The graph below shows the function of $y = g(x)$.



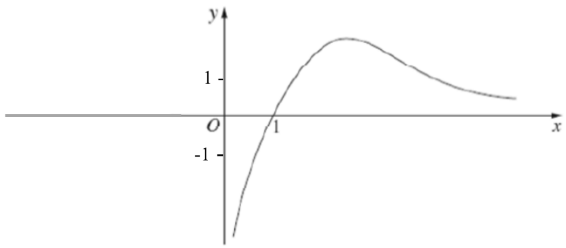

(i) Sketch the graph of $|y| = g(x)$. **2**




(ii) Sketch the graph of $y = \frac{1}{g(x)}$ . **2**




(iii) Sketch the graph of $y = f(x)$, where $f(x) = \begin{cases} g(x) & x \geq 1 \\ g(2-x) & x < 1 \end{cases}$ **2**






**Question 8 (7 Marks)**

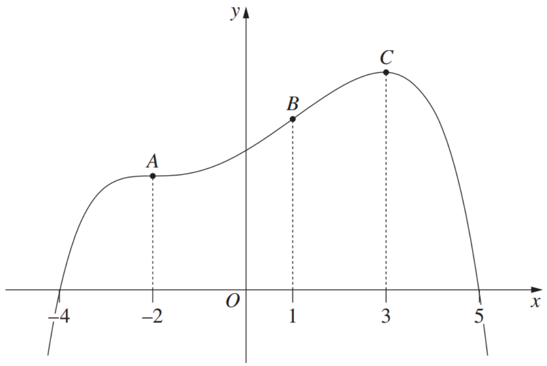
(a) Find the equation of the **normal** to the curve $y = \frac{1}{1+x^2}$ at the point on the curve where $x = 1$. **4**

(b) At what points on the curve $y = x^3 - 6x^2 + 2x - 9$ are the tangents parallel to the line $7x + y = 4$? **3**

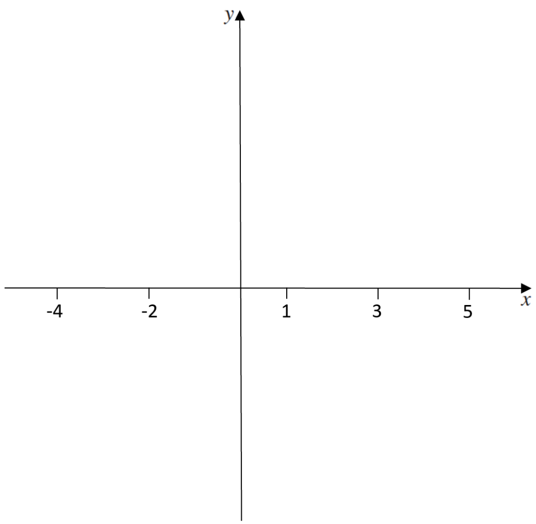
## Question 9 (3 marks)

The diagram shows the graph of a function $f(x)$.

The graph has horizontal gradients at points A and C. Between points A and C, the gradient is steepest at point B.




Sketch the graph of the derivative $f'(x)$ on the axis below. **3**

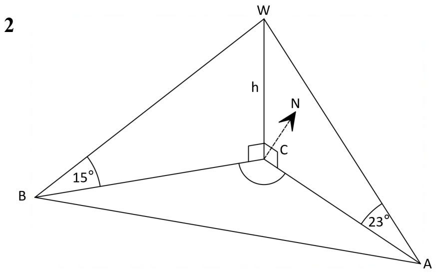





## Question 10 (6 marks)

A man walking along a straight road on a flat floodplain stops at point A and notices a windmill on a bearing of 300°. He measured the angle of elevation to the top of the windmill (W) to be 23°. After walking a further 200m along the straight road, he stops at point B and notices the windmill is now on a bearing of 025°. He measured the angle of elevation to the top of the windmill to be 15°.

(i) Show that $\angle ACB$ is 85°. **2**




(ii) Show the length of BC is $h\tan 75°$. **1**

(iii) Calculate the height, $h$, of the windmill to 1 decimal place. **3**

End of Paper

Q1. Year 11 Assessment Task 2 Friday 26th June

(a) $\log_a(15a)$

$= \log_a 15 + \log_a a$

$= \log_a 3 + \log_a 5 + \log_a a$

$= 1.7 + 2.5 + 1$

$= 5.2$

(b) $\log_a\left(\frac{9}{5}\right)$

$= \log_a 3^2 - \log_a 5$

$= 2(\log_a 3) - \log_a 5$

$= 2 \times 1.7 - 2.5$

$= 0.9$

Q2. (i) $f(x+h)$

$= 3(x+h)^2 - (x+h)$

$= 3x^2 + 6xh + 3h^2 - x - h$

(ii) $f'(x) = \lim_{h \to 0}\left[\frac{3x^2+6xh+3h^2-x-h-(3x^2-x)}{h}\right]$

$= \lim_{h \to 0}\left[\frac{6xh+3h^2-h}{h}\right]$

$= \lim_{h \to 0}\left[\frac{h(6x-1+3h)}{h}\right]$

$= \lim_{h \to 0}(6x-1+3h)$

$= 6x - 1$



Q3. (i)

$LHS = \frac{\sin^2\theta}{1-\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta}$

$= \frac{\sin^2\theta(1+\cos\theta) - \sin^2\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{2\sin^2\theta\cos\theta}{1-\cos^2\theta}$

$= \frac{2\sin^2\theta\cos\theta}{\sin^2\theta}$

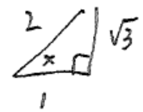
$= 2\cos\theta$

$= RHS$

(ii) Solve $\frac{\sin^2\theta}{1-\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta} = 1$

$\therefore 2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$




$\theta = -60°, 60°$

Q4

(a) $y = 1 - 3x^2$

$$\frac{dy}{dx} = -6x$$

(b) $y = \frac{4}{\sqrt{x}} - \sqrt[3]{x}$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{3}x^{-\frac{2}{3}}$$

(c) $y = x^2\sqrt{1-x}$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{2}(1-x)^{-\frac{1}{2}} \times (-1) + \sqrt{1-x} \times 2x$$

$$\frac{dy}{dx} = 2x\sqrt{1-x} - \frac{x^2}{2\sqrt{1-x}}$$

Q5 $f(x) = \frac{2x}{x^2-1}$

(i) $f(-x) = \frac{2(-x)}{(-x)^2 - 1}$

$$= \frac{-2x}{x^2-1}$$

$$= -\left[\frac{2x}{x^2-1}\right]$$

$$= -f(x)$$

$\therefore$ an odd function

Q5 (ii) Vertical asymptotes at $x = 1$ and $x = -1$.

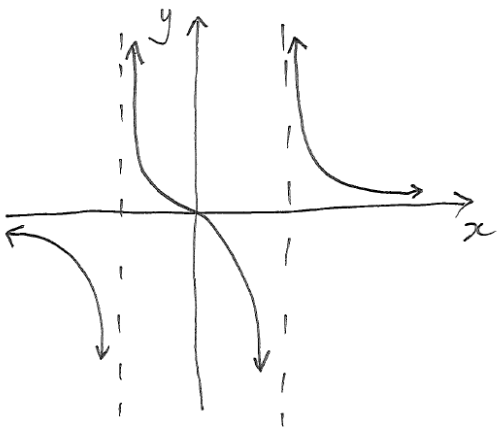
(iii) $\frac{d}{dx}\left[\frac{2x}{x^2-1}\right]$

$$= \frac{(x^2-1)\times 2 - 2x \times (2x)}{(x^2-1)^2}$$

$$= \frac{-2-2x^2}{(x^2-1)^2}$$

Now $(x^2-1)^2$ is always positive and the numerator is always $< -2$ (negative). Hence the derivative is a negative value for all $x$.

(iv) Hence the graph:




Q6

(a) $\log_{10}(x+4) - \log_{10}(x-5) = 1$

$$\log_{10}\left(\frac{x+4}{x-5}\right) = 1$$

$$\frac{x + \cancel{4} 4}{x-5} = 10$$

$$x + 4 = 10x - 50$$

$$54 = 9x$$

$$x = 6$$

(b) $2(4^x) - 17(2^x) + 8 = 0$

Let $m = 2^x$

$$m^2 = (2^x)^2 = 2^{2x} = 4^x$$

$$2m^2 - 17m + 8 = 0$$

$$(2m-1)(m-8) = 0$$

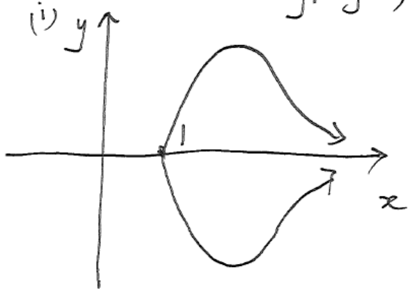
$$m = 8 \text{ or } \frac{1}{2}$$

$$\therefore 2^x = 8 \text{ or } \frac{1}{2}$$

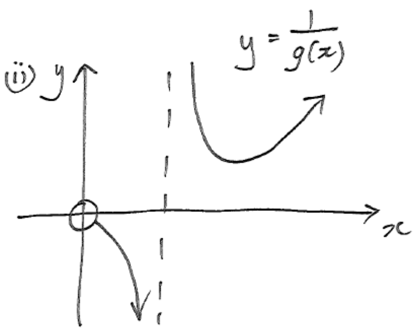
$$m = 3 \text{ or } -1$$

Q7

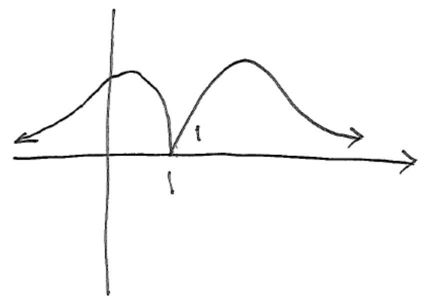
(i)




(ii)




(iii)



Q8//

(a) $y = \frac{1}{1+x^2}$

$f'(x) = -1(1+x^2)^{-2} \cdot 2x$

$= \frac{-2x}{(1+x^2)^2}$

$\therefore f'(1) = \frac{-2}{4} = -\frac{1}{2}$

So m of normal is 2.

$\therefore$ Equation of normal is

$y - \frac{1}{2} = 2(x-1)$

$y = 2x - 1\frac{1}{2}$

(b) $7x + y = 4$

has $m = -7$

$\frac{dy}{dx} = 3x^2 - 12x + 2$

Solve $3x^2 - 12x + 2 = -7$

$3x^2 - 12x + 9 = 0$

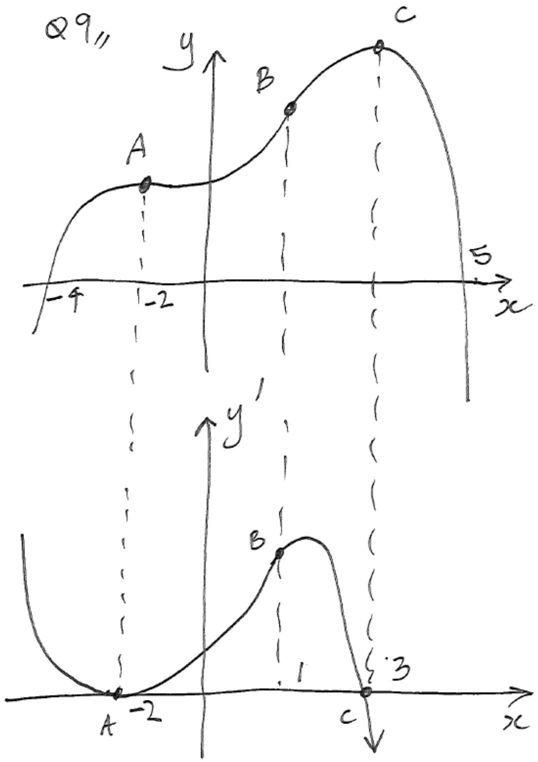
$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1$ and $3$

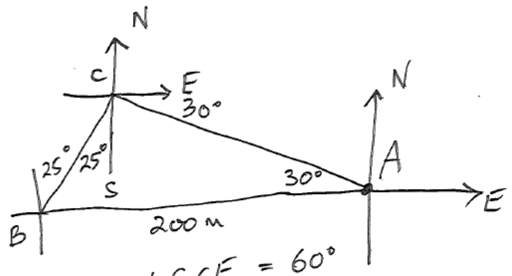
So $(1, -12)$ and $(3, -30)$ are the required points.

Q9//




Q10//

(i)




Now $\angle SCE = 60°$

and $\angle SCB = 25°$

$\therefore \angle ACB = 85°$, as required.

(ii) In $\Delta$ WBC,

$\tan 15° = \frac{h}{BC}$ and $\tan 75° = \frac{BC}{h}$

$\therefore BC = \frac{h}{\tan 15°}$ So $BC = h \tan 75°$, as required.

(iii) Similarly, $AC = h \tan 67°$

In $\Delta$ ABC, by the Cosine Rule,

$200^2 = (h \tan 75°)^2 + (h \tan 67°)^2 - 2(h \tan 75°)(h \tan 67°) \times \cos 85°$

$40000 = h^2[\tan^2 75° + \tan^2 67° - 2 \times \tan 75° \times \tan 67° \times \cos 85°]$

$40\,000 = h^2 \times [17.9457]$

$h^2 = 2228.946$

$h = 47.2$ metres.