Year 11 Mathematics Extension 1
Assessment Task June 2007

Use a pen. Show all necessary working. Do not use liquid paper. Put one line through your deletions.

Marks

Q1 Find all angles θ, for $0^o \leq \theta \leq 360^o$, for which $\sin 2\theta = \frac{1}{2}\cos\theta$. 4

Q2 Given that $\cos 2\theta = 1 - 2\sin^2\theta$, show that $\frac{\cos x - \cos(x+2\theta)}{2\sin\theta} = \sin(x+\theta)$. 3

Q3
a. Show that: $\cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + 60^o)$. 3
b. Hence solve the equation $\cos\theta - \sqrt{3}\sin\theta = 1$ for θ in the interval $0^o \leq \theta \leq 360^o$. 2

Q4 If $\tan(A+B) = \frac{1}{2}$ and $\tan B = \sqrt{2}$, find the value of $\tan A$. 3

Q5
a. Show that $\frac{1+\cos 2A}{\sin 2A} = \cot A$. 2
b. Hence find the exact value of $\cot 15^o$. 1

Q6 $A(-1, 5)$ and $B(5, -4)$ are two points. Find the coordinates of the point P which divides the interval AB internally in the ratio $2 : 1$. 2

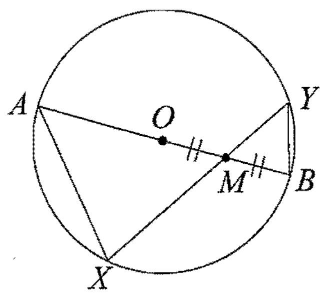
Q7 A is the point $(-2, 1)$ and B is the point (x, y). The point $P(13, -9)$ divides AB externally in the ratio $5 : 3$. Find the values of x and y. 3

Q8 Find the acute angle between the lines $2x - y = 0$ and $x + 3y = 0$, giving the answer correct to the nearest minute. 3

Q9 The acute angle between the line $x - 2y + 3 = 0$ and the line $y = mx$ is 45^o.
a. Show that $\left|\frac{2m-1}{m+2}\right| = 1$. 2
b. Find the possible values of m. 2

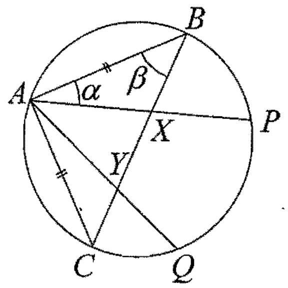
Q10 Two points A and B are on a circle and C is the other end of diameter through A. AE is the line from A perpendicular to the tangent at B.
a. Draw a careful diagram showing this information.
b. Prove that AB bisects <CAE 3

Q11 In the diagram below, AB is a diameter of a circle, whose centre is the point O. The chord XY passes through M, the mid-point of OB. AX and BY are joined.



a. Prove the two triangles formed (triangles AXM and MYB) are similar. 2
b. If $XM = 8$ cm and $YM = 6$ cm, find the length of the radius of the circle. 3

Q12



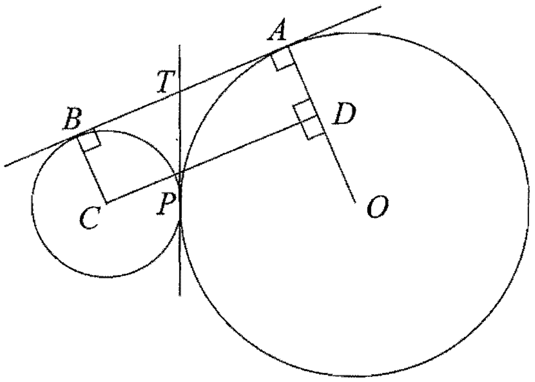
Let $ABPQC$ be a circle such that $AB = AC$, AP meets BC at X, and AQ meets BC at Y, as in the diagram. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$.

a. Prove that $\angle BQP = \alpha$. 1

b. Prove that $\angle BQA = \beta$. 2

c. Prove that $PQYX$ is a cyclic quadrilateral. 3

Q13



Two circles, one with centre C and radius 1 cm and the other with centre O and radius 3 cm, touch each other externally at P. AB is a common tangent to the two circles. CD is drawn perpendicular to OA to complete the rectangle $ABCD$. The common tangent to the two circles at P meets AB at T.

a. Find the exact length of AB giving reasons for our answer. 2

b. Find the exact length of PT giving reasons for our answer. 2

Total 48

YR 11 EXT 1 JUNE 2002 SOLNS

Q1. $\sin 2\theta = \frac{1}{2}\cos\theta$

$2\sin\theta\cos\theta - \frac{1}{2}\cos\theta = 0$ — 1

$\cos\theta(2\sin\theta - \frac{1}{2}) = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{4}$ — 1

$\therefore \theta = 90^\circ, 270^\circ$ — 1

or $\theta = 14^\circ 29', 165^\circ 31'$ — 1

Q2. $LHS = \dfrac{\cos x - (\cos x\cos 2\theta - \sin x\sin 2\theta)}{2\sin\theta}$ — 1

$= \dfrac{\cos x - \cos x(1-2\sin^2\theta) + 2\sin x\sin\theta\cos\theta}{2\sin\theta}$ — 1

$= \dfrac{\cos x - \cos x + 2\sin^2\theta\cos x + 2\sin x\sin\theta\cos\theta}{2\sin\theta}$

$= \dfrac{2\sin\theta(\sin\theta\cos x + \cos\theta\sin x)}{2\sin\theta}$ — 1

$= \sin(x+\theta)$

$= RHS$

Q3 (a) let $\cos\theta - \sqrt{3}\sin\theta = R\cos(\theta+\alpha)$

$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ — 1

$\therefore R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ $\}$ $\therefore \tan\alpha = \sqrt{3}$ $\therefore \alpha = 60^\circ$ — 1

$R = \sqrt{1+3} = 2$ — 1

$\therefore \cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + 60^\circ)$

(b) $\therefore 2\cos(\theta+60^\circ) = 1$

$\cos(\theta+60^\circ) = \frac{1}{2}$

$\theta + 60^\circ = 60^\circ, 300^\circ, 420^\circ$ — 1

$\therefore \theta = 0, 240^\circ, 360^\circ$ — 1

Q4 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ — 1

$\dfrac{1}{2} = \dfrac{T+\sqrt{2}}{1-T\sqrt{2}}$

$1 - T\sqrt{2} = 2T + 2\sqrt{2}$ — 1

$2T + T\sqrt{2} = 1 - 2\sqrt{2}$

$T(2+\sqrt{2}) = 1 - 2\sqrt{2}$

$T = \dfrac{1-2\sqrt{2}}{2+\sqrt{2}}$ — 1

Q5. (a) $LHS = \dfrac{\cos^2 A + \sin^2 A + \cos^2 A - \sin^2 A}{2\sin A\cos A}$ — 1 — 1

$= \dfrac{2\cos^2 A}{2\sin A\cos A}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A$

$= RHS$

(b) $\cot 15^\circ = \dfrac{1+\cos 30^\circ}{\sin 30^\circ}$

$= \dfrac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ — 1

$= 2+\sqrt{3}$

Q6 $m:n = 2:1$

$P = \left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$ — 1

$= \left(\dfrac{2\times 5 + 1\times -1}{2+1}, \dfrac{2\times -4 + 1\times 5}{2+1}\right)$

$= \left(\frac{9}{3}, \frac{-3}{3}\right)$

$= (3, -1)$ — 1

Q7. $m:n = 5:-3$

$P = \left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$

$\therefore 13 = \dfrac{5x - 3\times 2}{2}$, $-9 = \dfrac{5y - 3\times 1}{2}$ — 1

$26 = 5x + 6$ $\quad$ $-18 = 5y - 3$

$5x = 20$ $\quad$ $5y = -15$

$x = 4$ $\quad$ $y = -3$ — 1 — 1

Q8. $y = 2x \therefore m_1 = 2$

$y = -\frac{1}{3}x \therefore m_2 = -\frac{1}{3}$ $\}$ — 1

$\tan\alpha = \left|\dfrac{m_1 - m_2}{1+m_1m_2}\right|$ — 1

$= \left|\dfrac{2 - -\frac{1}{3}}{1 + 2\times -\frac{1}{3}}\right|$

$= \left|\dfrac{\frac{7}{3}}{\frac{1}{3}}\right|$

$= 7$

$\therefore \alpha \doteqdot 81^\circ 52'$ — 1

Q9 (a) $2y = x+3$

$y = \frac{1}{2}x + 1\frac{1}{2}$

$m_2 = \frac{1}{2}$

$\tan 45^\circ = 1$ — 1

$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore 1 = \left|\frac{m - \frac{1}{2}}{1 + m \times \frac{1}{2}}\right|$ — 1

$\therefore \left|\frac{2m-1}{m+2}\right| = 1$

(b) $2m - 1 = m + 2$

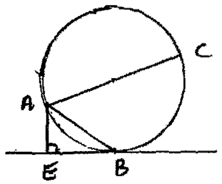
$\underline{m = 3}$ — 1

or $2m - 1 = -(m+2)$

$3m = -1$

$\underline{m = -\frac{1}{3}}$ — 1

Q10. (a)



(b) Draw BC

Let $\angle BAE = x^\circ$

$\therefore \angle ABE = (90 - x)^\circ$ ($\angle$s sum of right $\triangle ABE$) — 1

$\therefore \angle ACB = (90 - x)^\circ$ ($\angle$ bet. tang & chord $= \angle$ in alt. seg.) — 1

$\angle ABC = 90^\circ$ ($\angle$ in a semi-circle) — 1

$\therefore \angle CAB = x^\circ$ ($\angle$ sum of $\triangle ABC$)

$\therefore$ AB bisects $\angle CAE$ (Both $= x^\circ$)

Q11 (a) In $\triangle$'s AXM, MYB

$\angle X = \angle B$ ($\angle$'s on circ. stand on same arc are =)

$\angle A = \angle Y$ (" ") — 1

$\angle AMX = \angle BMY$ (Vert. opp. $\angle$'s)

$\therefore \underline{\triangle AXM \,|||\, \triangle MYB \text{ (Matching } \angle\text{'s equal)}}$ — 1

(b) $AM . MB = XM . YM$ — 1

(Prod. of Intercepts of chords are =)

$= 48$

$\therefore (AO + OM) \times OM = 48$

$AO = OB = 2 \times OM$

$\therefore (2OM + OM) . OM = 48$ — 1

$3OM^2 = 48$

$OM^2 = 16$

$OM = 4$

$\therefore$ $\underline{\text{radius} = 8 \text{ cm}}$ — 1

Q12 (a) $\angle BAP = \alpha$ given

$\therefore \angle BQP = \alpha$ ($\angle$'s on circ. st. on same arc are =) — 1

(b) $AB = AC$ (given) — 1

$\therefore \angle ABC = \beta$ (given)

$\therefore \underline{\angle BQA = \beta}$ ($\angle$'s on circ. st. on same arc are =) — 1

(c) $\angle AQP = \angle BQP + \angle BQA$

$= \alpha + \beta$ (from above) — 1

$\angle AXC = \alpha + \beta$ (Ext. $\angle$ of $\triangle ABX$ = sum of int. opp) — 1

$\therefore$ $\underline{\text{PQYX is cyclic (Ext. } \angle \text{ of cyc. quad = int. opp. } \angle\text{'s)}}$ — 1

Q13. (a) C, P, O are collinear

(centres & point of contact are collinear)

$\therefore CD^2 = CO^2 - DO^2$ (Pyth)

$= (1+3)^2 - (3-1)^2$

$= 4^2 - 2^2$ — 1

$= 12$

$\underline{AB = CD = \sqrt{12} = 2\sqrt{3} \text{ cm}}$ — 1

(b) $PT = TB$ (Tangs from ext pt are =)

$PT = TA$ (Tangs from ext pt are =) — 1

$\therefore BT = TA$ (Both $= PT$)

$= \frac{1}{2} BA = \frac{1}{2} CD = \sqrt{3}$

$\therefore PT = \sqrt{3}$ — 1

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