BAULKHAM HILLS HIGH SCHOOL

HALF YEARLY EXAMINATION

2009

YEAR 11

# MATHEMATICS
# Extension 1

Time Allowed:

90 minutes
*(plus 5 mins reading time)*

**Instructions:**

- Write on the paper provided
- Do not write on this question paper
- Show all working
- Use black or blue pens only
- Start a new page for each question
- Write your name and your teacher's name at the top of each page
- Board approved calculators may be used

**QUESTION 1** **Start on a new page**

**Marks**

a) Solve $(x+4)^2 = 3(x+4)$ **2**

b) Solve $\dfrac{1}{|x+5|} \geq 1$ **2**

c) Find the largest possible domain of **3**

$$f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$$

d) Solve $4^{2x+5} = 8$ **2**

e) State the range of $f(x) = \dfrac{|x|}{\sqrt{x^2+4}}$ **2**

**QUESTION 2** **Start on a new page**

a) If $2x + y + 3 + \sqrt{4x-3y} = 5 + 2\sqrt{6}$ , find the value of $x$ and $y$ **3**

b) Is the function $f(x) = x^3 \cos x$ odd, even or neither? Justify your answer with appropriate working. **2**

c) If $a + b = 8$ and $ab = 17$, find the value of $\dfrac{2}{a}+\dfrac{2}{b}$ **2**

d) Find the exact value of $\cos 105^\circ$ **2**

e) Prove $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2 \quad (\sin\theta \neq 0\ ,\ \cos\theta \neq 0)$ **3**

## QUESTION 3 Start on a new page

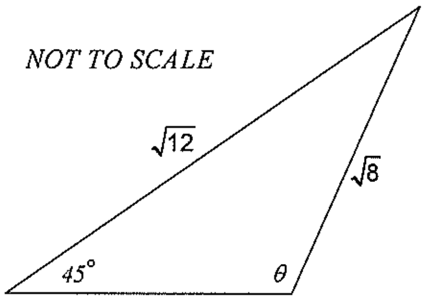
**Marks**

a) Solve $\dfrac{2x+1}{2x-1} \geq 2$ **3**

b) Find the values of $\tan x$ when $\tan^2 x + \sec^2 x = 7$ **2**

c) Fully simplify $\sqrt{\dfrac{\sqrt{50}-\sqrt{32}}{\sqrt{8}}}$ without the aid of a calculator. Show all necessary working. **2**

d) Use the sine rule to find the value of $\theta$ where $\theta$ is obtuse **3**

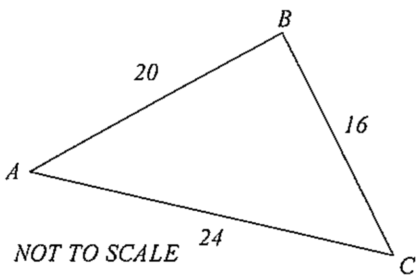



## QUESTION 4 Start on a new page

a) Solve for $0° \leq x \leq 360^0$ **3**

$\cos 2x = \sin x$

b) The sides of a triangle ABC are 16, 20, 24 cm




i) Find the exact value of cosA **1**

ii) Hence, find the exact area of the triangle **2**

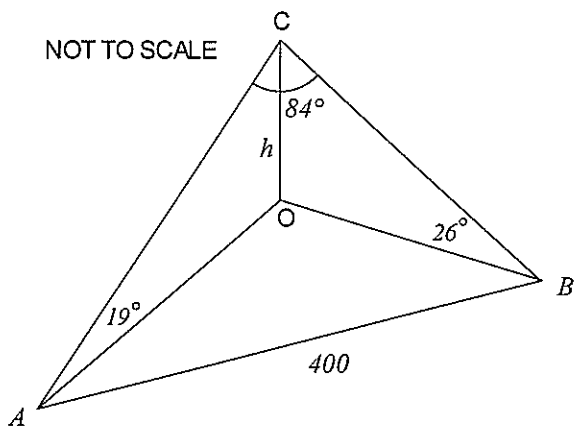
ii) Hence, find the distance from B to AC **2**

**QUESTION 5 Start on a new page**

a) Simplify $\dfrac{5^n + 5^{n-2}}{5^{n-1}}$ **2**

b) Factorise $a^2 - b^2 + 2b - 1$ **2**

c) A road runs by the side of a hill. From two points, A and B on the road, 400m apart, the angles of elevation of the top of the hill are $19^\circ$ and $26^\circ$ respectively. If the angle subtended by AB at the top of the hill is $84^\circ$, find the height of the hill to the nearest metre. **3**




d) If $2\sin(\theta - 60^0) = \cos(\theta - 60^0)$ express $\tan\theta$ in the form $a + b\sqrt{3}$ where $a$ and $b$ are integers. **4**

**QUESTION 6 Start on a new page**

a) Prove that $\dfrac{2\tan A}{1 + \tan^2 A} = \sin 2A$ **2**

b) i) Sketch the curves $y = |x - 3|$ and $y = 5 - |x|$, on the same diagram, showing all necessary features. **2**

ii) Hence, or otherwise, solve $|x - 3| + |x| < 5$ **2**

c) Sketch the graph of $y = \dfrac{x+2}{|x|}$ **3**

d) If $\cos x \cos y = \frac{1}{4}\left(\sqrt{3} - \sqrt{2}\right)$ and $\sin x \sin y = \frac{1}{4}\left(\sqrt{3} + \sqrt{2}\right)$ find the smallest positive value of $(x + y)$. **3**

**END OF EXAMINATION.**

# YEAR 11 EXTENSION 1 HALF YEARLY 2009

1 a) $(x+4)^2 = 3(x+4)$

$(x+4)(x+4-3) = 0$

$(x+4)(x+1) = 0$

$\therefore x = -4, -1$

(1) (1)

b) $\dfrac{1}{|x+5|} \geq 1 \quad (x \neq -5)$

$1 \geq |x+5|$

$-1 \leq x+5 \leq 1$

$-6 \leq x \leq -4 \quad (x \neq -5)$

c) $9 - x^2 \geq 0 \qquad x+1 > 0$

$x > -1$


$-3 \leq x \leq 3 \qquad x > -1$

Both exist when $-1 < x \leq 3$

d) $2^{2(2x+5)} = 2^3$

$\therefore 2x+5 = 3/2$

$2x = -7/2$

$x = -7/4$

e) $y \geq 0$ and as

$x \to \infty \quad f(x) \to 1$

$x \to -\infty \quad f(x) \to 1$

$\therefore 0 \leq f(x) < 1$

2 a) $2x+y+3 = 5$ (1)

$4x-3y = 24$ (2)

(1)×2 $4x+2y = 4$ (3)

(3)−(2) $5y = -20$

$y = -4$

sub in (1) $2x-4 = 2$

$2x = 6$

$x = 3$

(1) for set up of both eq'ns

$\boxed{5}$(b) $a^2 - (b^2 - 2b + 1)$

$= a^2 - (b-1)^2$

$= (a+b-1)(a-(b-1))$

$= (a+b-1)(a-b+1)$

(c) $\dfrac{2}{a} + \dfrac{2}{b}$

$= \dfrac{2b+2a}{ab}$

$= \dfrac{2(a+b)}{ab}$

$= \dfrac{2 \times 8}{17}$

$= \dfrac{16}{17}$

d) $\cos 105° = \cos(60+45)°$

$= \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{3}}{2} \dfrac{1}{\sqrt{2}}$

$= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$ or $\dfrac{\sqrt{2}(1-\sqrt{3})}{4}$

e) LHS $= \dfrac{\sin 3\theta \cos\theta}{\sin\theta\cos\theta} - \dfrac{\cos 3\theta \sin\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

$= \dfrac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= 2$

= RHS

$\therefore \dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$

3(a) $\frac{2x+1}{2x-1} \geq 2 \quad (x \neq \frac{1}{2})$

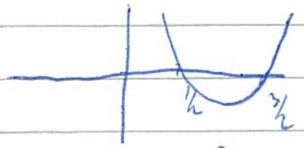
$(2x+1)(2x-1) \geq 2(2x-1)^2$

$4x^2 - 1 \geq 2(4x^2 - 4x + 1)$

$4x^2 - 1 \geq 8x^2 - 8x + 2$

$0 \geq 4x^2 - 8x + 3$

$(2x-1)(2x-3) \leq 0$




$\frac{1}{2} < x \leq \frac{3}{2}$

(b) $\tan^2 x + \sec^2 x = 7$

$\tan^2 x + \tan^2 x + 1 = 7$

$2\tan^2 x = 6$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

(c) $\sqrt{\frac{5\sqrt{2} - 4\sqrt{2}}{2\sqrt{2}}}$

$= \sqrt{\frac{1\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}}}$

$= \frac{1}{\sqrt{2}}$

(d) $\frac{\sin\theta}{\sqrt{12}} = \frac{\sin 45^\circ}{\sqrt{8}}$

$\sin\theta = \frac{\cancel{2}\sqrt{3}\ \frac{1}{\sqrt{2}}}{\cancel{2}\sqrt{2}}$

$\sin\theta = \frac{\sqrt{3}}{2}$

acute $\angle = 60^\circ$

$\theta = 180 - 60$

$= 120^\circ$

4a) $1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

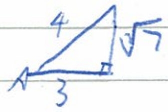
$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = 30^\circ, 150^\circ \qquad x = 270^\circ$

$\therefore x = 30^\circ, 150^\circ, 270^\circ$

(b) $\cos A = \frac{20^2 + 24^2 - 16^2}{2 \times 20 \times 24}$

$\cos A = 0.75$




$\therefore$ Area $= \frac{1}{2}\, 20 \times 24 \times \sin A$

$= 240 \times \frac{\sqrt{7}}{4}$

$= 60\sqrt{7}$

(ii) Area $= \frac{24 \times h}{2}$

$60\sqrt{7} = 12h$

$h = \frac{60\sqrt{7}}{12}$

$h = 5\sqrt{7}$

5(a) $\frac{5^x + 5^{x-2}}{5^{x-1}}$

$= \frac{\cancel{5^{x-2}}(5^2 + 1)}{\cancel{5^{x-2}} \times 5^1}$

$= \frac{26}{5}$

$\boxed{Q\,2\,(b)}$ $f(-x) = (-x)^3 \cos(-x)$

$= -x^3 \cos x$ since $\cos(-x) = \cos$

$= -f(x)$

$\therefore$ function is odd

5(c) $\sin 19^\circ = \frac{h}{AC}$ ✓

$\frac{h}{\sin 19^\circ} = AC$

(1) expressions for both BC and AC

Also $\sin 26^\circ = \frac{h}{BC}$

$\frac{h}{\sin 26^\circ} = BC$

In $\triangle ABC$ ✓

$$400^2 = \left(\frac{h}{\sin 26^\circ}\right)^2 + \left(\frac{h}{\sin 19^\circ}\right)^2 - \frac{2h}{\sin 26^\circ} \times \frac{h}{\sin 19^\circ}$$

$$160000 = \frac{h^2}{\sin^2 26^\circ} + \frac{h^2}{\sin^2 19^\circ} - \frac{2\cos 84^\circ h^2}{\sin 26^\circ \sin 19^\circ}$$

$$160000 = h^2\left(\frac{1}{\sin^2 26^\circ} + \frac{1}{\sin^2 19^\circ} - \frac{2\cos 84^\circ}{\sin 26^\circ \sin 19^\circ}\right)$$

$$h^2 = \frac{160000}{\operatorname{cosec}^2 26^\circ + \operatorname{cosec}^2 19^\circ - 2\cos 84^\circ \operatorname{cosec} 26^\circ \operatorname{cosec} 19^\circ}$$

$h^2 = 12145.707\ldots$

$h = 110.207\ldots$ ✓

$h = 110\text{m}$

5(d) $2\sin(\theta - 60^\circ) = \cos(\theta - 60^\circ)$

$2\tan(\theta - 60^\circ) = 1$

$\tan(\theta - 60^\circ) = \frac{1}{2}$ ✓

$\frac{\tan\theta - \tan 60^\circ}{1 - \tan\theta \tan 60^\circ} = \frac{1}{2}$ ✓

$\frac{\tan\theta - \sqrt{3}}{1 - \tan\theta\sqrt{3}} = \frac{1}{2}$

$2\tan\theta - 2\sqrt{3} = 1 - \sqrt{3}\tan\theta$

$\tan\theta(2+\sqrt{3}) = 1 + 2\sqrt{3}$ ✓

$\tan\theta = \frac{1+2\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2+3\sqrt{3}-6}{4-3}$

$\tan\theta = -4+3\sqrt{3}$ ✓

6(a) LHS $= \frac{\frac{2\sin A}{\cos A}}{1+\frac{\sin^2 A}{\cos^2 A}}$ ✓

$= \frac{2\sin A\cos A}{\cos^2 A + \sin^2 A}$

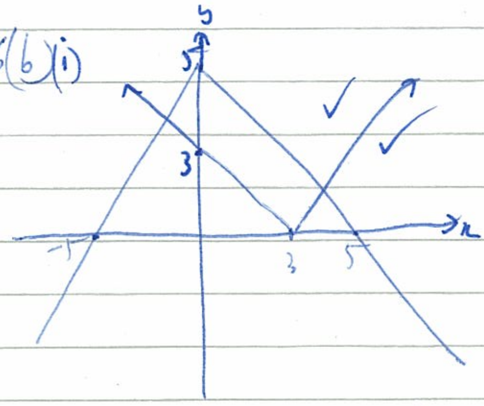
$= \frac{\sin 2A}{1}$ ✓

$= \sin 2A$

$=$ RHS

$\therefore \frac{2\tan A}{1+\tan^2 A} = \sin 2A$

6(b)(i)




1 each for graphs with intercept or 1 for both graphs without intercepts

6(b)(ii) $|x-3| < 5-|x|$

Pts of intersection:

$-(x-3) = 5-(-x)$ & $x-3 = 5-x$

$-x+3 = 5+x$ $\quad$ $2x = 8$

$-2 = 2x$ $\quad$ $x = 4$

$x = -1$ ✓

$\therefore -1 < x < 4$ is solution ✓

6(c) If $x<0$

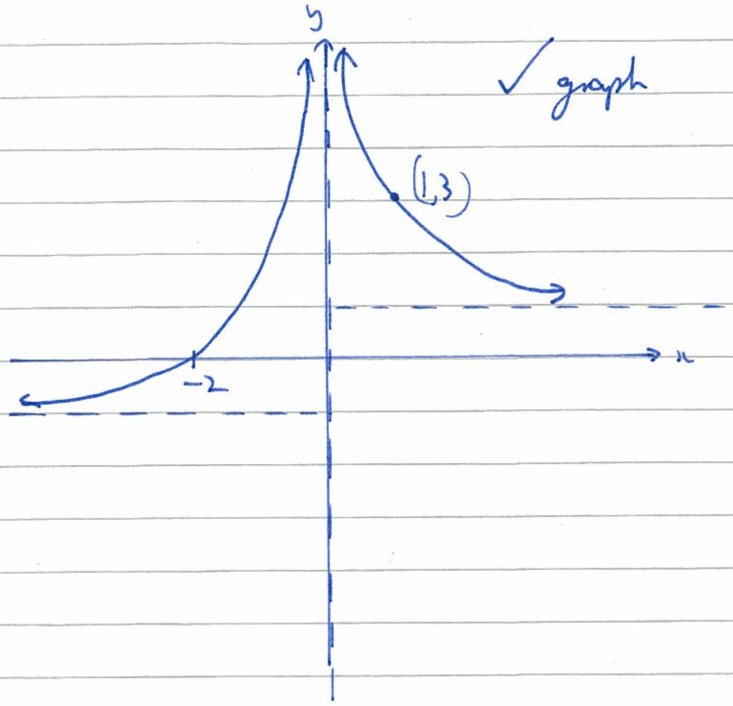
$y = \frac{x+2}{-x}$ ✓

$= -1 - \frac{2}{x}$

If $x>0$

$y = \frac{x+2}{x}$

$y = 1 + \frac{2}{x}$ ✓




✓ graph

6(d) $\cos(x+y) = \cos x \cos y - \sin x \sin y$ ✓

$= \frac{1}{4}(\sqrt{3}-\sqrt{2}) - \frac{1}{4}(\sqrt{3}+\sqrt{2})$

$= \frac{-2\sqrt{2}}{4}$

$= \frac{-\sqrt{2}}{2}$

$= -\frac{1}{\sqrt{2}}$ ✓

$x+y = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$x+y = 135°$ is smallest positive value ✓