Section I - (5 marks). Attempt Questions 1-5.
Use the multiple-choice answer sheet on the previous page.

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1 Which one of the following vectors is parallel to vector $\begin{bmatrix}-3\\7\end{bmatrix}$ and has magnitude of $2\sqrt{58}$?

(A) $\begin{bmatrix}-24\\28\end{bmatrix}$ (B) $\frac{-3}{2}\mathbf{i}+\frac{7}{2}\mathbf{j}$ (C) $\begin{bmatrix}3\\-7\end{bmatrix}$ (D) $-6\mathbf{i}+14\mathbf{j}$

2 The vectors $\mathbf{a}=3\mathbf{i}-x\mathbf{j}$ *and* $\mathbf{b}=2\mathbf{i}+5\mathbf{j}$ are perpendicular. What is the value of x?

(A) $\frac{-15}{2}$ (B) $\frac{-6}{5}$ (C) $\frac{6}{5}$ (D) $\frac{15}{2}$

3 The angle between the vectors $\mathbf{a}=2\mathbf{i}+3\mathbf{j}$ *and* $\mathbf{b}=3\mathbf{i}-\mathbf{j}$ is closest to:

(A) 89° (B) 91° (C) 75° (D) 105°

4 In the parallelogram $ABCD$, the point of intersection of their diagonals is O. $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{BC}=\mathbf{b}$.
The vector $\overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ is equal to:

(A) $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ (B) $\frac{1}{2}(\mathbf{a}-\mathbf{b})$ (C) $\frac{1}{2}\mathbf{b}+\mathbf{a}$ (D) $\frac{1}{2}(\mathbf{b}-\mathbf{a})$

5 The vector projection of $\mathbf{a}=2\mathbf{i}-5\mathbf{j}$ onto $\mathbf{b}=-3\mathbf{i}-\mathbf{j}$ is equal to:

(A) $\frac{3}{\sqrt{10}}\mathbf{i}+\frac{1}{\sqrt{10}}\mathbf{j}$ (B) $\frac{-3}{10}\mathbf{i}-\frac{1}{10}\mathbf{j}$ (C) $\frac{3}{10}\mathbf{i}+\frac{1}{10}\mathbf{j}$ (D) $\frac{-3}{\sqrt{10}}\mathbf{i}-\frac{1}{\sqrt{10}}\mathbf{j}$

End of Section I

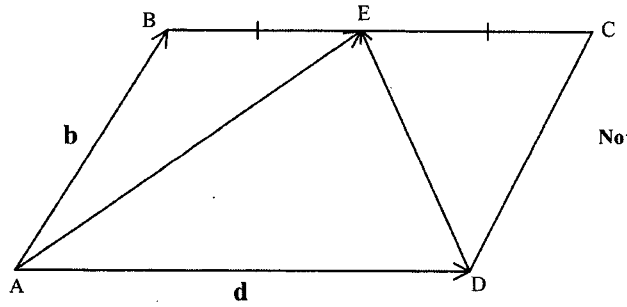
Section II - (28 marks). Attempt Questions 6-10. Answer each question in the space provided. Extra writing space is provided at the back. If you use this space, clearly indicate which question you are answering.

Question 6 (11 marks) | **Marks**

a) The point $C(-7,13)$ has a position vector of **c**. Find the vector $\overrightarrow{OA}$ if $\overrightarrow{AB}$ is equal to **c** and B is the point $(12,-2)$. **2**

b) Find a vector **a** that is perpendicular to vector $\mathbf{b} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ and has a magnitude of 10. **3**

c) $ABCD$ is a parallelogram in which $\overrightarrow{AB} = \mathbf{b}$, $\overrightarrow{AD} = \mathbf{d}$ and E is the midpoint of BC.



i) Express $\overrightarrow{AE}$ and $\overrightarrow{DE}$ in terms of $\mathbf{b}$ and $\mathbf{d}$. **2**

ii) If F is a point on $\overrightarrow{DE}$ and $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DE}$, express $\overrightarrow{DF}$ in terms of $\mathbf{b}$ and $\mathbf{d}$. **1**

iii) Find $\overrightarrow{AF}$ in terms of $\mathbf{b}$ and $\mathbf{d}$ and hence show that F lies on $\overrightarrow{AC}$. **3**

End of question 6

Question 7 (3 marks)

Prove by mathematical induction that for all positive integers $n \geq 1$ **3**

$$1 \times 3 + 2 \times 4 + 3 \times 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$$

End of question 7

Question 8 (3 marks)

Prove by mathematical induction that for all integers $n \geq 0$ **3**

$3^{2n} + 7$ is divisible by 8.

End of Question 8

Question 9 (7 marks)

The points A, B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - \mathbf{j}$.

a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$. **3**

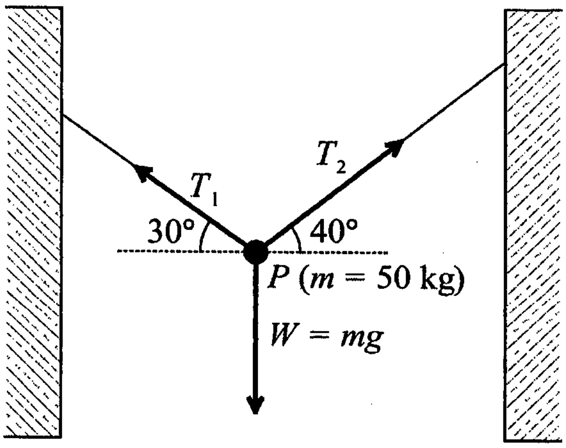
b) Show that ΔABC is a right-angled isosceles triangle with a right angle at B. **2**

c) Find the position vector of the point D such that $ABCD$ forms a square. **2**

End of Question 9

Question 10 (4 marks)

The diagram shows an object, P, of mass 50 kg being suspended by two light inextensible strings of different lengths. The strings are at different angles to the horizontal. One is at 30° and the other is at 40°. The tension in these strings are T_1 and T_2 respectively. Take $g = 9.8\ \text{m/s}^2$.



a) Show that if the object is stationary, $T_1 = \frac{\cos(40°)}{\cos(30°)} T_2$. 1

b) Hence find the tension in each string if the object is stationary, correct to the nearest newton. **3**

End of Paper

Extra writing space

XI- 2019 December Task

Section I - Multiple Choice (5 marks) Circle the correct answer.

1 Which one of the following vectors is parallel to vector $v\begin{bmatrix}-3\\7\end{bmatrix}$ and has magnitude of $2\sqrt{58}$?

(A) $\begin{bmatrix}-24\\28\end{bmatrix}$ (B) $\frac{-3}{2}i+\frac{7}{2}j$ (C) $\begin{bmatrix}3\\-7\end{bmatrix}$ (D) $-6i+14j$

B & D have same direction as $\underset{\sim}{V}$, but D has magnitude $\sqrt{(-6)^2+14^2}=2\sqrt{58}$

2 The vectors $a = 3i - xj$ and $b = 2i + 5j$ are perpendicular. What is the value of x?

(A) $\frac{-15}{2}$ (B) $\frac{-6}{5}$ (C) $\frac{6}{5}$ (D) $\frac{15}{2}$

$a \perp b$ if $a \cdot b = 0 \therefore 3\times 2 + (-x)\times 5 = 0 \therefore x = 6/5$

3 The angle between the vectors $a = 2i + 3j$ and $b = 3i - j$ is closest to:

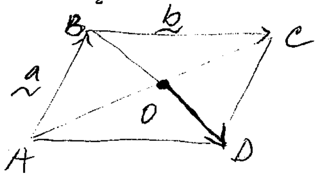
(A) 89° (B) 91° (C) 75° (D) 105°

$\cos\theta = \frac{a \cdot b}{|a|\times|b|} = \frac{2\times 3 + 3\times -1}{\sqrt{13}\times\sqrt{10}} \therefore \theta = 74.74°$

4 In the parallelogram $ABCD$, the point of intersection of their diagonals is O. $\overrightarrow{AB} = a, \overrightarrow{BC} = b$.

The vector $\overrightarrow{OD}$ in terms of a and b is equal to:

(A) $\frac{1}{2}(a+b)$ (B) $\frac{1}{2}(a-b)$ (C) $\frac{1}{2}(b+a)$ (D) $\frac{1}{2}(b-a)$



$\overrightarrow{OD} = \frac{1}{2}(\overrightarrow{BD}) = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{CD})$

$\overrightarrow{OD} = \frac{1}{2}(b - a)$

5 The vector projection of $a = 2i - 5j$ onto $b = -3i - j$ is equal to:

(A) $\frac{1}{\sqrt{10}}(3i+j)$ (B) $\frac{1}{10}(-3i-j)$ (C) $\frac{1}{10}(3i+j)$ (D) $\frac{1}{\sqrt{10}}(-3i-j)$

$proj_b\, a = (a \cdot \hat{b})(\hat{b}) = \frac{(2i-5j)\cdot(-3i-j)}{\sqrt{10}} \times \frac{(-3i-j)}{\sqrt{10}}$

$= \frac{-6+5}{10}(-3i-j) = -\frac{1}{10}(-3i-j) = \frac{1}{10}(3i+j)$

End of Section I

Section II - (28 marks). Attempt Questions 6-10. Answer each question in the space provided. Extra writing space is provided at the back. If you use this space, clearly indicate which question you are answering.

Question 6 (11 marks) **Marks**

a) The point $C(-7,13)$ has a position vector of **c**. Find the vector $\overrightarrow{OA}$ if $\overrightarrow{AB}$ is equal to **c** and B is the point $(12, -2)$. 2

$c = -7i + 13j = \overrightarrow{OC}$ $\quad \overrightarrow{OB} = 12i - 2j$

since $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, let $\overrightarrow{OA} = xi + yj$

$\therefore -7i + 13j = (12i - 2j) - (xi + yj)$ (1)

$\therefore -7 = 12 - x$ & $13 = -2 - y$

$x = 19 \quad \therefore y = -15$

$\therefore \overrightarrow{OA} = 19i - 15j$ or $\begin{bmatrix}19\\-15\end{bmatrix}$ (1)

or $\overrightarrow{OA}$ is a position vector of $(19, -15)$

Marking criteria	Marks
• correct solns	2
• Establishes $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ into equation	1
• Finds coord. of A correctly	1

b) Find a vector **a** that is perpendicular to vector $\mathbf{b} = \begin{bmatrix}3\\-4\end{bmatrix}$ and has a magnitude of 10. 3

let $a = \begin{bmatrix}x\\y\end{bmatrix}$ and given $b = \begin{bmatrix}3\\-4\end{bmatrix}$

since $a \perp b$

$\therefore$ (1) $a \cdot b = 0$

$\therefore 3x - 4y = 0$

and since $|a| = 10$

$\therefore$ (2) $\sqrt{x^2 + y^2} = 10$ (1)

Now solve simultaneously

(1) $x = 4/3\, y$

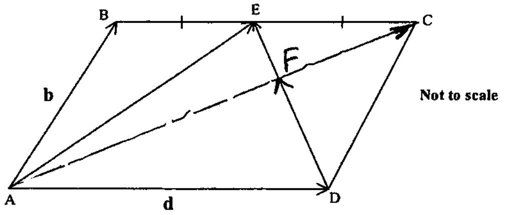
$\therefore$ (2) $\frac{16}{9}y^2 + y^2 = 10^2$

$y^2 = 36 \therefore y = \pm 6$

if $y = 6$	OR if $y = -6$
$\therefore x = 8$	$x = -8$
$\therefore a = 8i + 6j$	$\therefore a = -8i - 6j$ (1)

Marking criteria	Marks
• correct solns	3
• establishes $a \cdot b = 0$ and $\lvert a\rvert = 10$ into simulti eqns & solves the eqns	2
• finds one of the solns.	1

c) $ABCD$ is a parallelogram in which $\overrightarrow{AB} = \mathbf{b}$, $\overrightarrow{AD} = \mathbf{d}$ and E is the midpoint of BC.



i) Express $\overrightarrow{AE}$ and $\overrightarrow{DE}$ in terms of **b** and **d**. **2**

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \underset{\sim}{b} + \frac{1}{2}\underset{\sim}{d}$ (1)

$\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE} = \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{d}$ (1)

Marking criteria	Marks
• correct solns.	2
• finds $\overrightarrow{AE}$ or $\overrightarrow{DE}$ correctly	1

ii) If F is a point on $\overrightarrow{DE}$ and $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DE}$, express $\overrightarrow{DF}$ in terms of **b** and **d**. **1**

since $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DE}$ (given)

and $\overrightarrow{DE} = \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{d}$ (from (i))

$\therefore \overrightarrow{DF} = \frac{2}{3}\left(\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{d}\right) = \frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{d}$

Marking criteria	Marks
correct solution	1

iii) Find $\overrightarrow{AF}$ in terms of **b** and **d** and hence show that F lies on $\overrightarrow{AC}$. **3**

$\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF} = \underset{\sim}{d} + \left(\frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{d}\right)$

$\therefore \overrightarrow{AF} = \frac{2}{3}\underset{\sim}{d} + \frac{2}{3}\underset{\sim}{b}$ (1)

Now $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \underset{\sim}{d} + \underset{\sim}{b}$

so to show that F lies on $\overrightarrow{AC}$

is to show that

$\overrightarrow{AF} = k \times \overrightarrow{AC}$ for $0 < k < 1$

or $\overrightarrow{FC} = \lambda \times \overrightarrow{AC}$ for $0 < \lambda < 1$

Now $\overrightarrow{FC} = \overrightarrow{FD} + \overrightarrow{DC} = -\overrightarrow{DF} + \overrightarrow{DC}$

$\therefore \overrightarrow{FC} = \underbrace{-\left(\frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{d}\right)}_{\text{from (ii)}} + \underset{\sim}{b}$

$\therefore \overrightarrow{FC} = \frac{1}{3}\underset{\sim}{b} + \frac{1}{3}\underset{\sim}{d} = \frac{1}{3}\underbrace{(\underset{\sim}{b} + \underset{\sim}{d})}_{\overrightarrow{AC}}$

$\therefore \overrightarrow{FC} = \frac{1}{3} \times \overrightarrow{AC}$

$\therefore F$ lies on $\overrightarrow{AC}$

OR

similarly $\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF}$

$= \frac{2}{3}\underset{\sim}{d} + \frac{2}{3}\underset{\sim}{b}$ (shown above)

$\therefore \overrightarrow{AF} = \frac{2}{3}\underbrace{(\underset{\sim}{d} + \underset{\sim}{b})}_{\overrightarrow{AC}}$

$\therefore \overrightarrow{AF} = \frac{2}{3}\overrightarrow{AC}$ $\therefore F$ lies on $\overrightarrow{AC}$

Marking criteria	Marks
• correct solution	3
• finds $\overrightarrow{AF}$ and $\overrightarrow{AC}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$	2
• finds $\overrightarrow{AF}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$	1
• finds $\overrightarrow{AC}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{d}$	1

End of question 6

Question 7 (3 marks)

Prove by mathematical induction that for all positive integers $n \geq 1$ **3**

$$1 \times 3 + 2 \times 4 + 3 \times 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$$

STEP 1: prove identity true for $n=1$

$LHS = 1 \times (1+2) = 3$

$RHS = \frac{1(2)(9)}{6} = 3$

$\cancel{LHS = RHS}$ $\therefore$ identity is true for $n=1$.

STEP 2: Now assume that identity is true for some integer $k \geq 1$.

ie. $1\times 3 + 2\times 4 + .. + k(k+2) = \frac{k(k+1)(2k+7)}{6}$

STEP 3: Now prove the identity is true for $n = k+1$.

ie: $1\times3 + 2\times4 + .. + (k+1)(k+3) = \frac{(k+1)(k+2)(2k+9)}{6}$

Proof:

$LHS = \underbrace{1\times3 + \ldots + k(k+2)}_{\text{by assumption}} + (k+1)(k+3) =$

$= \frac{k(k+1)(2k+7)}{6} + (k+1)(k+3)$

$= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6}$

$= \frac{(k+1)[k(2k+7) + 6(k+3)]}{6}$

$= \frac{(k+1)(2k^2 + 13k + 18)}{6} = \frac{(k+1)(2k+9)(k+2)}{6}$

$= RHS$ $\therefore$ identity is proven true for $n = k+1$, if it's true for $n=k$ and since it's proven true for $n=1$ $\therefore$ by mathematical induction it's true for all integers $n \geq 1$.

Marking criteria

There are 4 key parts of the induction

1. Proves identity for $n=1$
2. clearly states the assumption and what is to be proven
3. uses the assumption clearly in the proof
4. correctly proves identity and concludes it

Criteria	Marks
All 4 parts correct	3
Any 3 of the 4 parts correct	2
Any 2 of the 4 parts correct	1

Question 8 (3 marks)

Prove by mathematical induction that for all integers $n \geq 0$
$3^{2n} + 7$ is divisible by 8. **3**

STEP 1: Prove $3^{2n}+7$ is divisible by 8 for $n=0$

$\therefore 3^{2(0)} + 7 = 1 + 7 = 8$ which is divisible by 8 $\therefore 3^{2n}+7$ is div. by 8 for $n=0$

STEP 2: Assume $3^{2n}+7$ is div. by 8 for some integer $k \geq 0$.

ie: $3^{2k} + 7 = 8m$ (m - integer)

STEP 3: Now prove $3^{2n}+7$ is div. by 8 for $n = k+1$ ie: $3^{2(k+1)} + 7 = 8W$ (W - integer)

Proof: $3^{2(k+1)} + 7 = 3^{2k} \times 3^2 + 7$

but from assumption $3^{2k} = 8m - 7$

$\therefore 3^{2k} \times 3^2 + 7 = 3^2(8m-7) + 7$

$= 9\times 8m - 9\times 7 + 7 = 9\times 8m - 8\times 7$

$= 8(9m - 7) = 8W$ (W - integer since m is integer)

$\therefore 3^{2n}+7$ is div. by 8 for $n = k+1$ if it is div. by 8 for $n=k$ and since $3^{2n}+7$ is div. by 8 for $n=1$ (proven) $\therefore$ by mathematical induction $3^{2n}+7$ is divisible by 8 for all integers $n \geq 0$

Marking criteria – same 4 key parts as in Q.7

Criteria	Marks
• All 4 parts correct	3
• Any 3 of the 4 parts correct	2
• Any 2 of the 4 parts correct	1

End of Question 8

Question 9 (7 marks)

The points A, B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} - 3\mathbf{j}$, $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - \mathbf{j}$.

a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$. 3

	Criteria
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4\underset{\sim}{i} + 6\underset{\sim}{j}$	1 mark for each correct
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 6\underset{\sim}{i} - 4\underset{\sim}{j}$	solution
$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 10\underset{\sim}{i} + 2\underset{\sim}{j}$	

b) Show that ΔABC is a right-angled isosceles triangle with a right angle at B. 2

Show first $\overrightarrow{AB} \perp \overrightarrow{BC}$ which means

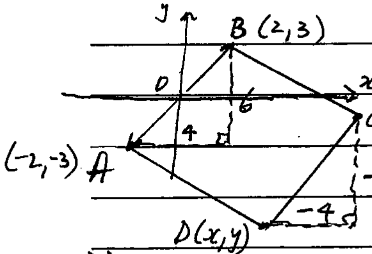
$\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$ (1)

$\therefore (4\underset{\sim}{i} + 6\underset{\sim}{j}) \cdot (6\underset{\sim}{i} - 4\underset{\sim}{j}) = 4 \times 6 + 6 \times -4 = 0$

$\therefore \overrightarrow{AB} \perp \overrightarrow{BC} \therefore \Delta ABC$ is right angled

	Criteria	Marks
Now show $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{BC}\rvert$	Shows $\overrightarrow{AB} \perp \overrightarrow{BC}$	1
$\lvert\overrightarrow{AB}\rvert = \sqrt{4^2 + 6^2} = \sqrt{52}$	(using dot product	
$\lvert\overrightarrow{BC}\rvert = \sqrt{6^2 + (-4)^2} = \sqrt{52}$	or otherwise)	
$\therefore \Delta ABC$ is isosceles	shows $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{BC}\rvert$	1

c) Find the position vector of the point D such that $ABCD$ forms a square. 2



	Criteria	Marks
let $D(x, y)$	Correct soln.	2
	Finds correct coordinates of pt D	(1)
$\therefore D(4, -7)$		
$\therefore \overrightarrow{OD} = 4\underset{\sim}{i} - 7\underset{\sim}{j}$	Finds $\overrightarrow{OD}$ correctly	1

OK $\overrightarrow{AB} = \overrightarrow{DC}$

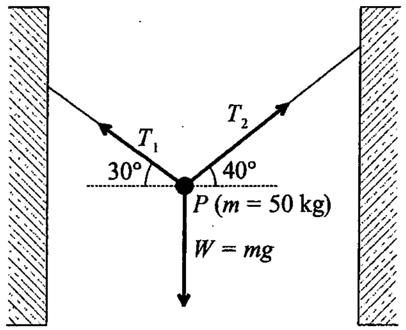
$\therefore 4\underset{\sim}{i} + 6\underset{\sim}{j} = (8\underset{\sim}{i} - \underset{\sim}{j}) - (x\underset{\sim}{i} + y\underset{\sim}{j})$

$\therefore 4 = 8 - x$ & $6 = -1 - y$

$\therefore 4 = x$ $\quad -7 = y$

Question 10 (4 marks)

The diagram shows an object, P, of mass 50 kg being suspended by two light inextensible strings of different lengths. The strings are at different angles to the horizontal. One is at 30° and the other is at 40°. The tension in these strings are T_1 and T_2 respectively. Take $g = 9.8$ m/s².



a) Show that $T_1 = \frac{\cos(40°)}{\cos(30°)} T_2$. 1

let $\vec{T_1} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} = T_1(\cos 30°\underset{\sim}{i} + \sin 30°\underset{\sim}{j})$

and $\vec{T_2} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} = T_2(\cos 40°\underset{\sim}{i} + \sin 40°\underset{\sim}{j})$

where $\lvert\vec{T_1}\rvert = T_1$ and $\lvert\vec{T_2}\rvert = T_2$

Now since object is stationary

$x_1 + x_2 = 0$

$\therefore -T_1 \cos 30° + T_2 \cos 40° = 0$

$\therefore T_1 \cos 30° = T_2 \cos 40° \therefore T_1 = \frac{\cos 40°}{\cos 30°} T_2$

$\therefore$ shown

b) Hence find the tension in each string if the object is stationary, correct to the nearest newton. **3**

Now $W = mg$ where $g = 9.8 m/s^2$

$\therefore W = 50 \times 9.8 = \underline{\underline{490 N}}$ 1

and since object is stationary

$\therefore y_1 + y_2 = W$

(2) $\therefore T_1 \sin 30° + T_2 \sin 40° = 490$

~~from~~ (a) $T_1 = \frac{\cos 40°}{\cos 30°} T_2$

$\therefore$ sub. T_1 into (2) 1

$\therefore$ (2) $\frac{\cos 40°}{\cos 30°} T_2 \times \sin 30° + T_2 \sin 40° = 490$

$T_2 \left(\frac{\cos 40°}{\cos 30°} \times \sin 30° + \sin 40° \right) = 490$

$T_2 = \frac{490}{\frac{\cos 40°}{\cos 30°} \times \sin 30° + \sin 40°}$

$\therefore T_2 = 451.586.. = 452 N$ (nearest newton)

now $T_1 = \frac{\cos 40°}{\cos 30°} \times T_2 = \frac{\cos 40°}{\cos 30°} \times 451.586$

$\therefore T_1 = 399.451.. = 399 N$

End of Paper

if used $T_2 = 452 N \therefore T_1 = 399.8174$

$\therefore T_1 = 400 N$

Extra writing space

	Criteria	Marks
Part a)	Correct solution	1
Part b)	Correct solns.	3
	- Finds correctly the value W and equates $y_1 + y_2 = W$ and correctly subst. T_1 into this equation	2
	- Finds tensions T_1 and T_2 correctly and finds correctly the value of W	2
	- Finds correctly the value of W	1
	- Equates $y_1 + y_2 = W$ correctly	1
	- Evaluates T_1 & T_2 using correct approach	1