2020 **YEAR 11 HALF YEARLY EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black pen.
- Calculators approved by NESA may be used.
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations.
- Answer all questions in the booklet provided.

Total marks: 65

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 7 minutes for this section.

Section II – 58 marks (pages 4 – 7)

- Attempt Questions 6 – 9
- Allow about 1 hour and 23 minutes for this section.

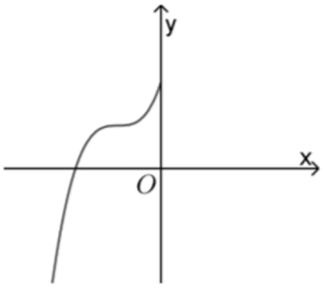
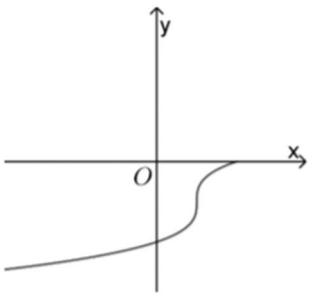
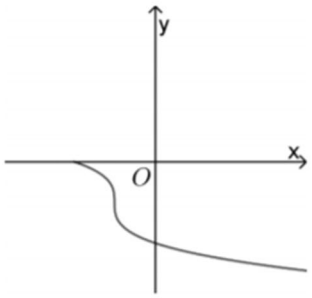
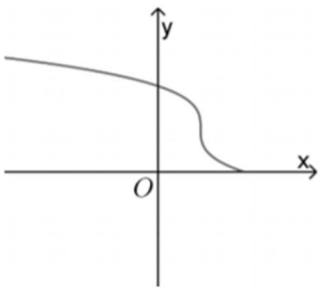
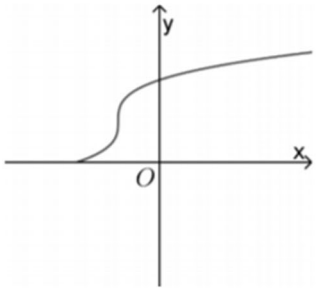
Section I

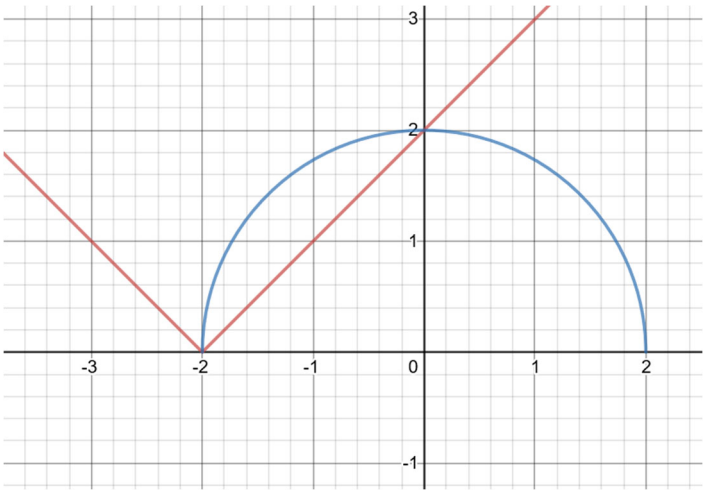
5 marks

Attempt questions 1-5

Allow about 7 minutes for this section

Use the multiple choice answer page in your booklet to answer Questions 1-5.

1	What are the asymptotes of $y = \frac{10x}{(x-5)(x+2)}$? (A) $y = 0,\ x = 2,\ x = -5$ (B) $y = 0,\ x = -2,\ x = 5$ (C) $y = 10,\ x = 2,\ x = -5$ (D) $y = 10,\ x = -2,\ x = 5$	**1**
2	The diagram shows the graph of $y = f(x)$.  Which diagram shows the graph $y = f^{-1}(x)$? A.  B.  C.  D.  	**1**

3	There are 10 contestants in a race and medals are awarded for 1st, 2nd and 3rd place. Andrew is a contestant in this race. How many ways could the medals have been awarded, given that Andrew won a medal? (A) 60 (B) 90 (C) 216 (D) 720	**1**
4	In an office, there are m boxes into which letters are sorted. This morning more than m items of mail were received for sorting. Consider the following statements. When the mail is sorted: I. There will be at least two boxes with the same number of letters. II. There will be at least one box with more than one letter. Which of these statements must be true? (A) I only (B) II only (C) Both I and II (D) Neither I nor II	**1**
5	The diagram shows the graphs $y = \|x + 2\|$ and $y = \sqrt{4 - x^2}$.  The solutions of $\sqrt{4 - x^2} \leq \|x + 2\|$ are the x −values given in interval notation by: (A) $[0, 2]$ (B) $[-2,0]$ (C) $[-2, -2] \cup [0,2]$ (D) $[0, \infty)$	**1**

End of Section I

Section II

58 marks

Attempt Questions 6-9.

Allow about 1 hour 23 minutes for this section.

Answer each question on the appropriate page of your answer booklet. Extra paper is available. Your responses should include relevant calculations and/or mathematical reasoning**.**

Question 6 (14 marks)

a) A curve has parametric equations $x = t - 3$ and $y = t^2 + 2$. **1**
What is the Cartesian equation of the curve?

b) Solve for x, and graph the solution on a number line: **3**

$$|5x - 4| > 9$$

c) Find the inverse function $y = f^{-1}(x)$ of **2**

$$f(x) = \frac{5}{2x-6} - 2 \quad .$$

d) Solve the inequality: **3**

$$\frac{1}{2x - 1} \leq 1$$

e) Solve the inequality: **3**

$$4x^2 - 6x^3 \leq 0$$

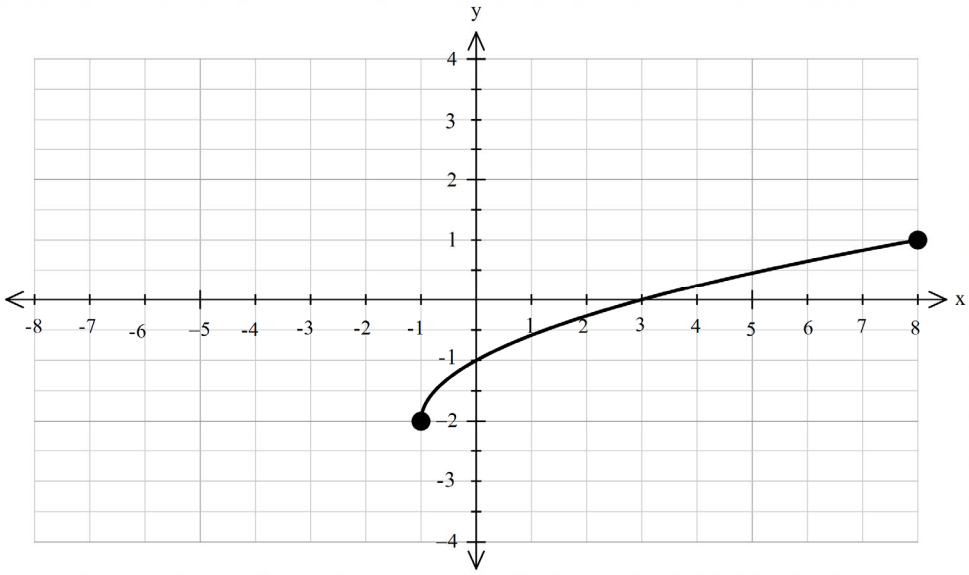
f) Find the number of distinct arrangements of the letters of the word PARRAMATTA. **1**

g) In a town, the names of all the residents are placed in a box and drawn randomly without replacement. What is the minimum number of names that must be drawn to ensure that three first names begin with the same letter? **1**

End of Question 6

Question 7 (16 marks)

a) The diagram below shows the graph of the function $y = f(x)$, where $f(x) = \sqrt{x+1} - 2$ and is defined for the domain $-1 \leq x \leq 8$.



Sketch the following graphs, clearly showing any intercepts with the coordinate axes and other important features. (You may use pencil to draw the graphs)

(i) $y = |f(x)|$ **2**

(ii) $y = [f(x)]^2$ **2**

(iii) $y^2 = f(x)$ **2**

(iv) $y = f(-x)$ **1**

b) Given the function $f(x) = \sqrt{a - x}$, where a is constant and $a > 1$.

(i) Sketch $y = f(x)$. **2**

(ii) Find its inverse function $y = f^{-1}(x)$, indicating any restrictions on its domain. **2**

(iii) If the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect when $x = 2$, find the value of a. **2**

c) How many even four-digit numbers can be formed from the set of digits {0,1,2,3,4,5,6} if each digit may only be used once? **3**

End of Question 7

Question 8 (16 marks)

a) Seven friends are to be seated at a circular table. In how many ways can this be done if:

(i) there are no restrictions? **1**

(ii) three people in the group (Anne, Ben and Chris) must be seated together? **2**

(iii) two people in the group, Paul and Qin, wish to have exactly one other person seated between them? **2**

(iv) Paul and Qin do not sit together? **2**

b) Consider the function $f(x) = x - \frac{1}{x}$:

(i) Write down the equation of each asymptote of $y = f(x)$. **2**

(ii) Find the intercepts of $y = f(x)$ with the coordinate axes. **2**

(iii) Sketch $y = f(x)$ showing all its important features. **2**

(iv) Using the restriction $x > 0$ on $y = f(x)$, find the equation of the inverse function in terms of x. **3**

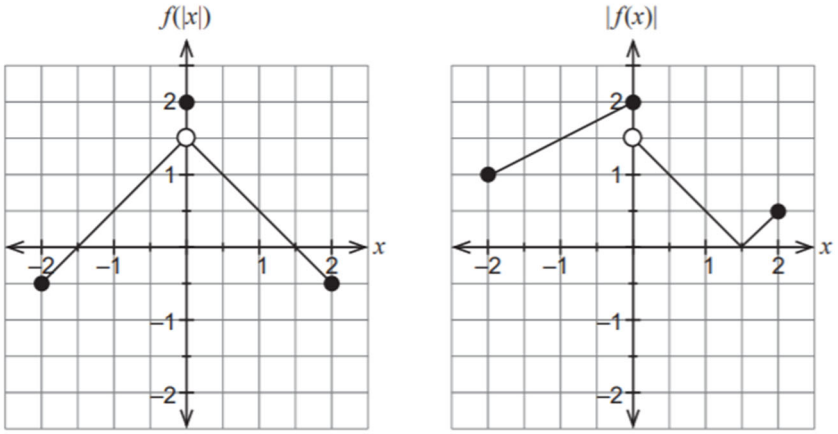
End of Question 8

Question 9 (12 marks)

a) (i) Show that $(k+1)! - k! = k \times k!$ **1**

(ii) Hence simplify $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n!$ **2**

b) The graphs of $y = f(|x|)$ and $y = |f(x)|$ are shown below. **4**



Sketch a possible graph of the function $y = f(x)$, given that its inverse is also a function.

c) There are six backpackers and four different rooms available for them to stay in.

(i) Suppose that each room can accommodate an unlimited number of people. How many ways can the backpackers be allocated rooms? **1**

(ii) How many ways can they be allocated to the rooms if one of the rooms is to have exactly five people and another is to have exactly one person? **2**

(iii) If each room can accommodate at most four people, find the number of ways of allocating the backpackers to the rooms. **2**

End of Examination

BAULKHAM HILLS HIGH SCHOOL

YEAR 11 – MATHEMATICS EXTENSION 1

JUNE 2020

SOLUTIONS.

Mathematics
Section I – Multiple Choice

1. $y=0$, $x=5$, $y=-2$ (A) (B)

2. Reflection in $y=x$ (A)

3. Award Andrew 1st, 2nd or 3rd : 3
   Award other 2 places : 9×8
   $\therefore 3 \times 9 \times 8 = 216$ (C)

4. I. is false since, we could have 1 in 1st box, 2 in 2nd box, 3 in 3rd box etc.
   II. is true by pigeonhole principle. (B)

5. $\sqrt{4-x^2} = |x+2|$ when $x = -2, 0$
   $\sqrt{4-x^2} < |x+2|$ when $0 < x \leq 2$
   $\therefore [-2, 2] \cup [0, 2]$ (C)

Start Here → 1.	A ○	B ●	C ○	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ●	D ○

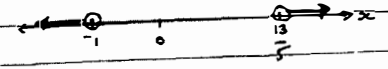
Section II

Question 6.

a) $x = t-3 \quad \therefore t = x+3$
$y = t^2 + 2 \quad \therefore y = (x+3)^2 + 2$
or $y = x^2 + 6x + 11$

(1) a correct eqn.

b) $|5x-4| > 9$
$5x - 4 < -9 \quad$ or $\quad 5x - 4 > 9$
$5x < -5 \quad 5x > 13$
$x < -1 \quad$ or $\quad x > \frac{13}{5}$



(1) Form inequalities with ± 9
(2) Solve correctly, ~~a~~ eqn.
(3) Solve + graph correctly.

c) $f: y = \frac{5}{2x-6} - 2$

$f^{-1}: x = \frac{5}{2y-6} - 2$

$x+2 = \frac{5}{2y-6}$

$2y-6 = \frac{5}{x+2} + 6$

$y = \frac{5^{-1}}{2x+4} + 3.$

(1) Significant progress after interchanging x and y
(1) correct soln.

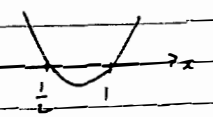
d) $\frac{1}{2x-1} \leq 1 \quad \left(\text{NOTE } x \neq \frac{1}{2}\right)$

Multiply by $(2x-1)^2$:
$2x-1 \leq (2x-1)^2$
$2x-1 \leq 4x^2 - 4x + 1$
$4x^2 - 6x + 2 \geq 0$
$2x^2 - 3x + 1 \geq 0$

$(2x-1)(x-1) \geq 0$

Roots : $x = \frac{1}{2}, 1$

Sketch :



Soln : $x \leq \frac{1}{2}$ or $x \geq 1$.

(1) obtain quadratic eqn. after multiplying by $(2x-1)^2$

(2) obtain a partially correct soln.

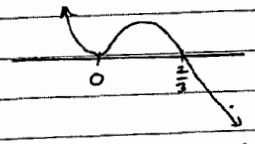
(3) correct soln.

e) $4x^2 - 6x^3 \leq 0$

$2x^2(2-3x) \leq 0$

Roots : $x = 0, 0, \frac{2}{3}$

Sketch :



Soln : $x = 0$ or $x \geq \frac{2}{3}$.

(1) correct roots

(2) partially correct soln

(3) correct soln.

f) PARMT

ART

A

A

$\frac{11!}{4!2!2!} = 415800$

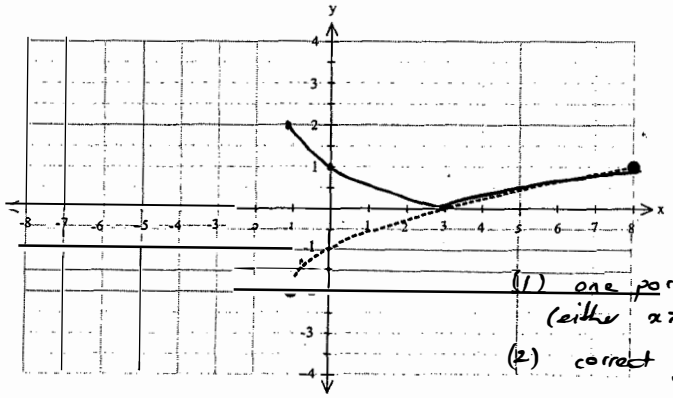
(1) correct soln.

g) $2 \times 26 + 1 = 53$

(1) correct answer.

Question 7 **Name:** _____

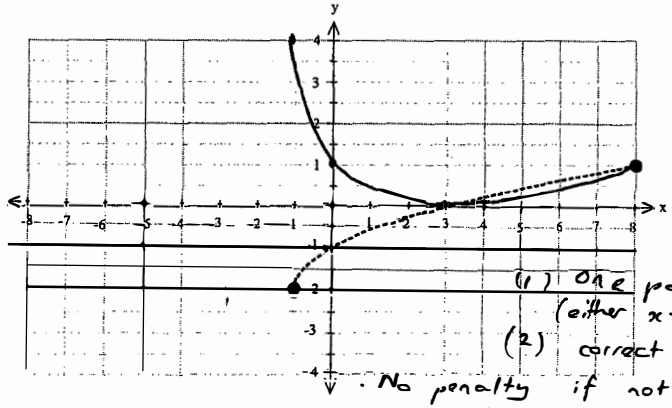
a)(i)



(1) one portion correct (either $x > 3$ or $x < 3$)

(2) correct graph.

(ii)

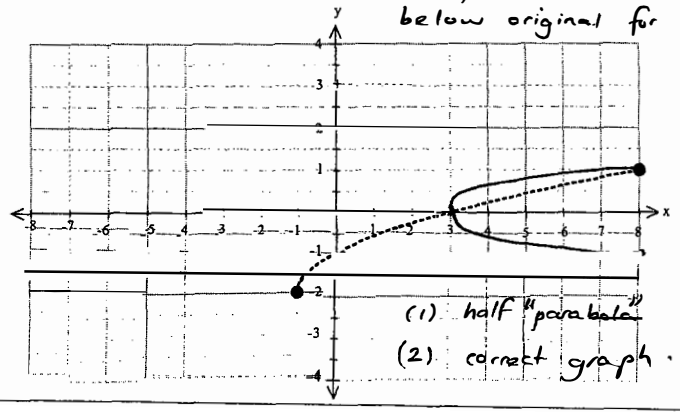


(1) one portion correct (either $x > 3$ or $x < 3$)

(2) correct graph.

· No penalty if not smooth at $x = 3$, but new curve must lie below original for $3 < x < 8$.

(iii)

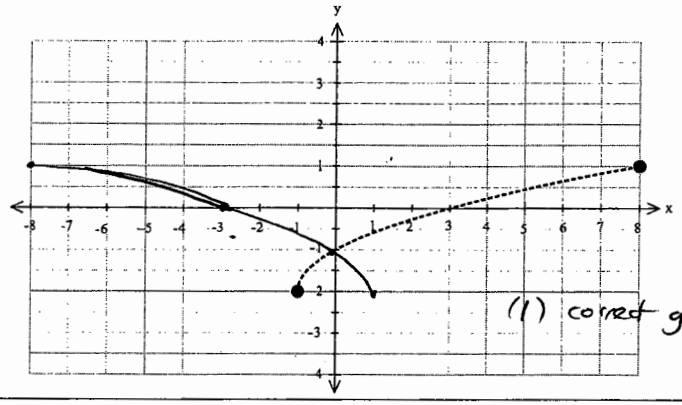


(1) half "parabola"

(2) correct graph.

7c) See last page.

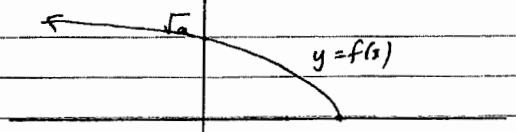
(iv)



(1) correct graph.

b) $f(x) = \sqrt{a-x}$

(i)



(1) Recognise it as a half-parabola
(2) Correct half-parabola.

(ii) $f: \quad y = \sqrt{a-x} \quad (y \geq 0)$

$f^{-1}: \quad x = \sqrt{a-y}$

$x^2 = a - y$

$\underline{y = a - x^2 \text{ with } x \geq 0}$

(1) correct eqn.
(2) correct eqn and correct restriction.

(iii) Since a function and its inverse always intersect on the line $y = x$, they must intersect at $(2,2)$.

Sub. $(2,2)$ into $y = \sqrt{a-x}$

$2 = \sqrt{a-2}$

$4 = a - 2$

$\underline{a = 6}$

(1) Sub. (2,2) or attempt to solve simult. eqns.
(2) correct soln

Question 8 **Name:** _____

a) (i) $6! = 720$ | (1) correct answer

(ii) [A/B/C] ----

Arrange the 5 groups in circle $4!$

Arrange A/B/C $3!$

$\therefore 4! \times 3! = 144$ ways | (1) significant progress (2) correct solution

(iii) Seat P 1

Seat Q 2

Seat others $5!$

$\therefore 1 \times 2 \times 5! = 240$ | (1) sig. progress (2) correct soln.

(iv) Seat P/Q together 2

Seat other 5 $5!$

$\therefore 2 \times 5! = 240$ ways together

$\therefore 720 - 240 = 480$ ways apart | (1) sig. progress (2) correct soln

b) i) $x = 0$, since denominator $\neq 0$

$y = x$, since as $x \to \pm\infty$, $\frac{1}{x} \to 0$ and $y \to x$

(1) 1 correct asymptote (2) 2 correct asymptotes.

ii) If $x = 0$, there is no y-intercept

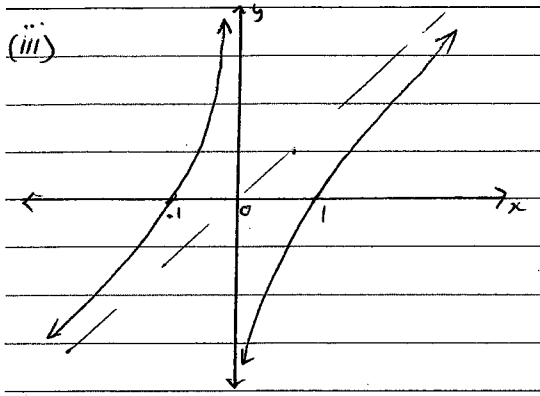
If $y = 0$, $0 = x - \frac{1}{x}$

$x = \frac{1}{x}$

$x^2 = 1 \qquad \underline{x = \pm 1}$

(1) 1 correct intercept (2) 2 correct interce

(iii)



(1) one correct branch
(2) correct graph

(iv) $f: \quad y = x - \frac{1}{x}$

$f^{-1}: \quad x = y - \frac{1}{y}$

$xy = y^2 - 1$

$y^2 - xy - 1 = 0$ (a quadratic in y)

$\Delta = b^2 - 4ac$

$= (-x)^2 - 4.1.-1$

$= x^2 + 4$

$\therefore y = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{x \pm \sqrt{x^2+4}}{2}$

but since $x > 0$ for f, we must have $y > 0$ for f^{-1}

$\therefore f^{-1}(x) = \frac{x + \sqrt{x^2+4}}{2}$

(1) uses a correct method to obtain a quadratic equation in y
(2) solve for y
(3) Correct solution with a reason for $+\sqrt{}$

Question 9

a) (i) $(k+1)! - k! = (k+1) \times k! - k!$

$= k! \times ((k+1) - 1)$

$= k! \times k$

(1) correctly shown

(ii) $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n!$

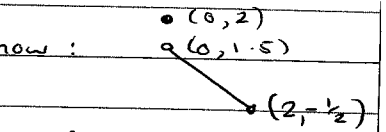
$= (2! - 1!) + (3! - 2!) + (4! - 3!) + \ldots + (n+1)! - n!$

$= -1! + (n+1)!$

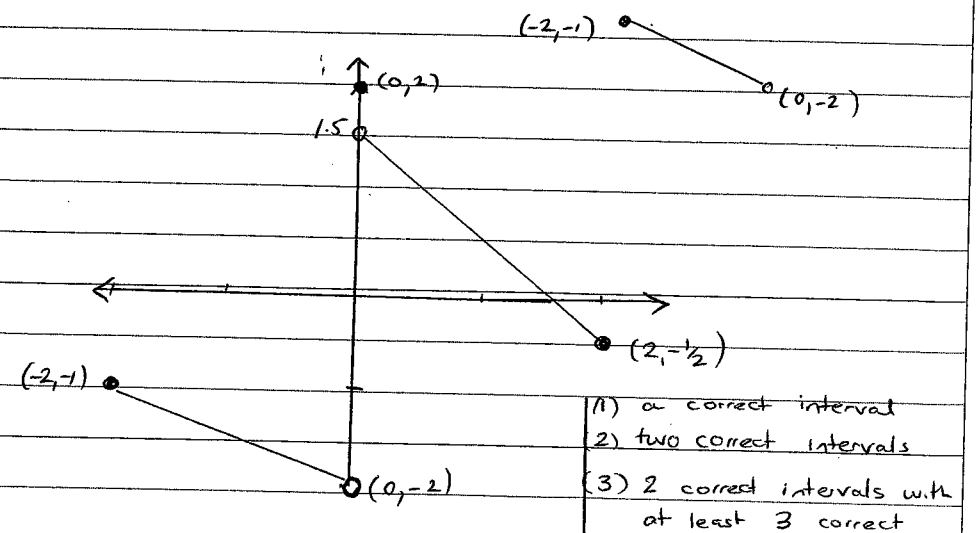
$= (n+1)! - 1$

(1) attempt to use result from (i) on each term
(2) correct solution

b) From the graph $y = f(|x|)$, we know:



From the graph $y = |f(x)|$ and the fact that the inverse is a function, we can have:



(1) a correct interval
(2) two correct intervals
(3) 2 correct intervals with at least 3 correct endpoints
(4) correct solution

c) (i) $4^6 = 4096$ | (1) correct answer

(ii) Choose 5 people ${}^6C_5$

Choose their room 4

Choose a room for the remaining person 3

$\therefore {}^6C_5 \times 4 \times 3 = 72$ | (1) sig. progress; (2) correct soln.

(iii) Total no. of ways = (Ways with 5 per room) − (Ways with all 6 in one room)

$= 4096 - 72 - 4$

$= 4020$

(1) subtraction from total no. of ways

(2) correct soln

Solution for 7c)

Case 1 : 0 in units place

Units digit 1

First digit 6

Other two 5×4

$6 \times 5 \times 4 = 120$

Case 2 : 2 4 or 6 in units place

Units digit 3

First digit 5

Other two 5×4

$3 \times 5 \times 5 \times 4 = 300$

$\therefore$ Total no. of ways $= 420$