BAULKHAM HILLS HIGH SCHOOL

YEAR 12 TASK 3 2021

Mathematics Extension 1

General Instructions
- Reading time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- Show all necessary working in questions 6-9
- Marks may be deducted for careless or badly arranged work
- A reference sheet is provided at the back of this paper

Total marks – 52
Exam consists of 9 pages.

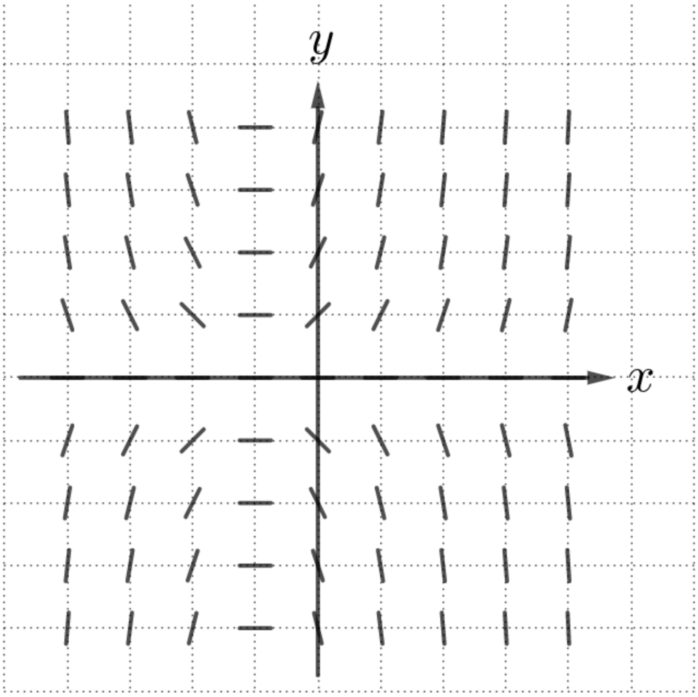
This paper consists of TWO sections.

<u>Section I</u> – (5 marks) Pages (2-3)
Questions 1-5
- Attempt Questions 1-5
- Answer on answer sheet provided
- Allow about 7 minutes for this section

<u>Section II</u> – (47 marks) Pages (4-9)
- Attempt Questions 6-9
- Allow about 1 hour and 23 minutes
- Answer each question on the appropriate page of your answer booklet

Section I - 5 marks
Use the multiple choice answer sheet for questions 1-5

1.	Which of the following functions is a primitive of $\dfrac{1}{\sqrt{4-9x^2}}$? A) $\dfrac{1}{3}\sin^{-1}\dfrac{2x}{3}$ B) $\dfrac{1}{9}\sin^{-1}\dfrac{3x}{2}$ C) $\dfrac{1}{9}\sin^{-1}\dfrac{2x}{3}$ D) $\dfrac{1}{3}\sin^{-1}\dfrac{3x}{2}$
2.	A cricket ball is hit from the origin so that its position vector at time t is given by $\underset{\sim}{r}(t)=15t\underset{\sim}{i}+(20t-5t^2)\underset{\sim}{j}$ for $t\geq 0$, where $\underset{\sim}{i}$ is the unit vector in the forward direction and $\underset{\sim}{j}$ is a unit vector vertically up. When the cricket ball reaches its maximum height, what is its position vector? A) $\underset{\sim}{r}=30\underset{\sim}{i}+20\underset{\sim}{j}$ B) $\underset{\sim}{r}=15\underset{\sim}{i}+20\underset{\sim}{j}$ C) $\underset{\sim}{r}=60\underset{\sim}{i}$ D) $\underset{\sim}{r}=30\underset{\sim}{i}+10\underset{\sim}{j}$
3.	If $\sin(\alpha+\beta)=a$ and $\sin(\alpha-\beta)=b$ then $\sin\alpha\cos\beta=$? A) $\sqrt{a^2+b^2}$ B) $\sqrt{ab}$ C) $\sqrt{a^2-b^2}$ D) $\dfrac{a+b}{2}$

4.	The direction (slope) field for a first order differential equation is shown.  Which of the following could be the differential equation represented? A) $\dfrac{dy}{dx}=(x+1)^3$ B) $\dfrac{dy}{dx}=x(y+1)$ C) $\dfrac{dy}{dx}=(x+1)y$ D) $\dfrac{dy}{dx}=(x-1)y$
5.	The equation $y=e^{ax}$ satisfies the differential equation $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-6y=0$. What are the possible values of a? A) $a=3$ or $a=-2$ B) $a=-6$ or $a=-1$ C) $a=1$ or $a=-6$ D) $a=2$ or $a=-3$

End of Section I

Section II

47 marks
Attempt Questions 6-9
Allow about 1 hour and 23 minutes for this section

Answer each question on the appropriate page of your answer booklet. Extra paper is available. Your responses should include relevant calculations and/or mathematical reasoning.

Question 6 (13 marks) **Marks**

a) Find the derivative of

i. $y = \tan^{-1}(2x)$. **1**

ii. $y = e^x\left(\cos^{-1} x\right)$ **2**

b) Use the substitution $u = x+1$ to find $\int \frac{x}{\sqrt{x+1}}\,dx$. **3**

c) Find the solution to differential equation **3**

$$\frac{dy}{dx} = \frac{2x}{3y^2} \text{ where } y(0) = 1$$

d) What is the exact value of k, given that **2**

$$\int_0^k \frac{dx}{9+x^2} = \frac{\pi}{9}?$$

e) Evaluate **2**

$$\int_0^{\frac{\pi}{4}} \sin 5x \sin x \, dx.$$

Question 7 (9 marks)

a) Consider the equation $\cos x - 2\sin x = 1$ for $-\pi \leq x \leq \pi$.

i. Show that the equation can be written as $t^2 + 2t = 0$, where $t = \tan\dfrac{x}{2}$. **1**

ii. Hence, solve the equation $\cos x - 2\sin x = 1$ for $-\pi \leq x \leq \pi$, giving solution to two decimal places where necessary. **2**

b) Solve $\sin 2\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$. **2**

c) Using the substitution $x = \cos 2\theta$, **4**

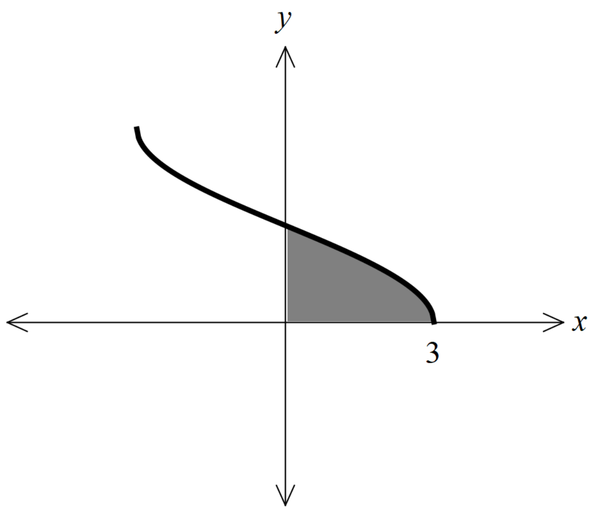
evaluate $\displaystyle\int_0^1 \sqrt{\frac{1+x}{1-x}}\,dx$ in simplest exact form.

Question 8 (14 marks) **Marks**

a) i. Express $2\sqrt{3}\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**

ii. Find the values of x for which $f(x) = 2\sqrt{3}\sin x - 2\cos x$ attains its maximum value over the domain $[-2\pi, 2\pi]$. **2**

b) The diagram below shows the graph of $y = 2\cos^{-1}\frac{x}{3}$.
The area under the curve for $0 \le x \le 3$ is shaded.

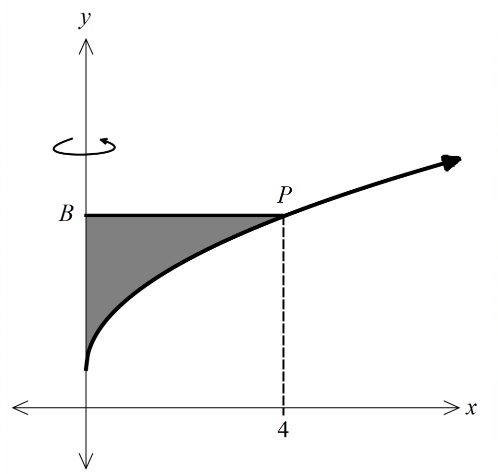


i. Find the y-intercept. **1**

ii. Calculate the area of the shaded region. **2**

Question 8 continued on next page

c)



The shaded region is bounded by the curve $y = 1 + 2\sqrt{x}$, the $y-$axis and the horizontal interval BP. The $x-$coordinate of P is 4. **3**

Calculate the exact volume of the solid of revolution formed when the shaded region is rotated about the $y-$axis.

d) i. Show that **2**

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

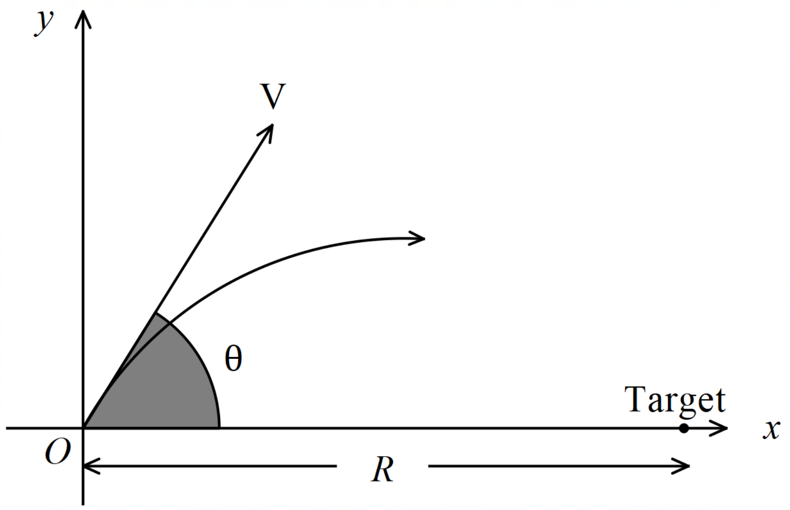
ii. Hence, or otherwise, solve for $0 \leq \theta \leq 2\pi$ **2**

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Question 9 (11 marks)

a) A projectile is fired from O at an angle θ to the horizontal with initial velocity V ms^{-1} to strike a target R metres right of O on the level ground. Assume the displacement of the projectile is given by the vector

$$\underset{\sim}{r}(t) = (Vt\cos\theta)\underset{\sim}{i} + \left(Vt\sin\theta - \frac{gt^2}{2}\right)\underset{\sim}{j}.$$ Do not prove it.



i. If the projectile hits the target, prove that: **2**

$$\tan^2\theta - \left(\frac{2V^2}{gR}\right)\tan\theta + 1 = 0$$

ii. Show that the target will be hit from two angles of projection, say θ_1 and θ_2, if **2**

$$R < \frac{V^2}{g}.$$

iii. Let the respective times of flight for each path be t_1 and t_2. By considering the roots of the equation in part (i) or otherwise, find the difference in the two times in terms of V, g and R. **2**

Question 9 continued on next page

b) A colony of bacteria occupied the area $A\,\text{cm}^2$. The colony has its growth modelled by the logistic equation $\dfrac{dA}{dt}=\dfrac{1}{25}A(50-A)$ where $t \geq 0$ and t is measured in days. At time $t = 0$, the area occupied by the bacteria colony is $\dfrac{1}{2}\text{cm}^2$.

i) Show that $\dfrac{1}{A(50-A)}=\dfrac{1}{50}\left(\dfrac{1}{A}+\dfrac{1}{50-A}\right)$. **1**

ii) Using the above result, solve the logistic equation and hence show that **3**

$A=\dfrac{50}{1+99e^{-2t}}$.

iii) According to this model, what is the limiting area of the bacteria colony? **1**

End of Exam

HSC Year12 Extension 1 Task3 2021(suggested marking guidline)

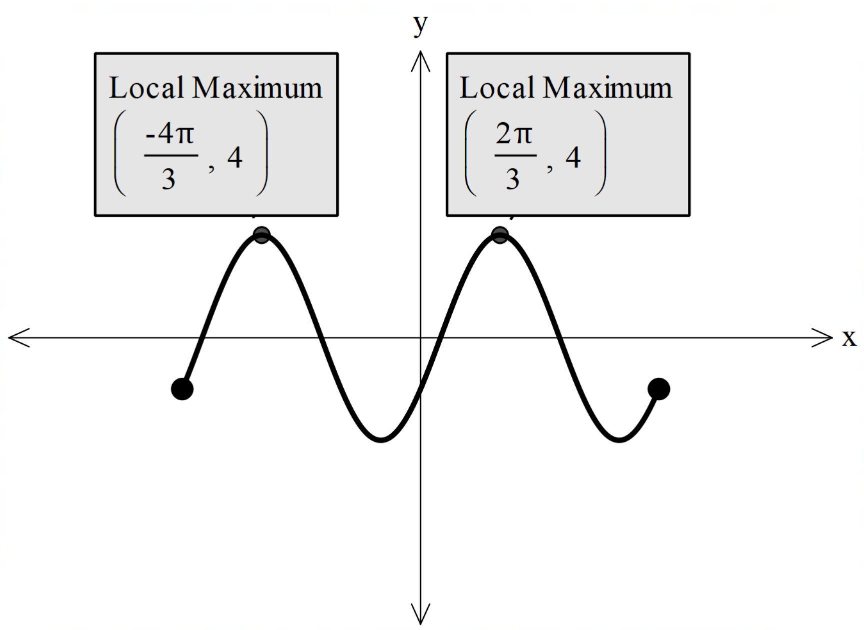
	Solutions	Mks	Comments
1	$\int \frac{dx}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}$ $\frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}$ $\frac{1}{3}\sin^{-1}\left(\frac{x}{2/3}\right)+c$ $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+c$	**1**	D
2	$\underset{\sim}{r}(t)=15t\underset{\sim}{i}+(20t-(20t-5t^2)\underset{\sim}{j}$ Max range when $(20t-5t^2)=0$ $5t(4-t)=0$ $\therefore$ max range occurs when $t=4$ max height when $t=2$ Position vector at maximum height is $\underset{\sim}{r}(2)=30\underset{\sim}{i}+20\underset{\sim}{j}$	**1**	A
3	$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta=a\ \ (1)$ $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=b\ \ (2)$ $(1)+(2)\ \ 2\sin\alpha\cos\beta=a+b$ $\sin\alpha\cos\beta=\frac{a+b}{2}$	**1**	D
4	At $(0,0)\ \frac{dy}{dx}=0\ \therefore$ not A At $(-1,1)\ \frac{dy}{dx}=0\ \therefore$ not B or D	**1**	C
5	$y=e^{ax}$ $\frac{dy}{dx}=ae^{ax}$ $\frac{d^2y}{dx^2}=a^2e^{ax}$ $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ $a^2e^{ax}+ae^{ax}-6=0$ $e^{ax}(a^2+a-6)=0$ $e^{ax}(a+3)(a-2)=0$ $a=2, a=-3$	**1**	D

	Solutions	Mks	Comments
6a(i)	$$\frac{dy}{dx}=\frac{2}{1+(2x)^2}=\frac{2}{1+4x^2}$$	1	**1 mark** • correct answer. **<u>Marker Comments</u>** - Well done by the vast majority of students. Of those students who did not score a mark, they either forgot to square the 2 on the denominator or forgot to multiply by the derivative of 2x
6a(ii)	$$\frac{dy}{dx}=e^x(\cos^{-1}x)+e^x\times-\frac{1}{\sqrt{1-(x)^2}}.$$ $$=e^x(\cos^{-1}x)-\frac{e^x}{\sqrt{1-x^2}}$$	2	**2 mark** • correct solution. **1 mark** • significant progress towards the solution. **<u>Marker Comments</u>** - Nearly all students scored full marks is this part.
6b	$u=x+1$ $x=u-1$ $\frac{du}{dx}=1 \quad dx=du$ $=\int\frac{u-1}{\sqrt{u}}du$ $=\int u^{\frac{1}{2}}-u^{-\frac{1}{2}}du$ $=(\frac{2u^{\frac{3}{2}}}{3}-2u^{\frac{1}{2}})+C$ $=\left[\frac{2(x+1)^{\frac{3}{2}}}{3}-2(x+1)^{\frac{1}{2}}\right]+C$	3	**3 mark** • correct solution. **2 mark** • significant progress towards solution **1 mark** • correct use of substitution **<u>Marker Comments</u>** - This was well done by the majority of students. Some students are still mixing variables (having both x and u on the same line of working.

	Solutions	Mks	Comments
6c	$\frac{dy}{dx}=\frac{2x}{3y^2}$ $\int 3y^2 dy=\int 2x\,dx$ $y^3=x^2+C$ $when\, x=0,\ y=1$ $1=0+C$ $\therefore y^3=x^2+1$ $y=\sqrt[3]{x^2+1}$	**3**	**3 mark** • correct solution. **2 mark** • significant progress to finding $y^3=x^2+C$ **1 mark** • Separating the variables correctly **<u>Marker Comments</u>** - Most students were awarded full marks. Some students made errors separating the varaibles and achieving all y on one side and x on the other.
6d	$\int_0^k \frac{dx}{9+x^2}=\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^k$ $=\frac{1}{3}(\tan^{-1}\frac{k}{3}-0)$ $\frac{1}{3}\tan^{-1}\frac{k}{3}=\frac{\pi}{9}$ $\tan^{-1}\frac{k}{3}=\frac{\pi}{3}$ $\frac{k}{3}=\sqrt{3}$ $\therefore k=3\sqrt{3}$	**2**	**2 mark** • correct solution. **1 mark** • correct integration **<u>Marker Comments</u>** Most students scored full marks but many students are reminded to use the Reference sheet as they made careless errors applying the result or using an incorrect result. A small number of students did not know or use the correct the of $\tan\frac{\pi}{3}$

	Solutions	Mks	Comments
6e	$\int_0^{\frac{\pi}{4}} \sin 5x \sin x\, dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 4x - \cos 6x)\, dx$ $= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 6x}{6}\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{2}\left[\frac{\sin \pi}{4} - \frac{\sin\left(\frac{3\pi}{2}\right)}{6} - 0\right]$ $= \frac{1}{2}\left[0 - \left(-\frac{1}{6}\right)\right]$ $= \frac{1}{12}$	**2**	**2 mark** • correct solution. **1 mark** • significant progress towards the solution **<u>Marker Comments</u>** This was the question that students struggled with most in Qn 6. Some students were recognised the need to use the product-to-sums results but could not accurately apply the result to the question.
7a(i)	$\cos x - 2\sin x = 1$ $\cos x = \frac{1-t^2}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$ $\therefore \frac{1-t^2}{1+t^2} - 2\times\frac{2t}{1+t^2} = 1$ $1 - t^2 - 4t = 1 + t^2$ $2t^2 + 4t = 0$ $t^2 + 2t = 0$	**1**	**1 mark** • correct solution. **Marker comments** - Well done by the grade
7a(ii)	$t(t+2) = 0$ $t = 0, \quad t = -2$ $\tan\frac{x}{2} = 0, -2$ $\therefore \frac{x}{2} = \tan^{-1} 0,\ \tan^{-1}(-2)$ $x = 0,\ 2\tan^{-1}(-2)$ $= 0,\ -2.21$ Checking for π and $-\pi$. Do not satisfies $\cos x - 2\sin x = 1$	**2**	**2 mark** • correct solution. **1 mark** • significant progress towards finding x. **Marker comments** Question s asked in terms of radians must be answered in terms of radians. Reminder that students need to check π and $-\pi$.
7b	$\sin 2\theta = \cos\theta$ $2\sin\theta\cos\theta - \cos\theta = 0$ $\cos\theta(2\sin\theta - 1) = 0$ $\cos\theta = 0 \quad \sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$	**2**	**2 mark** • correct solution. **1 mark** • significant progress towards the solution. **Marker comments** - Done well by majority. Some

	Solutions	Mks	Comments
			students divided by cos and lost solutions
7c(i))	$\frac{dT}{dt}=-k(I-A)e^{-kt}$ $=-k(T-A)$ $\therefore T=A+(I-A)e^{-kt}$ satisfies the differential eqution.	1	**1 mark** • correct solution. **Marker comments** - Done well by the majority of students
7c (ii)	$I=1500, A=20$ $T=20+1480e^{-kt}$ $when\, t=5, T=1200$ $1200=20+1480e^{-5k}$ $\frac{118}{148}=e^{-5k}$ $\therefore k=-\frac{1}{5}\ln\frac{59}{74}$ when $t=60$, $T=20+1480e^{-60k}$, where $k=-\frac{1}{5}\ln\frac{59}{74}$ $=118^{\circ}C$(3 sig. figs).	2	**2 mark** • correct solution. **1 mark** • finding the correct expression for e^{-5k}. **Marker comments** - Done well by the majority of students
	$x=\cos 2\theta \qquad 0\le\theta\le\frac{\pi}{2}$ $dx=-2\sin 2\theta\, d\theta=-4\sin\theta\cos\theta$ $x=1\ \ \theta=0$ $x=0\ \ \theta=\frac{\pi}{4}$ $\int_0^1\left(\sqrt{\frac{1+x}{1-x}}\right)dx=-\int_{\frac{\pi}{4}}^{0}\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\times 2\sin 2\theta\, d\theta$ $=\int_0^{\frac{\pi}{4}}\sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}\times 4\sin\theta\cos\theta\, d\theta$ $=\int_0^{\frac{\pi}{4}}\frac{\cos\theta}{\sin\theta}\times 4\sin\theta\cos\theta\, d\theta$ $=\int_0^{\frac{\pi}{4}}4\cos^2\theta\, d\theta$ $\int_0^{\frac{\pi}{4}}4\cos^2\theta\, d\theta=2\int_0^{\frac{\pi}{4}}(1+\cos 2\theta)\, d\theta$ $=2\left(\theta+\frac{1}{2}\cos 2\theta\right)_0^{\frac{\pi}{4}}$	4	**4mark** • correct solution. **3 mark** • Simplify integrand in terms of θ with correct boundaries and then finding the correct integration. **2 mark** • Simplify integrand in terms of θ with correct boundaries. **1 mark** • Significant progress. **Marker comments** - A small number of students did not correctly calculate the new limits - A few students did not correctly express $\cos^2\theta$ in terms of $\cos 2\theta$

	Solutions	Mks	Comments
	$= 2\left(\theta + \frac{1}{2}\sin 2\theta\right)_0^{\frac{\pi}{4}}$ $= 2\left(\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{2} - (0 + \frac{1}{2}\sin 0\right)$ $= 2(\frac{\pi}{4} + \frac{1}{2})$ $= \frac{\pi}{2} + 1$		
8a(i)	$2\sqrt{3}\sin x - 2\cos x$ $R\sin(x-\alpha) = R\sin x\cos\alpha - R\sin\alpha\cos x$ $R\cos\alpha = 2\sqrt{3} \qquad R\sin\alpha = 2$ $\tan\alpha = \frac{2}{2\sqrt{3}}$ $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ $R = \sqrt{(2\sqrt{3})^2 + 2^2}$ $= \sqrt{16} = 4$ $2\sqrt{3}\sin x - 2\cos x = 4\sin\left(x - \frac{\pi}{6}\right)$	**2**	**2 marks** • correct solution. **1 mark** • either R or α correct by correct method. **<u>Marker comments</u>** -Most students were awarded full marks. Some students made errors in finding the value of α.
8a(ii)	From graph  Maximum at $\left(-\frac{4\pi}{3}, 4\right)$ *and* $\left(\frac{2\pi}{3}, 4\right)$ **<u>OR</u>** For sine curve- maximum occurs at $\sin\theta = 1$	**2**	**2 mark** • correct solution. **1 mark** • correct ordinate but failure to check nature. • one max only with nature confirmed. **<u>Marker comments</u>** - Most students scored full marks but some forgot to write the nature of the points.

	Solutions	Mks	Comments
	$y=4\sin\left(x-\frac{\pi}{6}\right)$ max at $\sin\left(x-\frac{\pi}{6}\right)=1, \quad -2\pi\le x\le 2\pi$ $x-\frac{\pi}{6}=\frac{\pi}{2},-\frac{3\pi}{2}$ $x=\frac{2\pi}{3},-\frac{4\pi}{3}$ $\therefore \max at\left(\frac{2\pi}{3},4\right)\ and\ \left(-\frac{4\pi}{3},4\right)$		
8b(i)	y −intercept: $x=0$ $y=2\cos^{-1}0$ $=2\times\frac{\pi}{2}$ $=\pi$	1	**1 marks** • correct solution. **Marker comments** - Most students scored full marks.
8b(ii)	Shaded area=$\int_0^{\pi}3\cos\frac{y}{2}dy$ $=3\int_0^{\pi}\cos\frac{y}{2}dy$ $=6\left[\sin\frac{y}{2}\right]_0^{\pi}$ $=6\sin\frac{\pi}{2}$ $=6\,\text{unit}^2$	2	**2 marks** • correct solution. **1 mark** • finds the correct expression for the integrand with correct limits. **Marker comments** - Most students scored full marks but some of the did not convert the limits in terms of y.
8c	$y=1+2\sqrt{x}$ $2\sqrt{x}=y-1$ $\sqrt{x}=\frac{y-1}{2}$ $x=\left(\frac{y-1}{2}\right)^2$ $x=4$ at point B $y=1+2\sqrt{4}=5$	3	**3 mark** • correct solution. **2 mark** • significant progress to finding correct primitive function for volume with the correct boundaries **1 mark** • Finds $x=\left(\frac{y-1}{2}\right)^2$ **Marker comments** - Most students scored full marks but some students are making errors

	Solutions	Mks	Comments
	$V=\pi\int_1^5\left(\frac{(y-1)^2}{4}\right)^2 dy$ $=\frac{\pi}{16}\int_1^5 (y-1)^4 dy$ $=\frac{\pi}{16}\left[\frac{(y-1)^5}{5}\right]_1^5$ $=\frac{\pi}{16\times 5}(4^5)$ $=\frac{64\pi}{5}unit^3$		in doing integration or making x the subject.
8d(i)	$\tan(\theta+\frac{\pi}{6})=\frac{\tan\theta+\tan\frac{\pi}{6}}{1-\tan\theta\tan\frac{\pi}{6}}$ $=\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\tan\theta\times\frac{1}{\sqrt{3}}}$ $=\frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$	2	**2 mark** • correct solution. **1 mark** • significant progress towards the solution. **Marker comments** - Well done by grade.
8d(ii)	$1+\sqrt{3}\tan\theta=-\tan\theta(\sqrt{3}-\tan\theta)$ $=\sqrt{3}\tan\theta+\tan^2\theta$ $\tan^2\theta-2\sqrt{3}\tan\theta-1=0$ $\tan\theta=\frac{2\sqrt{3}\pm\sqrt{22+4}}{2}$ $\theta=\sqrt{3}\pm 2$ $\theta=\frac{5\pi}{12},\frac{11\pi}{12},\frac{17\pi}{12},\frac{23\pi}{12}$ OR $\tan(\theta+\frac{\pi}{6})=\tan(\pi-\theta)$ from(i) $\theta+\frac{\pi}{6}=\pi-\theta+k\pi$ where $k\in\mathbb{Z}$ $2\theta+\frac{\pi}{6}=(k+1)\pi \quad 0\le 2\theta\le 4\pi$ $2\theta=k\pi+\pi-\frac{\pi}{6}$ $\theta=\frac{(6k+5)\pi}{6} \quad k=0,1,2,3$	2	**2 mark** • correct solution. **1 mark** • 2 correct answers. **Marker comments** - Most students scored full marks but many students are not writing all the possible answers.

	Solutions	Mks	Comments
9a(i)	$y=-\frac{gt^2}{2}+Vt\sin\theta$..........(1) $x=Vt\cos\theta$..........(2) From (2) $t=\frac{R}{V\cos\theta}$ as $x=R$ and $y=0$ at the target. Substituting in $\underset{\sim}{r}(t)$ $0=-\frac{gR^2}{2V^2\cos^2\theta}+\frac{VR\sin\theta}{V\cos\theta}$ $0=-\frac{gR^2}{2V^2}(1+\tan^2\theta)+R\tan\theta$ $0=\frac{gR^2}{2V^2}\tan^2\theta+\frac{gR^2}{2V^2}-R\tan\theta$ $0=gR^2\tan^2\theta-2RV^2\tan\theta+gR^2$ $0=\tan^2\theta-\frac{2RV^2}{gR^2}\tan\theta+1$ $0=\tan^2\theta-\frac{2V^2}{gR}\tan\theta+1$	2	**2 mark** • correct solution. **1mark** • significant progress towards solution. *<u>Marker comments:</u>* Majority of students have done well in this question. Only a few were stuck on how to show the final equation.
9a(ii)	For 2 values θ, $\Delta>0$ $\left(-\frac{2V^2}{gR}\right)^2-4\times1\times1>0$ $\frac{4V^4}{g^2R^2}>4$ $\frac{V^4}{g^2}>R^2$ $R<\frac{V^2}{g}$ as $R>0$	2	**2 mark** • correct solution. **1 mark** • finding the correct expression using Δ *<u>Marker comments:</u>* Some had a different approach than the suggested solution. Majority knew to use the quadratic formula or discriminant.

9a(iii)	Time of flight: $t=\dfrac{2V\sin\theta}{g}\Rightarrow \sin\theta=\dfrac{gt}{2V}$ Horizontal range: $R=Vt\cos\theta\Rightarrow\cos\theta=\dfrac{R}{Vt}$ $\tan\theta=\dfrac{\left(\frac{gt}{2V}\right)}{\left(\frac{R}{Vt}\right)}$ $=\dfrac{gt^2}{2R}$ $t=\sqrt{\dfrac{2R\tan\theta}{g}}\quad(\text{as } t>0)$ Let the solutions to $\tan^2\theta-\dfrac{2V^2}{gR}\tan\theta+1=0$ be $\tan\theta_1$ and $\tan\theta_2$, where $\theta_2>\theta_1$ so that the angles of projection of θ_1 and θ_2 result in the flight times of t_1 and t_2 respectively, noting $t_2>t_1$. $t_2-t_1=\sqrt{\dfrac{2R\tan\theta_2}{g}}-\sqrt{\dfrac{2R\tan\theta_1}{g}}$ $=\sqrt{\dfrac{2R}{g}}\left(\sqrt{\tan\theta_2}-\sqrt{\tan\theta_1}\right)$ $(t_2-t_1)^2=\dfrac{2R}{g}\left(\sqrt{\tan\theta_2}-\sqrt{\tan\theta_1}\right)^2$ $=\dfrac{2R}{g}\left(\tan\theta_2+\tan\theta_1-2\sqrt{\tan\theta_1\tan\theta_2}\right)$ $=\dfrac{2R}{g}\left(\dfrac{2V^2}{gR}-2\sqrt{1}\right)$, by sum and product of roots $=\dfrac{4R}{g}\left(\dfrac{V^2}{gR}-1\right)$ $=\dfrac{4}{g^2}\left(V^2-gR\right)$ $t_2-t_1=\dfrac{2}{g}\sqrt{V^2-gR}\quad(\text{as } t_2-t_1>0)$ *Marker comments:* - Many candidates were able to solve the quadratic equation to obtain correction expressions for tanθ, poorer responses misinterpreted the solutions to the quadratic as θ, or the time of flight - More successful candidates recognised the use of the horizontal range formula or the time of flight formula, and attempted to convert tanθ into sinθ or cos θ, but made algebraic errors - Some students who chose to leave algebraic expressions unsimplified mid-response were able to obtain a correct expression for the time difference, albeit an unsimplified one	2	**2 mark** • correct solution **1 mark** • obtaining a suitable expression for $\tan\theta$ • solves the quadratic equation to yield correct expressions for $\tan\theta$ *Marker comments* See below solution in the column on the left

	- Responses most resemblant to the suggested solution made use of multiple projectile motion formulas and projectile motion properties (e.g. recognising the two angles of projection are complementary)		
9b(i)	$RHS = \frac{1}{50}\left(\frac{1}{A}+\frac{1}{50-A}\right)$ $= \frac{1}{50}\left(\frac{50-A+A}{A(50-A)}\right)$ $= \frac{1}{A(50-A)} = LHS$	1	**1 mark** • correct solution. *Marker comments:* Students did this question well and knew how to show the equation.
9b(ii)	$\frac{dA}{dt} = \frac{1}{25}A(50-A)$ $\frac{dt}{dA} = \frac{25}{A(50-A)}$ Split the integral $\int dt = \int \frac{25}{A(50-A)}dA$ $t = \frac{25}{50}\int\left(\frac{1}{A}+\frac{1}{50-A}\right)dA$ using part(i) $t = \frac{1}{2}\left[\ln A - \ln(50-A)\right]+C$ $= \frac{1}{2}\ln\left(\frac{A}{50-A}\right)+C$ $2t = \log_e\left(\frac{A}{50-A}\right)+2C$ $2t-2C = \log_e\left(\frac{A}{50-A}\right)$ $\therefore \frac{A}{50-A} = e^{2t-2C}$ $\frac{A}{50-A} = e^{-2C}e^{2t}$ $\frac{A}{50-A} = A_\circ e^{2t}$ (where $A_\circ = e^{2C}$) hence $A_\circ > 0$ When $t=0\ A = \frac{1}{2}\ so\ A_\circ = \frac{1}{99}$ $\frac{1}{99}e^{2t} = \frac{A}{50-A}$ $e^{2t} = \frac{99A}{50-A}$ $50e^{2t} - Ae^{2t} = 99A$ $50e^{2t} = A(99+e^{2t})$ $A = \frac{50e^{2t}}{99+e^{2t}}$ Dividing by e^{2t} $A = \frac{50}{1+99e^{-2t}}$	3	**3 mark** • correct solution. **2 mark** • Finding the expression $\frac{A}{50-A} = A_\circ e^{2t}$ **1 mark** • Finding correctly $t = \frac{25}{50}\int\left(\frac{1}{A}+\frac{1}{50-A}\right)$ *Marker comments:* Students did this well. A few different approaches to answer this question but majority got to the final solution.
9b(iii)	$t \to \infty\quad \frac{99}{e^{2t}} \to 0$ As $A = \frac{50}{1+99e^{-2t}}$ $= \frac{50}{1+0}$ $= 50\,cm^2$	1	**1 mark** • correct answer. *Marker comments:* Majority of students who attempted this, got it correct.