



BAULKHAM HILLS HIGH SCHOOL

YEAR 11 TASK 1 2021

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- Calculators approved by NESA may be used
- Show all necessary working in questions 6-9
- Marks may be deducted for careless or badly arranged work

Total marks – 61

Exam consists of 7 pages.

This paper consists of TWO sections.

Section I – (5 marks) Pages (2-3)

Questions 1-5

- Attempt Questions 1-5
- Answer on answer sheet provided
- Allow about 7 minutes for this section

Section II – (56 marks) Pages (4-7)

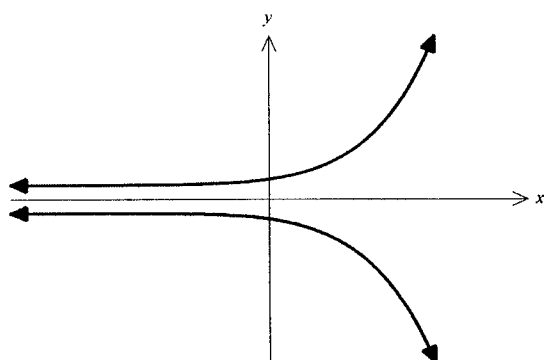
- Attempt Questions 6-9
- Allow about 1 hour and 23 minutes
- Answer each question on the appropriate page of your answer booklet

Section I - 5 marks

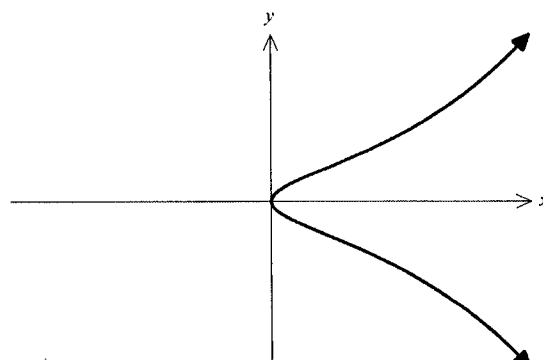
Use the multiple choice answer sheet for questions 1-5

1. How many different 3-digit numbers can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 where repetitions are not allowed?
- A) 648
B) 720
C) 900
D) 1000
2. A small class of 10 students (5 boys and 5 girls) are to be arranged in a single line. How many different arrangements are there if the girls have to be all together?
- A) 2 880
B) 14 400
C) 86 400
D) 362 888 000
3. Which graph best represents the curve $y^2 = 2^x - 1$?

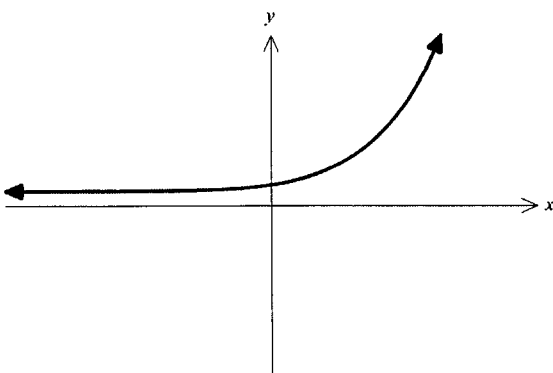
A)



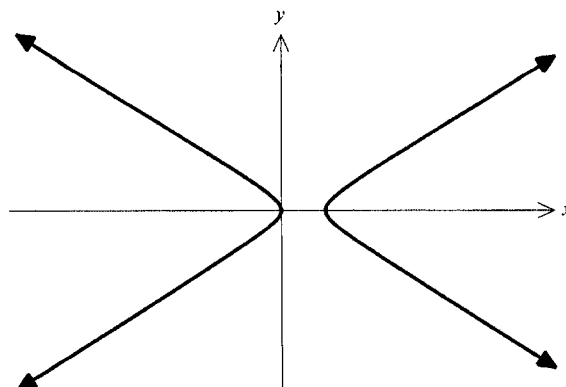
B)



C)



D)



4. What is the horizontal asymptote of $y = \frac{3-x^2}{4-x^2}$?

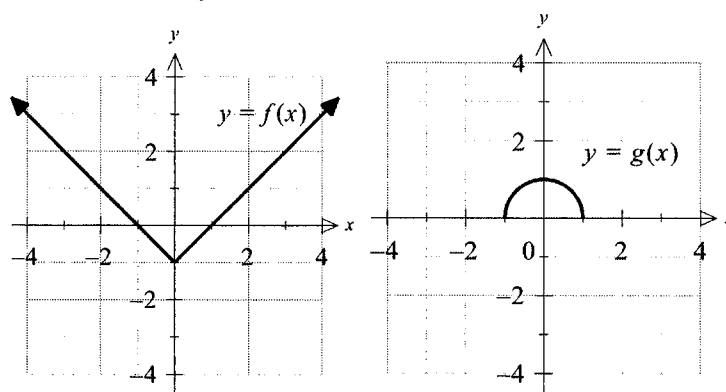
A) $y = 0$

B) $y = 1$

C) $y = \frac{\sqrt{3}}{2}$

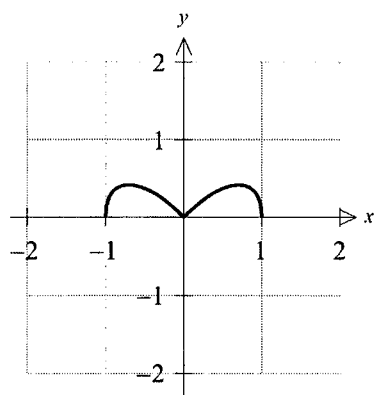
D) $y = \frac{3}{4}$

5. Consider these two curves on the same grid.

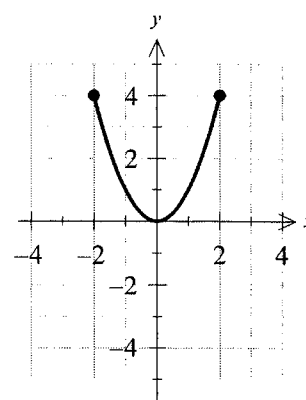


Which of the following represents the **addition** of these two curves?

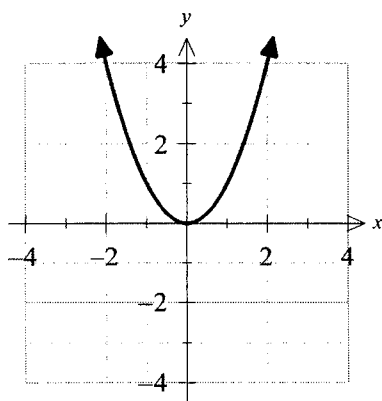
A)



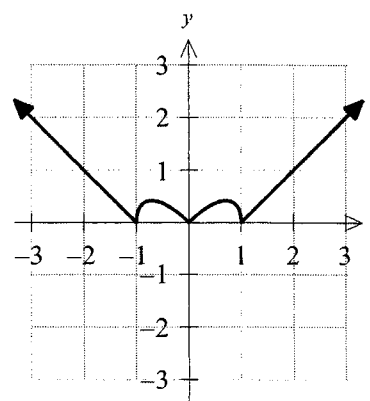
B)



C)



D)



End of Section I

Section II

56 marks

Attempt Questions 6-9

Allow about 1 hour and 23 minutes for this section

Answer each question on the appropriate page of your answer booklet. Extra paper is available. Your responses should include relevant calculations and/or mathematical reasoning.

Question 6 (14 marks)

Marks

- a) A conference has 600 attendees and all attendees are in one of eleven conference rooms.

1

Show that at least one room must have at least 55 people.

- b) A curve has parametric equations $x = 2 - t$ and $y = 3 + 6t - 4t^2$.

2

What is the Cartesian equation of the curve?

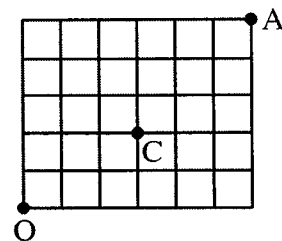
- c) Solve the inequality:

3

$$\frac{2}{x+1} \leq 4$$

- d) The streets of a section of the city of Melbourne form a rectangular grid, 6 blocks long and 5 blocks wide as shown. A man walks along the streets from O to A, always walking either due East or due North.

Determine:



- i. the number of these paths from O to A

1

- ii. the number of these paths from O to A passing through the point C

2

- e) Solve for x , and graph the solution on a number line:

3

$$|3x - 4| > 7$$

- f) Find the inverse function $y = f^{-1}(x)$ of

2

$$f(x) = \frac{2x-1}{x+2}$$

Question 7 (15 marks)

a) Consider the function, $f(x) = x^2 - 4x$.

i. Let $x \in [p, \infty)$ be a domain of $f(x)$ so that $f^{-1}(x)$ exists.

1

Find the smallest value of p .

ii. Find the equation of the inverse and hence sketch the graph of $y = f^{-1}(x)$.

3

b) A security lock has 10 buttons labelled as shown. Each person using the lock is given a 4-digit PIN code.

1	2	3
4	5	6
7	8	9
	0	

i. How many different codes are possible if numbers can be repeated and their order is important?

1

ii. How many different codes are possible if numbers cannot be repeated and their order is important?

1

iii. Now suppose that the lock operates by holding 4 buttons down together, so that the order is not important. How many different codes are possible?

1

c) The letters of the word REPORTS are arranged randomly in a line.

i. Find the total number of arrangements if there are no restrictions.

2

ii. Find the total number of arrangements if the R's are separated.

2

iii. If one word is chosen at random, find the probability that the letters of the word are in alphabetical order.

1

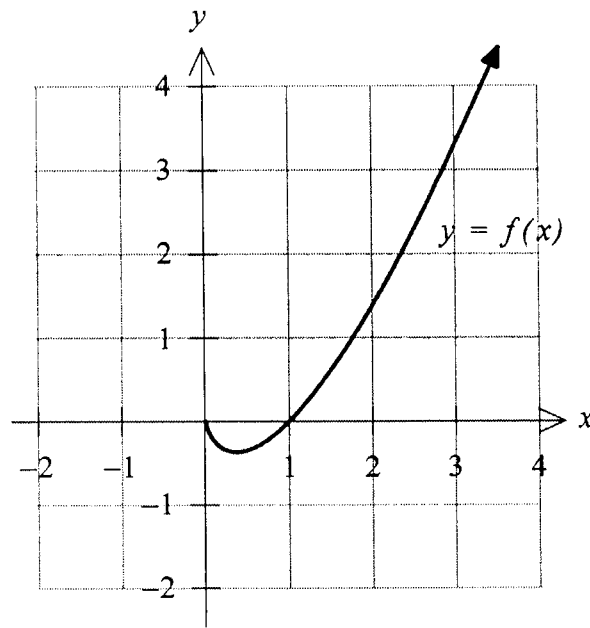
d) Solve the inequality:

3

$$\frac{1}{x} < \frac{1}{x+2}$$

Question 8 (14 marks)**Marks**

- a) The diagram below shows the graph of $y = f(x)$.



Graph the following:

- | | | |
|------|----------------------|---|
| i. | $y = f(x) $ | 2 |
| ii. | $y = \frac{1}{f(x)}$ | 2 |
| iii. | $y = \sqrt{f(x)}$ | 2 |
| iv. | $ y = f(x)$ | 2 |
- b) Ten marbles are to be arranged in a circle on the ground. In how many different ways can this be arranged when:
- | | | |
|-----|---|---|
| i. | the marbles are all of different colours? | 1 |
| ii. | there are 6 red, 3 yellow and 1 white marble? | 2 |
- c) How many different numbers less than 5000 that are divisible by 3 can be formed using the digits 1, 2, 3, 4, 5, if repetition is not allowed? 3

Question 9 (13 marks)

- a) Consider the following parametric equations, $x = 5 \sin \theta$, $y = 5 \cos \theta$.
- i. Find the Cartesian equation of the curve. **2**
 - ii. Sketch the graph of the curve. **2**
- b) Let $h(x) = |x - 4|$ and $g(x) = |x - 2|$. Write down the equations of the branches for **4**
 $f(x) = h(x) - g(x)$ and sketch the graph of $y = f(x)$.
- c) Suppose that a committee of 4 is to be chosen from 6 couples (6 men and 6 women).
- i. In how many ways can this be done? **1**
 - ii. Of the possible committees how many contain exactly 2 women? **1**
 - iii. Of the possible committees how many contain at least 2 women? **2**
 - iv. Of the possible committees how many contain at most 1 person from each couple? **1**

End of Exam

Baulkham Hills High School**Task 1 Examination 2021****Marking Guideline- Yr 11 Mathematics Extension 1****Section I (5 marks)**

Award 1 mark to each correct answer.

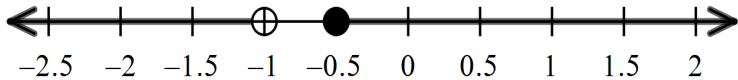
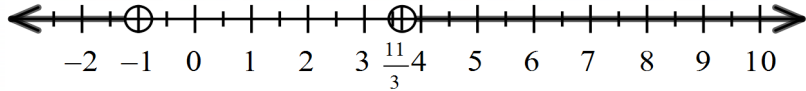
Answers: 1.A 2.C 3.B 4.B 5.A

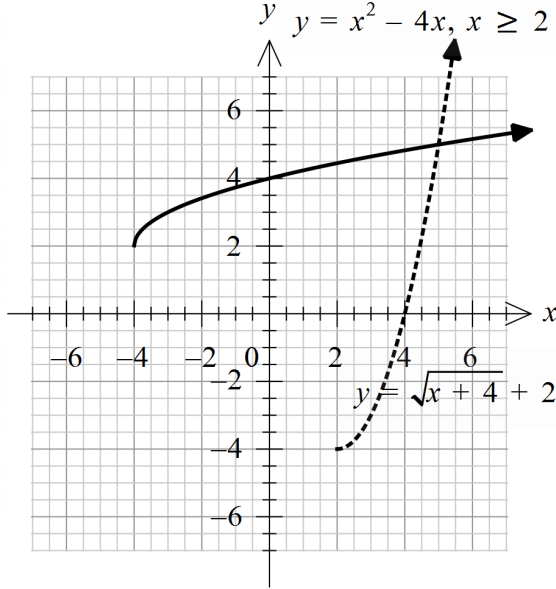
Question	Suggested solution	Answer
1	$9 \times 9 \times 8 = 648$ Note: 0 cannot be the first digit	A
2	$5! \times 6! = 86400$ The girls have to be together. Group the girls together. $5!$ ways of arranging the girls. $6!$ for the group of girls and the 5 boys.	C
3	Solve for y , gives $y = \pm\sqrt{2^x - 1}$. Graph $y = \sqrt{2^x - 1}$. Duplicate $y = \sqrt{2^x - 1}$ below the x -axis by reflection below the x -axis.	B
4	As $x \rightarrow \pm\infty, y \rightarrow 1$ $y = 1$ is the horizontal asymptote of $y = \frac{3 - x^2}{4 - x^2}$	B
5	The semicircle has a restricted domain, so should the sum of the two graphs. $\therefore$ Option C and D cannot be the answer Can't be option B since the ordinates at the x -intercept does not add up correctly $\therefore$ Hence option A.	A

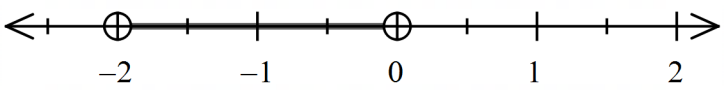
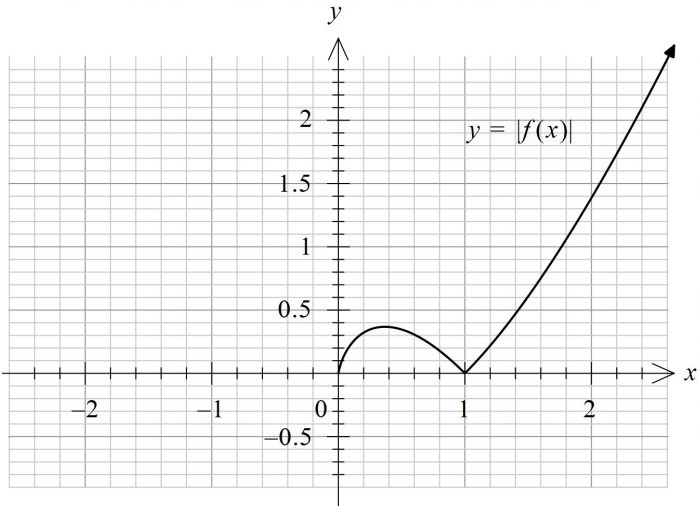
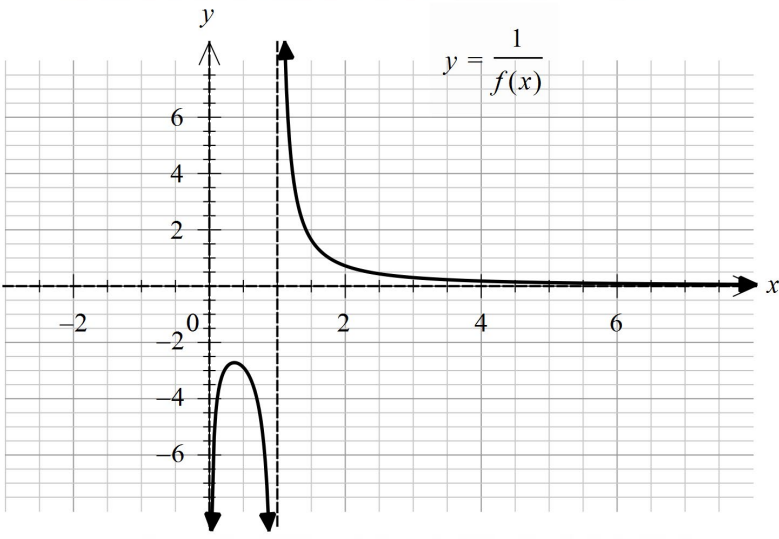
Section II (56 marks)

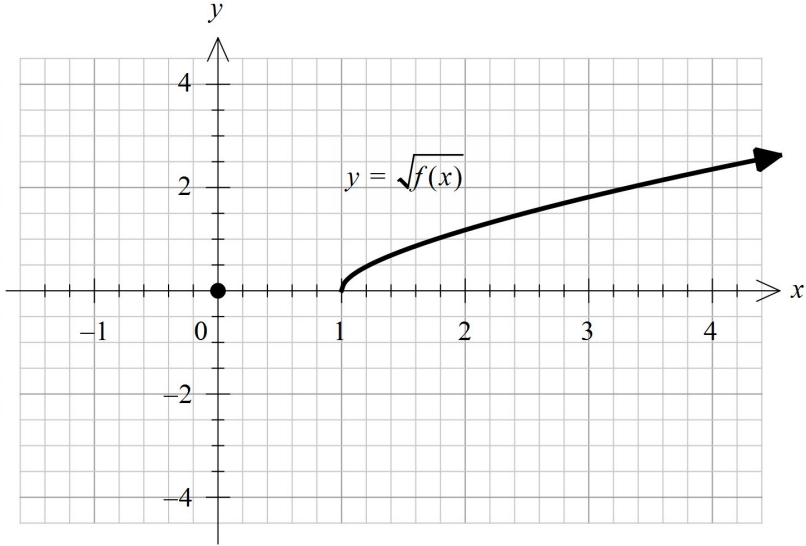
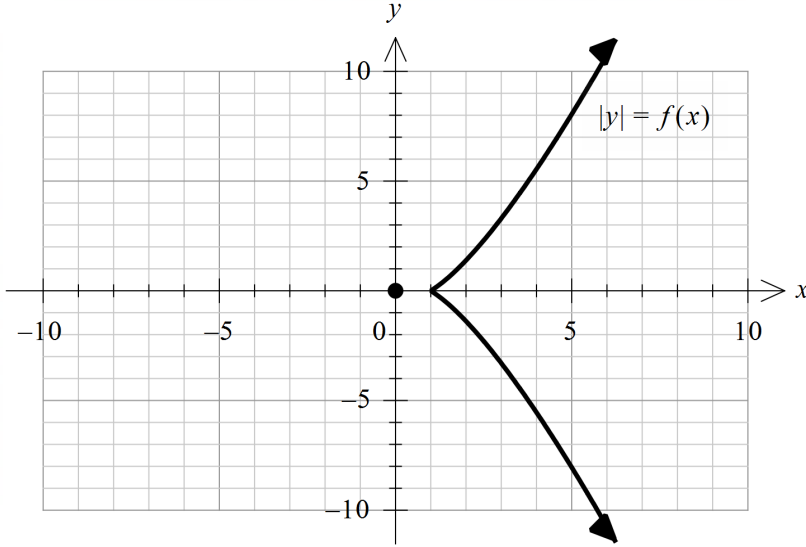
In all questions, award full marks for correct answers with necessary working.

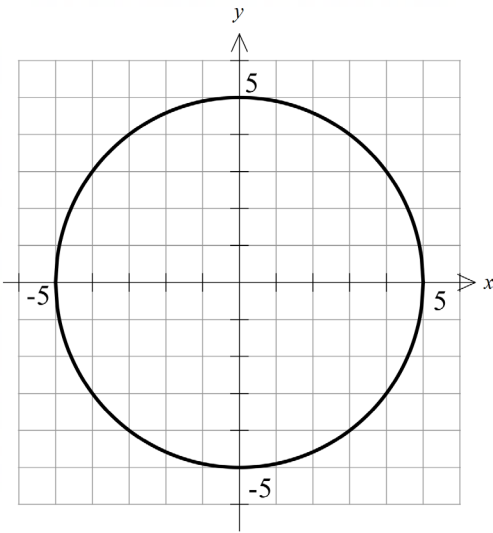
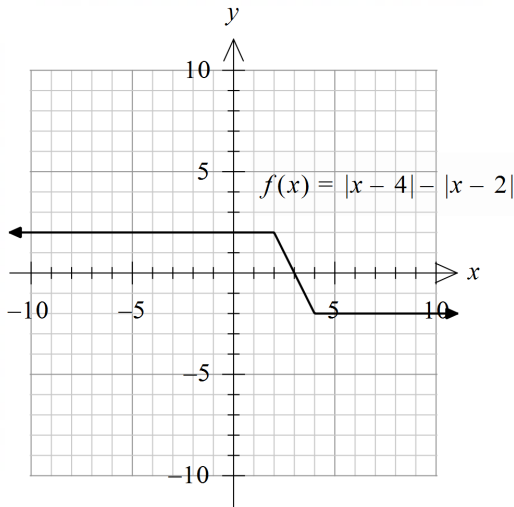
Use the suggested solutions in conjunction with the marking criteria.

Question	Suggested solutions	Marking criteria
6a)	$\frac{600}{11} = 54.\dot{5}\dot{4}$ $\therefore \text{At least one room must have at least 55 people}$	1- correct answer
6b)	$x = 2 - t \quad y = -4t^2 + 6t + 3$ $t = -x + 2 \quad y = -4(-x + 2)^2 + 6(-x + 2) + 3$ $\therefore y = -4x^2 + 10x - 1$	2- correct solution for y in terms of x 1- correct equation for t in terms of x
6c)	$2(x+1) \leq 4(x+1)^2$ $2x+2 \leq 4x^2+8x+4$ $2x^2+3x+1 \geq 0$ $(x+1)(2x+1) \geq 0$ $x = -1, -\frac{1}{2}$  $\therefore x < -1, x \geq -\frac{1}{2}$ $x \neq -1, \text{ as it would be undefined at that point}$	3- correct solution 2- correct factorised form before solving for x . OR correct solution but including $x = -1$ as a solution 1- multiplying both sides by $(x+1)^2$
6d)i	${}^{11}C_5 = \frac{11!}{5! \times 6!} = 462$	1- correct solution
6d)ii	$\frac{5!}{3! \times 2!} \times \frac{6!}{3! \times 3!} = 200$	2- correct solution 1- considered going either north or east only
6e)	$\pm(3x-4) > 7$ $3x-4 > 7 \quad 3x-4 < -7$ $3x > 11 \quad 3x < -3$ $x > \frac{11}{3} \quad x < -1$ $\therefore x < -1, x > \frac{11}{3}$ 	3- correct solution 2- correct solution with no number line 2- correct number line with no final solution 1- solve and/or plot $3x - 4 > 7$

6f)	$y = \frac{2x-1}{x+2}$ $x = \frac{2y-1}{y+2}$ $xy + 2x = 2y - 1$ $2x + 1 = y(2 - x)$ $y = \frac{2x+1}{2-x}$	2- correct solution 1- interchanging x and y
7a)i	$p = 2$	1- correct solution
7a)ii	$y = x^2 - 4x$ $x = y^2 - 4y$ $x = (y-2)^2 - 4$ $\therefore y = \sqrt{x+4} + 2$ 	3- correct inverse equation & graph 2- correct inverse equation but incorrect graph 1- completing the square or some other form to factorise 1- reflection graph Note: For graph to be correct, intercepts and vertex must be correct
7b)i	$10^4 = 10000$	1- correct solution
7b)ii	${}^{10}P_4 = 5040$	1- correct solution
7b)iii	${}^{10}C_4 = 210$	1- correct solution
7c)i	$\frac{7!}{2!} = 2520$	2- correct solution 1- 7!
7c)ii	$\frac{7!}{2} - 6! = 1800$	2- correct solution 1- some correct progress
7c)iii	$\frac{1}{\left(\frac{7!}{2!}\right)} = \frac{1}{2520}$	1- correct solution

7d)	$\frac{1}{x} < \frac{1}{x+2}$ $\frac{x+2-x}{x(x+2)} < 0$ $\frac{2}{x(x+2)} < 0$ $x \neq -2, 0$  $\therefore -2 < x < 0$	3- correct solution 2- significant progress towards solving the polynomial 1- identifying $x \neq -2, 0$ 1- valid polynomial equation
8a)i	 Everything below the x-axis is reflected above the x-axis.	2- correct graph 1- one portion correct (either $x < 1$ or $x > 1$)
8a)ii		2- correct graph 1- Only graphs the branch $x > 1$ or $0 < x < 1$

8a)iii	 Half parabola shape	2- correct graph 1- recognise the vertical tangent 1- half parabola shape starting at (1,0)
8a)iv		2- correct graph 1- including $x = 0$ to 1
8b)i	$9! = 362\,880$	1- correct solution
8b)ii	$\frac{9!}{6! \times 3!} = 84$	2- correct solution 1- some progress
8c)	1 digit- (3) 1 option 2 digit- (1,2)(1,5)(2,4)(4,5) 8 options 3 digit- (1,2,3)(2,3,4)(3,4,5)(5,3,1) 24 options 4 digit- (1,2,4,5) 18 options Total = 51 options	3- correct solution 2- correctly obtains any 3 cases 1- correctly obtains any 2 cases OR 1- correctly obtains the 4-digit case only
9a)i	$x = 5 \sin \theta$ $y = 5 \cos \theta$ $x^2 + y^2 = (5 \sin \theta)^2 + (5 \cos \theta)^2$ $= 25 \sin^2 \theta + 25 \cos^2 \theta$ $= 25(\sin^2 \theta + \cos^2 \theta)$ $x^2 + y^2 = 25$	2- correction cartesian equation 1- some progress towards it

9a)ii		2- correct solution 1- not showing the intercepts Note: Consider COE
9b)	$f(x) = \begin{cases} -(x-4) + (x-2), & \text{if } x < 2 \\ -(x-4) - (x-2), & \text{if } 2 \leq x \leq 4 \\ (x-4) - (x-2), & \text{if } x > 4 \end{cases}$ $f(x) = \begin{cases} 2, & \text{if } x < 2 \\ -2x + 6, & \text{if } 2 \leq x \leq 4 \\ -2, & \text{if } x > 4 \end{cases}$ 	4- correct equation and graph 3- graph all correct with only 2 equations 3- equation all correct with only 2 branches drawn 2- graph all correct (no equations) 2- equations all correct (no graph) 1- 2 branches graph only 1- 2 correct equations of branches only 1- 1 branch with 1 correct equation 1- draw $h(x)$ and $g(x)$ correctly
9c)i	${}^{12}C_4 = 495$	1- correct solution
9c)ii	${}^6C_2 \times {}^6C_2 = 225$	1- correct solution
9c)iii	exactly 2 + exactly 3 + exactly 4 $225 + {}^6C_3 \times {}^6C_1 + {}^6C_4 = 360$	2- correct solution 1- some correct progress
9c)iv	We can select 4 couples from the 6 couples $\times$ selecting 1 person from each couple ${}^6C_4 \times 2^4 = 240$	1- correct solution