BAULKHAM HILLS HS

PERSEVERE

2022 **YEAR 12 Assessment Task 1**

Mathematics Extension 1

General Instructions	• Reading time – 5 minutes • Working time – 1 hour • Write using black pen. • Calculators approved by NESA may be used. • In Questions 6 – 8, show relevant mathematical reasoning and/or calculations. • Answer all questions in the booklet provided.
Total marks: 34	**Section I – 5 marks** (pages 2 – 3) • Attempt Questions 1 – 5 • Allow about 8 minutes for this section. **Section II – 29 marks** (pages 4 – 6) • Attempt Questions 6 – 8 • Allow about 52 minutes for this section.

Section I - 5 marks
Answer on the multiple choice answer sheet

1. The points P, Q and R lie on a straight line. The vector $\overrightarrow{PQ}=\begin{pmatrix}-4\\6\end{pmatrix}$ and the vector $\overrightarrow{PR}=2\overrightarrow{PQ}$. The coordinates of P are $(2,-2)$. What are the coordinates of R?

(A) $(-2, 4)$

(B) $(-6, 8)$

(C) $(-6, 10)$

(D) $(6, -10)$

2. Consider the vectors $\underset{\sim}{a}=2\underset{\sim}{i}+3\underset{\sim}{j}$, $\underset{\sim}{b}=-3\underset{\sim}{i}+2\underset{\sim}{j}$ and $\underset{\sim}{c}=2\underset{\sim}{i}-\underset{\sim}{j}$. Which of the following vectors is parallel to $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}$?

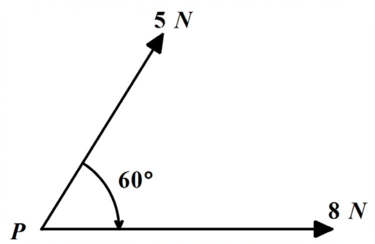
(A) $-2\underset{\sim}{i}-6\underset{\sim}{j}$

(B) $2\underset{\sim}{i}+8\underset{\sim}{j}$

(C) $2\underset{\sim}{i}-6\underset{\sim}{j}$

(D) $2\underset{\sim}{i}+6\underset{\sim}{j}$

3. Forces of magnitude $8\,N$ and $5N$ act on a particle P. The angle between the directions of the two forces is 60° as shown in the diagram.



Which of the following is correct magnitude and direction of the resultant force acting on P?

(A) $11.36\,N, 22^\circ 25'$ to the horizontal

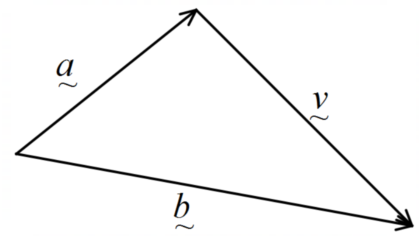
(B) $11.36\,N, 67^\circ 35'$ to the horizontal

(C) $12.58\,N, 22^\circ 25'$ to the horizontal

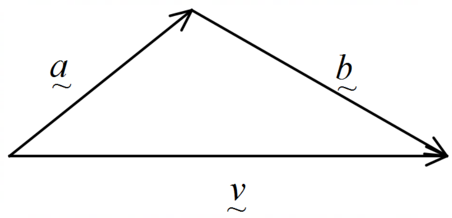
(D) $12.58\,N, 67^\circ 35'$ to the horizontal

4. Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$?

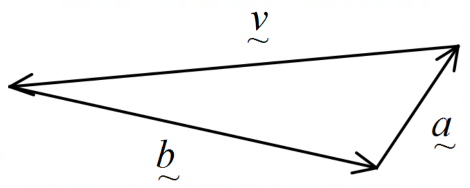
A



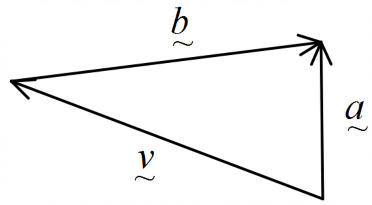
B



C



D



5. Consider the two non-zero vectors, $\underset{\sim}{u}$ and $\underset{\sim}{v}$, with the following properties:

- $\underset{\sim}{v} = k\underset{\sim}{u}$, where k is a constant and
- $\underset{\sim}{u} \cdot \underset{\sim}{v} < 0$.

Which of the following statements is true?

(A) $\underset{\sim}{u} \perp \underset{\sim}{v}$

(B) $\underset{\sim}{u} \parallel \underset{\sim}{v}$ in the same direction

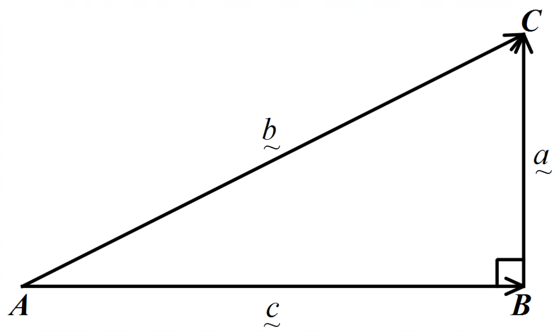
(C) $\underset{\sim}{u} \parallel \underset{\sim}{v}$ in opposite directions

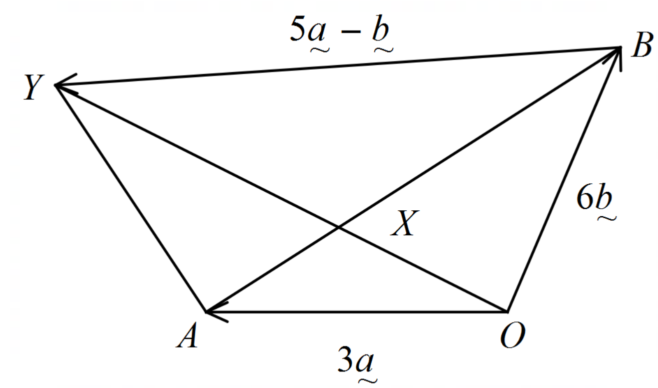
(D) The angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is acute

End of Section I

Section II – Extended Response

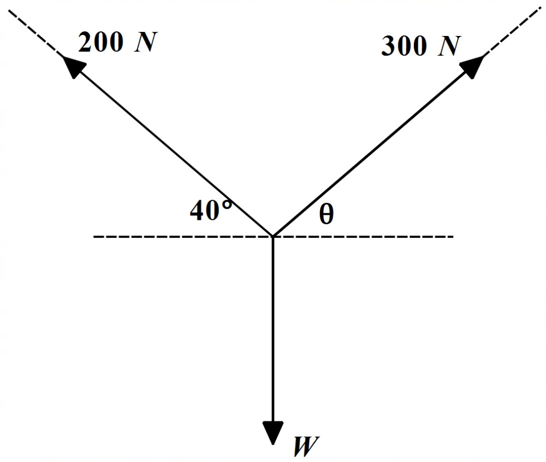
Attempt questions 6-8.
Answer each question on the appropriate pages of the answer booklet, showing all necessary working.

Question 6 (11 marks)		**Marks**
(a)	Consider the vectors $\overrightarrow{OP} = -2\underset{\sim}{i} + \underset{\sim}{j}$ and $\overrightarrow{OQ} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$.	
	(i) Find the vector $\underset{\sim}{u} = \overrightarrow{PQ}$.	**1**
	(ii) Given that $\underset{\sim}{v} = -\underset{\sim}{i} + 2\underset{\sim}{j}$, find $\underset{\sim}{u} \cdot \underset{\sim}{v}$.	**1**
	(iii) Find $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$.	**2**
(b)	Consider ΔABC below where $\overrightarrow{AB} = \underset{\sim}{c}$, $\overrightarrow{BC} = \underset{\sim}{a}$, $\overrightarrow{AC} = \underset{\sim}{b}$ and $\angle ABC = 90^\circ$. Let M be the midpoint of AC. 	
	(i) Explain why $\overrightarrow{MB} = \underset{\sim}{c} - \dfrac{\underset{\sim}{b}}{2}$.	**1**
	(ii) Find the expression for $\overrightarrow{MC}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.	**1**
(c)	(i) Prove by mathematical induction that for all integers $n \geq 1$, $$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$	**3**
	(ii) Use the above result to show that $2^2 + 4^2 + 6^2 + \ldots + 100^2 = 171700$.	**1**
	(iii) Hence evaluate $1^2 + 3^2 + 5^2 + \ldots + 99^2$.	**1**
	End of Question 6	

Question 7 (8 marks)		Marks
a)	Two vectors are given by $\underset{\sim}{a} = 3\underset{\sim}{i} - m\underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + n\underset{\sim}{j}$, where m and n are real with $m > 0$. If $\lvert \underset{\sim}{a} \rvert = 5$ and $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, find the values of m and n.	**2**
b)	In the quadrilateral $OAYB$, $\overrightarrow{OA} = 3\underset{\sim}{a}$, $\overrightarrow{OB} = 6\underset{\sim}{b}$ *and* $\overrightarrow{BY} = 5\underset{\sim}{a} - \underset{\sim}{b}$. AB intersects OY at X. 	
	(i) Express $\overrightarrow{AB}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.	**1**
	(ii) X is the point on AB such that $\overrightarrow{AX} = \frac{1}{2}\overrightarrow{XB}$. Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$.	**2**
c)	Use the method of mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for all integer, $n \geq 1$.	**3**
	End of Question 7	

Question 8 (10 marks) **Marks**

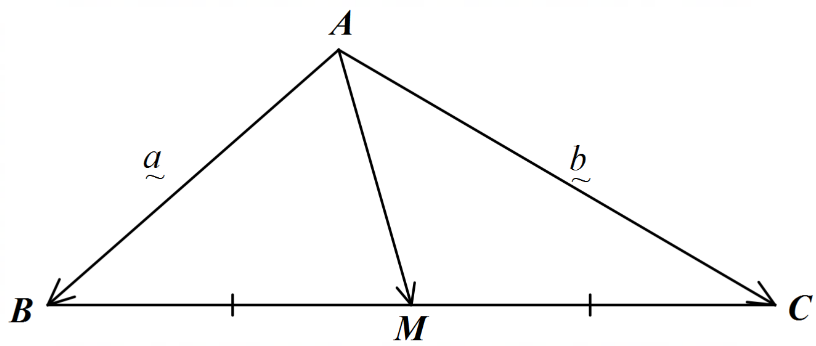
a) The diagram shows the weight force, W, that pulls straight down on an object. Two forces of 200 N and 300 N are exerted by cables as shown. If the resultant force is zero, find the size of θ to the nearest degree and magnitude of W to 2 decimal places. **3**



b) Use mathematical induction to prove that for integers $n \geq 1$, **3**

$$1+2\times\frac{1}{2}+3\times\left(\frac{1}{2}\right)^2+4\times\left(\frac{1}{2}\right)^3+\ldots+n\times\left(\frac{1}{2}\right)^{n-1}=4-\left(\frac{n+2}{2^{n-1}}\right).$$

c) In the diagram, M is the midpoint of the side BC of triangle ABC.
Let $\overrightarrow{AB}=\underset{\sim}{a}$ and $\overrightarrow{AC}=\underset{\sim}{b}$.

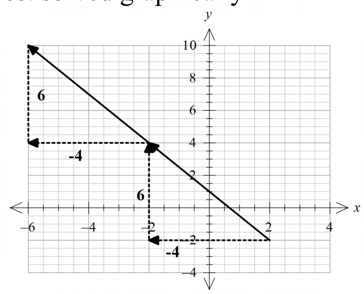


(i) Express the vector $\overrightarrow{AM}$ and $\overrightarrow{CM}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Show that $\left|\overrightarrow{AM}\right|^2+\left|\overrightarrow{CM}\right|^2=\frac{1}{2}\left(\left|\overrightarrow{AB}\right|^2+\left|\overrightarrow{AC}\right|^2\right)$. **2**

End of Exam

HSC Year12 Extension 1 Task 1 2022(suggested marking guidelines)

	Solutions	Mks	Comments
1	This problem could be best solved graphically  We are given that the coordinates of P are (2, -2). The vector $\overrightarrow{PQ}=(-4,6)$, so Q has coordinates (-2, 4). Given $\overrightarrow{PR}=2\overrightarrow{PQ},\ \overrightarrow{QR}=\overrightarrow{PQ}$ so R will have coordinates (-6, 10).	1	C
2	$\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=2\underset{\sim}{i}+3\underset{\sim}{j}-3\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{i}-\underset{\sim}{j}$ $=\underset{\sim}{i}+4\underset{\sim}{j}$ $2\underset{\sim}{i}+8\underset{\sim}{j}=2(\underset{\sim}{i}+4\underset{\sim}{j})$ $=2(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c})$ then gives option B	1	B
3	Horizontal components: $x=5\cos 60^\circ+8$ Vertical components: $y=5\sin 60^\circ+0$ $\text{Magnitude}=\sqrt{x^2+y^2}$ $=\sqrt{(5\cos 60^\circ+8)^2+(5\sin 60^\circ)^2}$ $=11.3578..$ $=11.36\,N$ $Direction:\tan\theta=\frac{y}{x}$ $=\frac{5\sin 60^\circ}{5\cos 60^\circ+8}$ $\therefore\theta=22^\circ24'39.28''$ $=22^\circ25'$ (This step is not necessary as θ must be less than 60° , but you can use it as a check)	1	A
4	$A.\ \underset{\sim}{a}+\underset{\sim}{v}=\underset{\sim}{b}$ $\underset{\sim}{v}=\underset{\sim}{b}-\underset{\sim}{a}$ $B.\ \underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{v}$ $C.\ \underset{\sim}{b}+\underset{\sim}{a}=-\underset{\sim}{v}$ $\underset{\sim}{v}=-\underset{\sim}{a}-\underset{\sim}{b}$ $D.\ \underset{\sim}{v}+\underset{\sim}{b}=\underset{\sim}{a}$ $\underset{\sim}{v}=\underset{\sim}{a}-\underset{\sim}{b}$	1	A

	Solutions	Mks	Comments
5	$\underset{\sim}{u}+\underset{\sim}{v}=k\underset{\sim}{u}$ means $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are parallel vectors $\underset{\sim}{u}\cdot\underset{\sim}{v}<0$ means angle between then is $>90^\circ$ $\therefore$ in opposite direction	**1**	C
6a(i)	$\overrightarrow{OP}=-2\underset{\sim}{i}+\underset{\sim}{j}\quad \overrightarrow{OQ}=4\underset{\sim}{i}-3\underset{\sim}{j}$ $\underset{\sim}{u}=\overrightarrow{PO}+\overrightarrow{OQ}$ $=-(\overrightarrow{OP})+\overrightarrow{OQ}$ $=-(-2\underset{\sim}{i}+\underset{\sim}{j})+(4\underset{\sim}{i}-3\underset{\sim}{j})$ $=6\underset{\sim}{i}-4\underset{\sim}{j}$	**1**	**1 mark** • correct answer.
6a(ii)	$\underset{\sim}{u}\cdot\underset{\sim}{v}=(6\underset{\sim}{i}-4\underset{\sim}{j})\cdot(-\underset{\sim}{i}+2\underset{\sim}{j})$ $=6\times-1+(-4)\times 2$ $=-14$	**1**	**1 mark** • correct solution.
6a(iii)	$\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}=\dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert}\underset{\sim}{u}$ $=\dfrac{-14}{\left(\sqrt{6^2+(-4)^2}\right)^2}(6\underset{\sim}{i}-4\underset{\sim}{j})$ $=\dfrac{-84}{52}\underset{\sim}{i}+\dfrac{56}{52}\underset{\sim}{j}$ $=\dfrac{-21}{13}\underset{\sim}{i}+\dfrac{14}{13}\underset{\sim}{j}$	**2**	**2 mark** • correct solution. **1 mark** Adequately using the formula for projection.
6b(i)	We know that $\overrightarrow{AB}=\underset{\sim}{c}$. Since M is the midpoint of $\overrightarrow{AC}$, then $\overrightarrow{AM}=\dfrac{b}{2}$ $\overrightarrow{MB}=\overrightarrow{MA}+\overrightarrow{AB}$ $=-\dfrac{\underset{\sim}{b}}{2}+\underset{\sim}{c}$ (as required)	**1**	**1 mark** • correct explanation with vector notation.
6(b)(ii)	$\overrightarrow{MC}=\overrightarrow{MB}+\overrightarrow{BC}$ $=\underset{\sim}{c}-\dfrac{\underset{\sim}{b}}{2}+\underset{\sim}{a}$	**1**	**1 mark** • correct answer.
6c(i)	Let $n=1$ $LHS=1^2$ $RHS=\dfrac{1}{6}\times1\times(1+1)\times(2\times1+1)$ $=\dfrac{1}{6}\times6$ $=1=LHS$ $\therefore$ true for $n=1$ Assume true for $n=k$	**3**	There are 4 key parts of the induction; 1. Proving the result true for $n=1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof

	Solutions	Mks	Comments
	i.e. $1^2+2^2+3^2+...+k^2=\frac{1}{6}k(k+1)(2k+1)$ Prove true for $n=k+1$ $1^2+2^2+3^2+...+k^2+(k+1)^2=\frac{1}{6}(k+1)(k+2)(2k+3)$ LHS $1^2+2^2+3^2+...+k^2+(k+1)^2$ $=\frac{1}{6}(k)(k+1)(2k+1)+(k+1)^2$ (from assumption) i.e. $=\frac{1}{6}(k+1)\left[k(2k+1)+6(k+1)\right]$ $=\frac{1}{6}(k+1)(2k^2+7k+6)$ $=\frac{1}{6}(k+1)(k+2)(2k+3)$ $=RHS$ $\therefore$ if true for $n=k$, then true for $n=k+1$. But it is true for $n=1$ Hence it is true for $n=1+1=2$, since true for $n=2$, it is true for $n=3$ and so on, then it is true for all positive integer n.		4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 marks** • Successfully does 2 of the 4 key parts
6c(ii)	$2^2+4^2+6^2+...+100^2=2^2(1^2+2^2+3^2+...+50^2)$ $=4\times\frac{1}{6}\times 50\times(50+1)(2\times 50+1)$ using the result from part(i) $=171700$	**1**	**1 mark** • Correct solution.
6c(iii)	$1^2+3^2+5^2+...+99^2$ $=(1^2+2^2+3^2+...+100^2)-(2^2+4^2+6^2+...+100^2)$ $=\frac{1}{6}\times 100\times(100+1)(2\times 100+1)-171700$ $=166650$	**1**	**1 mark** • Correct solution.
7a	$\underset{\sim}{a}=3\underset{\sim}{i}-m\underset{\sim}{j}$ $\underset{\sim}{b}=-2\underset{\sim}{i}+n\underset{\sim}{j}$ $\lvert\underset{\sim}{a}\rvert=5=\sqrt{3^2+m^2}$ $m^2=16$ $as\ m>0\ \therefore\ m=4$ $\underset{\sim}{a}\perp\underset{\sim}{b}\Rightarrow\underset{\sim}{a}\cdot\underset{\sim}{b}=0$ $\therefore 3(-2)-4n=0$ $4n=-6$ $n=-\frac{3}{2}$	**2**	**2 mark** • correct solution. **1 mark** • either finds n or m.
7b(i)	$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$ $=-\overrightarrow{OA}+\overrightarrow{OB}$ $=6\underset{\sim}{b}-3\underset{\sim}{a}$	**1**	**1 mark** • Correct answer.

	Solutions	Mks	Comments
7b (ii)	$\overrightarrow{OX}=3\underset{\sim}{a}+\frac{1}{3}(6\underset{\sim}{b}-3\underset{\sim}{a})$ $=2\underset{\sim}{a}+2\underset{\sim}{b}$ $\frac{2}{5}\overrightarrow{OY}=\frac{2}{5}(6\underset{\sim}{b}+5\underset{\sim}{a}-\underset{\sim}{b})$ $=2\underset{\sim}{a}+2\underset{\sim}{b}$ $\therefore \overrightarrow{OX}=\frac{2}{5}\overrightarrow{OY}.$	**2**	**2 marks** • correct solution. **1 mark** • significant progress towards solution.
7c	<u>Test for $n=1$</u> $2^{3(1)}-3=8-3=5$ $\therefore$ divisible by 5 $\therefore$ true for $n=1$. <u>Assume true for $n=k$</u> i.e $2^{3k}-3^k=5P$, where P is a positive integer Prove true for $n=k+1$ i,e $2^{3(k+1)}-3^{k+1}=5Q$ where Q is a positive integer LHS= $2^{3k}.2^3-3^k.3$ $(5P+3^k).8-3.3^k$ $40P+8.3^k-3.3^k$ (from assumption) $40P-5.3^k$ $5(8P-3^k)$ $5Q$ where Q is a positive integer Which is divisible by 5. Hence the statement is true for $n=k+1$ if it is true for $n=k$.. By Induction, since it is true for $n=1$, then it is also true for $n=2,3,4\ldots$ all positive integer of n	**3**	There are 4 key parts of the induction; 1. Proving the result true for $n=1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 marks** • Successfully does 2 of the 4 key parts Note: Without using the assumption, it is not the process of mathematical induction

	Solutions	Mks	Comments
8a	Equating up and down and left and right $200\sin 40 + 300\sin\theta = \lvert w\rvert \qquad (1)$ $300\cos\theta - 200\cos 40 = 0 \qquad (2)$ $300\cos\theta = 200\cos 40$ $\theta = \cos^{-1}\left(\frac{2}{3}\cos 40\right)$ $\theta = 59.28...$ $sub\ \theta = 59.28...\text{into}\,(1)$ 200sin40+300sin59.28..=w $w = -386.49...$ $\therefore \theta = 59.28 \quad and \quad \lvert w\rvert = 386.49(to\,2\,dp)$	**3**	**3 mark** • correct solution. **2 mark** • Correct θ. **1 mark** • one correct horizontal/vertical equation.
8b	Let $n = 1$ $LHS = 1$ $RHS = 4 - \frac{1+2}{2^{1-1}}$ $= 4 - 3$ $= 1 = LHS$ $\therefore$ true for $n = 1$ Assume true for $n = k$ i.e. $1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 + ... + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$ Prove true for $n = k+1$ $1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 + ... + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^{k+1-1} = 4 - \frac{(k+1)+2}{2^{k+1-1}} = 4 - \frac{k+}{2^k}$ LHS $1 + 2\times\frac{1}{2} + 3\times\left(\frac{1}{2}\right)^2 + ... + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^{k}$ $= 4 - \frac{(k+2)}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$ (from assumption) $= 4 - \frac{(k+2)}{2^{k-1}} + (k+1)\frac{1}{2^k}$ $= 4 + \frac{-2(k+2)+k+1}{2^k}$ $= 4 + \frac{-k-3}{2^k}$ $= 4 - \frac{k+3}{2^k} = RHS$ If it is true for $n = k$, then it is also true for $n = k+1$. By Induction, since it is true for $n = 1$, then it is also true for $n = 2, 3, 4\ldots$ all positive integer of n	**3**	There are 4 key parts of the induction; 1. Proving the result true for n = 1 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 marks** • Successfully does 2 of the 4 key parts Note: Without using the assumption, it is not the process of mathematical induction

	Solutions	Mks	Comments
8c(i)	$\overrightarrow{AM}=\underset{\sim}{a}=\overrightarrow{BM} \quad (1)$ $also\ \overrightarrow{AM}=\underset{\sim}{b}+\overrightarrow{CM}\ and\ as$ $\overrightarrow{CM}=-\overrightarrow{BM}\ then$ $\overrightarrow{AM}=\underset{\sim}{b}-\overrightarrow{BM} \quad (2)$ $By\ adding\ (1)\ and\ (2)$ $2\overrightarrow{AM}=\underset{\sim}{a}+\underset{\sim}{b}$ $\overrightarrow{AM}=\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$ $\overrightarrow{CM}=-\underset{\sim}{b}+\overrightarrow{AM}$ $\overrightarrow{CM}=-\underset{\sim}{b}+\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$ $=-\underset{\sim}{b}+\frac{1}{2}\underset{\sim}{a}+\frac{1}{2}\underset{\sim}{b}$ $=\frac{1}{2}\underset{\sim}{a}-\frac{1}{2}\underset{\sim}{b}$ $=\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b})$	**2**	**2 mark** • correct solution. **1 mark** • finds the correct expression for $\overrightarrow{AM}\ or\ \overrightarrow{CM}$..
8c(ii)	$=\left\|\overrightarrow{AM}\right\|^2+\left\|\overrightarrow{CM}\right\|^2$ $=\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b}).\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})+\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b}).\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b})$ $=\frac{1}{4}(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})+\frac{1}{4}(\underset{\sim}{a}-\underset{\sim}{b})\cdot(\underset{\sim}{a}-b)$ LHS $=\frac{1}{4}\left(\left\|\underset{\sim}{a}\right\|^2+2\underset{\sim}{a}\cdot\underset{\sim}{b}+\left\|\underset{\sim}{b}\right\|^2+\left\|\underset{\sim}{a}\right\|^2-2\underset{\sim}{a}\cdot\underset{\sim}{b}+\left\|\underset{\sim}{b}\right\|^2\right)$ $=\frac{1}{4}\left(2\left\|\underset{\sim}{a}\right\|^2+2\left\|\underset{\sim}{b}\right\|^2\right)$ $=\frac{1}{2}\left(\left\|\underset{\sim}{a}\right\|^2+\left\|\underset{\sim}{b}\right\|^2\right)$ $=\frac{1}{2}\left(\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{AC}\right\|^2\right)$ $=RHS$	**2**	**2 mark** • correct solution. **1 mark** • finds the correct expression for $\left\|\overrightarrow{AM}\right\|^2+\left\|\overrightarrow{CM}\right\|^2$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.