BAULKHAM HILLS
HS
PERSEVERE

2023 YEAR 12 ASSESSMENT TASK 1

Mathematics Extension 1

General Instructions	• Reading time – 5 minutes • Working time – 1 hour • Write using black pen. • Calculators approved by NESA may be used. • Answer all questions in the booklet provided. • In Questions 6 – 8, show relevant mathematical reasoning and/or calculations. • Relevant formulas are provided in the answer booklet
Total marks: 38	**Section I – 5 marks** (pages 2 – 3) • Attempt Questions 1 – 5 • Allow about 8 minutes for this section. **Section II – 33 marks** (pages 4 – 8) • Attempt Questions 6 – 8 • Allow about 52 minutes for this section.

Section I
5 marks
Attempt questions 1 - 5
Allow about 8 minutes for this section

Use the multiple-choice answer page in your booklet to answer Questions 1 – 5.

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1) If a vector $\underset{\sim}{u}$ is multiplied by a scalar λ, which of the following expressions represents its magnitude?

A) $|\underset{\sim}{u}|+\lambda$

B) $|\underset{\sim}{u}|^2+\lambda^2$

C) $\underset{\sim}{u}|\lambda|$

D) $|\underset{\sim}{u}|\lambda$

2) If $\underset{\sim}{a}=\begin{pmatrix}2\\1\end{pmatrix}$ and $\underset{\sim}{b}=\begin{pmatrix}-4\\-8\end{pmatrix}$ then which of the following is the length of the projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$?

A) $\dfrac{16}{\sqrt{5}}$

B) $-\dfrac{16}{\sqrt{5}}$

C) $\dfrac{4}{\sqrt{5}}$

D) $-\dfrac{4}{\sqrt{5}}$

3) A force of magnitude 18 newtons acts on a body at an angle of 150° in the anticlockwise direction to the vector $\underset{\sim}{i}$. Which of the following is the vector representation of this force?

A) $-9\underset{\sim}{i}+9\sqrt{3}\underset{\sim}{j}$

B) $-9\sqrt{3}\underset{\sim}{i}+9\underset{\sim}{j}$

C) $-9\underset{\sim}{i}-9\sqrt{3}\underset{\sim}{j}$

D) $9\sqrt{3}\underset{\sim}{i}-9\underset{\sim}{j}$

4) Vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are such that $\left|\underset{\sim}{p}\right| = 3$ and $\left|\underset{\sim}{q}\right| = 4$ and $\underset{\sim}{p} \cdot \underset{\sim}{q} = 10$. What is the value of $\underset{\sim}{q} \cdot (\underset{\sim}{p} + \underset{\sim}{q})$?

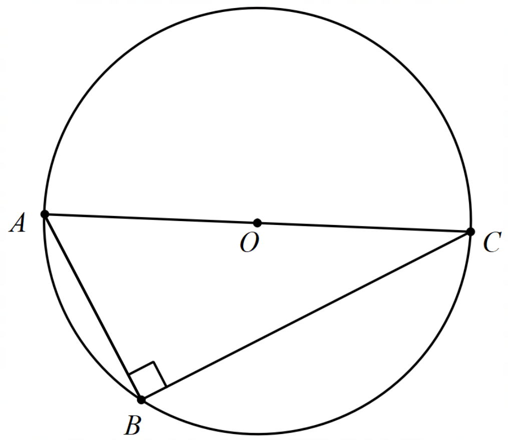
A) 0

B) 14

C) 26

D) 28

5) A circle with centre O has radius $\overrightarrow{OA} = \underset{\sim}{a}$, the points B and C are on the circle such that $\overrightarrow{BC} = \underset{\sim}{b}$.



Which one of the following statements must be true?

A) $\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{b}$

B) $\underset{\sim}{a} = -\frac{1}{2}\underset{\sim}{b}$

C) $2\underset{\sim}{a} \cdot \underset{\sim}{b} = -\underset{\sim}{b} \cdot \underset{\sim}{b}$

D) $\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{b} \cdot \underset{\sim}{b}$

End of Section I

Section II (33 marks)

Attempt Questions 6 - 8
Allow about 52 minutes for this section

Answer each question on the appropriate pages of your answer booklet. Extra paper is available.
Your responses should include relevant calculations and/or mathematical reasoning.

Question 6 (12 marks)

a) Given that $OACB$ is a parallelogram, where $\overrightarrow{OA} = \underset{\sim}{a} = -2\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{OB} = \underset{\sim}{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$. Find;

i) $3\underset{\sim}{b} - 2\underset{\sim}{a}$ **1**

ii) the magnitude of $\overrightarrow{OA}$ **1**

iii) the position vector $\overrightarrow{OC}$ **1**

iv) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

v) the angle θ between vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ correct to the nearest degree. **1**

b) Evaluate p and q if $4\begin{pmatrix} p \\ -2 \end{pmatrix} - 3\begin{pmatrix} 2 \\ q \end{pmatrix} = \begin{pmatrix} 14 \\ -8 \end{pmatrix}$. **2**

c) Consider the vector $\underset{\sim}{a} = 2\underset{\sim}{i} + 2\sqrt{3}\underset{\sim}{j}$.

i) Find the unit vector in the direction of $\underset{\sim}{a}$. **1**

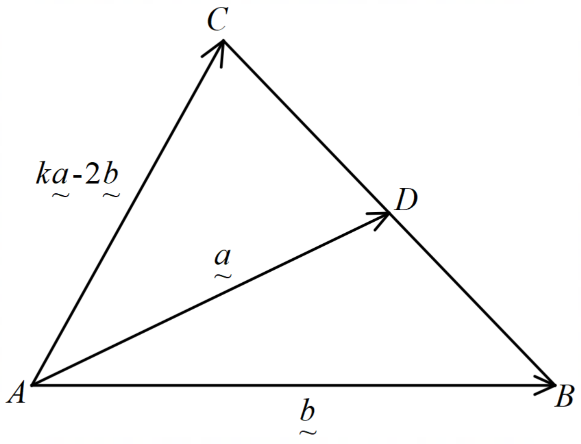
ii) Given that vector $\underset{\sim}{b} = m\underset{\sim}{i} - 2\underset{\sim}{j}$ is perpendicular to $\underset{\sim}{a}$, find the value of m. **1**

d) Use the method of mathematical induction to prove that $5^{2n} + 5^n + 2$ is divisible by 4, where n is positive integer. **3**

End of Question 6

Question 7 (9 marks)

a) ABC is a triangle where $\overrightarrow{AC} = k\underset{\sim}{a} - 2\underset{\sim}{b}$, $\overrightarrow{AD} = \underset{\sim}{a}$ and $\overrightarrow{AB} = \underset{\sim}{b}$. Find the value of k, if point D lies on side BC of a triangle. **3**



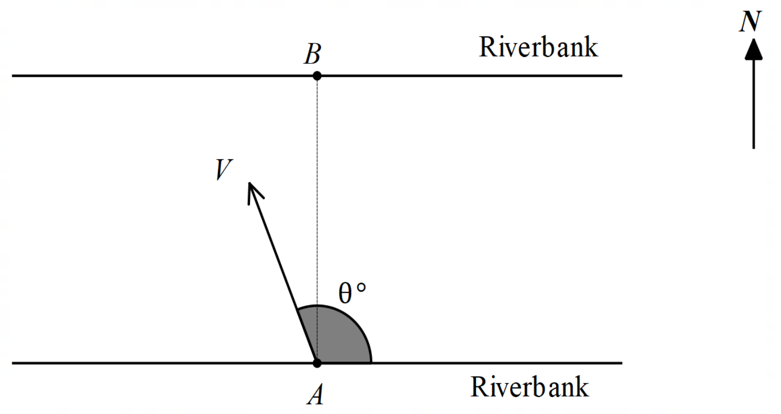
b) Consider the vectors $\underset{\sim}{u} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$.

i) Find the vector projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$. **2**

ii) Find a possible vector $\underset{\sim}{w}$ such that $\left|proj_{\underset{\sim}{u}}\underset{\sim}{w}\right| = 2$. **1**

Question 7 continues on next page

c) Andy wants to swim across the river from position A to position B shown on the diagram below. The riverbanks are parallel straight lines, and $\overrightarrow{AB}$ is perpendicular to the riverbanks. The speed of the river flow is 0.5m/s from west to east and the river is 0.7 km wide. Assume Andy swims at a constant speed of 1.4 m/s in still water.



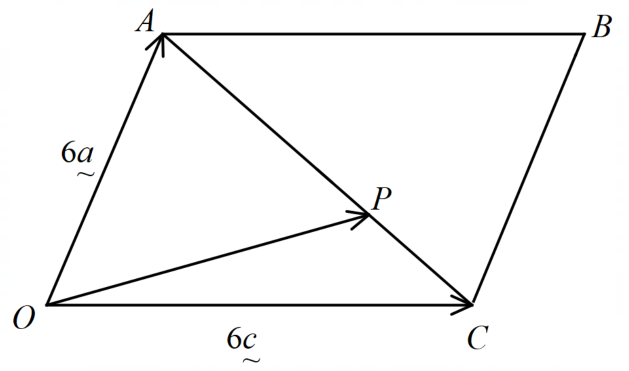
i) Find the angle θ at which Andy should swim to cross the river in the shortest time from point A to point B. Draw a vector diagram to demonstrate the velocities and show all necessary working out. Give your answer in degrees correct to two decimal places. **2**

ii) Calculate the time Andy would take to travel from point A to point B. Give your answer to the nearest second. **1**

End of Question 7

Question 8 (12 marks)

a) $OABC$ is a parallelogram, where $\overrightarrow{OA} = 6\underset{\sim}{a}$, $\overrightarrow{OC} = 6\underset{\sim}{c}$ and point P is on AC such that $AP : PC = 2 : 1$.



i) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$. **2**

ii) Given that the midpoint of CB is M, prove that OPM is a straight line. **2**

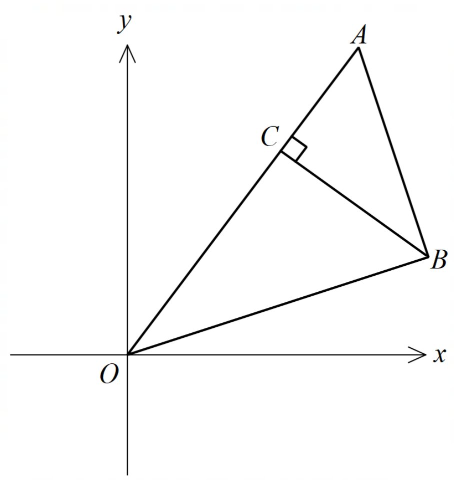
b) If $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are unit vectors such that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = \underset{\sim}{0}$ then find the value of $\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{c} + \underset{\sim}{c} \cdot \underset{\sim}{a}$. **2**

c) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$\frac{3}{1\times 2\times 2} + \frac{4}{2\times 3\times 2^2} + \ldots + \frac{n+2}{n(n+1)2^n} = 1 - \frac{1}{(n+1)2^n}$$

Question 8 continues on next page

d) The diagram below shows ΔOAB where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.



Point C lies on OA such that CB is perpendicular to OA.

i) Show that $\overrightarrow{BC} = \dfrac{1}{\underset{\sim}{a} \cdot \underset{\sim}{a}}\left[\left(\underset{\sim}{a} \cdot \underset{\sim}{b}\right)\underset{\sim}{a} - \left(\underset{\sim}{a} \cdot \underset{\sim}{a}\right)\underset{\sim}{b}\right]$. **2**

ii) Hence, show that the area of ΔOAB is $\dfrac{1}{2\left|\underset{\sim}{a}\right|}\left|\left(\underset{\sim}{a} \cdot \underset{\sim}{b}\right)\underset{\sim}{a} - \left(\underset{\sim}{a} \cdot \underset{\sim}{a}\right)\underset{\sim}{b}\right|$ **1**

End of Exam

2023 Year 12 Assessment Task 1 Solutions

Section 1 Multiple Choice

1 D 2 A 3 B 4C 5C

1 Magnitude $= |\underset{\sim}{u}|\lambda$ (D)

2 Length of projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$ $= \left|\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}}\right|$

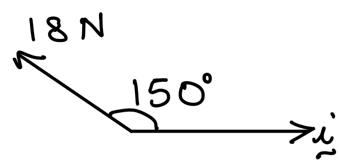
$$= \left|\frac{2(-4)+1(-8)}{\sqrt{2^2+1^2}}\right|$$

$$= \left|\frac{-8-8}{\sqrt{5}}\right|$$

$$= \frac{16}{\sqrt{5}}$$

(A)

3 Vector representation of force is:



$$(18\cos 150°)\underset{\sim}{i} + (18\sin 150°)\underset{\sim}{j}$$

$$18\left(-\frac{\sqrt{3}}{2}\right)\underset{\sim}{i} + 18\left(\frac{1}{2}\right)\underset{\sim}{j}$$

$$-9\sqrt{3}\,\underset{\sim}{i} + 9\underset{\sim}{j}$$

(B)

4 $\underset{\sim}{q}\cdot(\underset{\sim}{p}+\underset{\sim}{q}) = \underset{\sim}{q}\cdot\underset{\sim}{p} + \underset{\sim}{q}\cdot\underset{\sim}{q}$

$$= 10 + 16$$

$$= 26$$

(C)

5 $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{b}$

$\overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$

$= \underset{\sim}{b} + 2\underset{\sim}{a}$ $\left(\because \overrightarrow{CA} = 2\overrightarrow{OA}\right)$

$\overrightarrow{BA} \perp \overrightarrow{BC}$

$\therefore \overrightarrow{BA} \cdot \overrightarrow{BC} = 0$

$(\underset{\sim}{b} + 2\underset{\sim}{a}) \cdot \underset{\sim}{b} = 0$

$\underset{\sim}{b} \cdot \underset{\sim}{b} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$2\underset{\sim}{a} \cdot \underset{\sim}{b} = -\underset{\sim}{b} \cdot \underset{\sim}{b}$

(C)

END OF Multiple Choice Section.

Section-2

Question-6 a)

i) $3\underset{\sim}{b} - 2\underset{\sim}{a} = 3(3\underset{\sim}{i} + 5\underset{\sim}{j}) - 2(-2\underset{\sim}{i} + 4\underset{\sim}{j})$

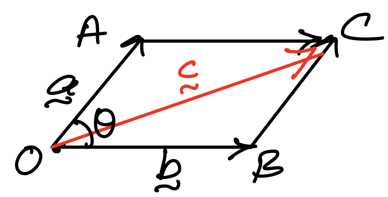
$= 13\underset{\sim}{i} + 7\underset{\sim}{j}$

1 mark correct answer

ii) $|\overrightarrow{OA}| = \sqrt{(-2)^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$

1 mark correct answer

iii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ or $\overrightarrow{OB} + \overrightarrow{BC}$



$= \overrightarrow{OA} + \overrightarrow{OB}$

$= (-2\underset{\sim}{i} + 4\underset{\sim}{j}) + (3\underset{\sim}{i} + 5\underset{\sim}{j})$

$= \underset{\sim}{i} + 9\underset{\sim}{j}$

1 mark correct answer

iv) $\underset{\sim}{a} \cdot \underset{\sim}{b} = (-2)(3) + (4)(5)$

$= -6 + 20 = 14$

1 mark correct answer

v) $\cos\theta = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

$= \dfrac{14}{\sqrt{20}\,\sqrt{34}}$

$\theta = 57.5288$

$\theta = 58^\circ$

1 mark correct answer

Question-6

b) $4\begin{pmatrix} p \\ -2 \end{pmatrix} - 3\begin{pmatrix} 2 \\ q \end{pmatrix} = \begin{pmatrix} 14 \\ -8 \end{pmatrix}$

$\begin{pmatrix} 4p-6 \\ -8-3q \end{pmatrix} = \begin{pmatrix} 14 \\ -8 \end{pmatrix}$

$4p-6=14$ & $-8-3q=-8$

$p=5$ $\quad$ $q=0$

2 marks correct solution

1 mark for correct value of either p or q.

(OR)

Correct process to multiply and equating two sides of the vector to perform equations.

c)

i) $\underset{\sim}{a} = 2\underset{\sim}{i} + 2\sqrt{3}\underset{\sim}{j}$

$|\underset{\sim}{a}| = \sqrt{(2)^2+(2\sqrt{3})^2} = 4$

$\hat{\underset{\sim}{a}} = \dfrac{2\underset{\sim}{i}+2\sqrt{3}\underset{\sim}{j}}{4} = \frac{1}{2}(\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j})$

or $\dfrac{\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}}{2}$

1 mark correct solution

ii) Since $\underset{\sim}{a} \perp \underset{\sim}{b}$ $\therefore$ $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$2m - 4\sqrt{3} = 0$

$m = 2\sqrt{3}$

1 mark correct solution

d) Let $S(n)$ be the statement that $5^{2n}+5^{n}+2$ is divisible by 4.

Show $S(1)$ is true

$5^{2}+5+2 = 25+5+2 = 32$, which is divisible by 4.

$\therefore S(1)$ is true.

Assume $S(k)$ is true

i.e $5^{2k}+5^{k}+2 = 4M$, for some integer M.

Show $S(k+1)$ is true.

RTP : $5^{2(k+1)}+5^{k+1}+2 = 4N$, for some integer N.

$$
\begin{aligned}
L.H.S &= 5^{2(k+1)}+5^{k+1}+2 \\
&= 25(5^{2k})+5(5^{k})+2 \\
&= 25(4M-5^{k}-2)+5(5^{k})+2 \quad \text{(using assumption)} \\
&= 100M-20(5^{k})-48 \\
&= 4(25M-5(5)^{k}-12) \\
&= 4N \quad \text{where } N = 25M-5^{k+1}-12 \text{ is integer.} \\
&= R.H.S
\end{aligned}
$$

$\therefore S(k+1)$ is true if $S(k)$ is assumed true.

Since $S(1)$ is true, by the principle of mathematical induction, $S(n)$ is true for all positive integers n.

3 marks correct induction.
2 marks to use the assumption to prove the required result.
1 mark correct base case.

Question 7 a)

Since D lies on BC $\therefore \overrightarrow{BC} \parallel \overrightarrow{BD}$

$\therefore \overrightarrow{BC} = \lambda \overrightarrow{BD}$ for some real λ.

$$\overrightarrow{AC} - \overrightarrow{AB} = \lambda(\overrightarrow{AD} - \overrightarrow{AB})$$

$$k\underset{\sim}{a} - 2\underset{\sim}{b} - \underset{\sim}{b} = \lambda(\underset{\sim}{a} - \underset{\sim}{b})$$

$$k\underset{\sim}{a} - 3b = \lambda \underset{\sim}{a} - \lambda \underset{\sim}{b}$$

Coeff. of $\underset{\sim}{b}$, $\Rightarrow \lambda = 3$

& Coeff. of $\underset{\sim}{a} \Rightarrow k = \lambda = 3$.

3 marks correct solution.

2 marks for correctly finds two equations from $\overrightarrow{BC}$, $\overrightarrow{BD}$ or $\overrightarrow{DC}$.

1 mark to find $\overrightarrow{BC}$ or $\overrightarrow{BD}$ or realising C, D B are collinear points.

b) i) $\underset{\sim}{u} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$, $\quad \underset{\sim}{v} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = -6 - 8 = -14 \qquad \underset{\sim}{v} \cdot \underset{\sim}{v} = 4 + 4 = 8$

$$\text{proj}_{\underset{\sim}{v}} \underset{\sim}{u} = \left(\frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\underset{\sim}{v} \cdot \underset{\sim}{v}}\right) \underset{\sim}{v}$$

$$= -\frac{14}{8}(2\underset{\sim}{i} - 2\underset{\sim}{j})$$

$$= -\frac{7}{4}(2\underset{\sim}{i} - 2\underset{\sim}{j})$$

$$= -\frac{7}{2}(\underset{\sim}{i} - \underset{\sim}{j})$$

2 marks correct solution

1 mark to attempt the correct process to find the projection.

ii) Let $\underset{\sim}{w} = x\underset{\sim}{i} + y\underset{\sim}{j}$

$$\text{scal}_{\underset{\sim}{u}} \underset{\sim}{w} = \frac{\underset{\sim}{w} \cdot \underset{\sim}{u}}{|\underset{\sim}{u}|} = \frac{-3x + 4y}{5}$$

Since $\text{scal}_{\underset{\sim}{u}} \underset{\sim}{w} = 2$

$$\therefore -3x + 4y = 10$$

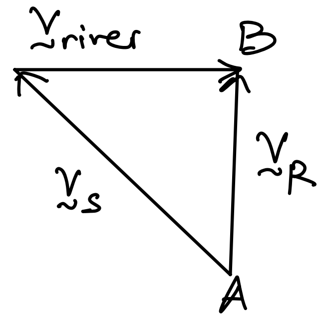
1 mark correct answer

$\therefore \underset{\sim}{w} = -2\underset{\sim}{i} + \underset{\sim}{j}$ or any possible set of x & y that statisfies the equation.

c) River flow velocity vector $\underset{\sim}{V}_{river}$

Let swimmers velocity vector $\underset{\sim}{V}_s$

and Resultant velocity vector $\underset{\sim}{V}_R$



$$\therefore \underset{\sim}{V}_s + \underset{\sim}{V}_{river} = \underset{\sim}{V}_R \quad \text{——— (i)}$$

R is the speed in m/s.

$$\underset{\sim}{V}_s = \begin{pmatrix} 1.4\cos\theta \\ 1.4\sin\theta \end{pmatrix} \qquad \underset{\sim}{V}_{river} = \begin{pmatrix} 0.5\cos 0^\circ \\ 0.5\sin 0^\circ \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$

$$\underset{\sim}{V}_R = \begin{pmatrix} 0 \\ R \end{pmatrix}$$

Using (i);
$$\begin{pmatrix} 0 \\ R \end{pmatrix} = \begin{pmatrix} 1.4\cos\theta \\ 1.4\sin\theta \end{pmatrix} + \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$

$$1.4\cos\theta + 0.5 = 0 \quad \text{——— (2)}$$

$$R = 1.4\sin\theta \quad \text{——— (3)}$$

2 marks correct solution

1 mark attempt to create vector equations

from (2); $\cos\theta = -\dfrac{0.5}{1.4}$

$$\theta = 110.924..$$
$$= 110.92^\circ$$

$\therefore$ Swimmer should swim at an angle of $110^\circ 55'$ on the south bank at point A.

ii) from (3); $R = 1.4\sin 110^\circ 55' = 1.307669...$ m/s

$\therefore$ Time taken to cross river and reach point B is

$$\frac{700}{1.307669..} \approx 535.3033 \text{ seconds}$$

8 mins 55 seconds

1 mark correct solution

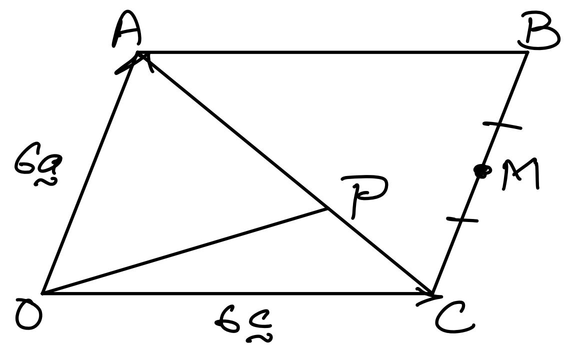
Question 8

a) i) $AP : PC = 2 : 1$

$\therefore \ \vec{AP} = \frac{2}{3}\vec{AC}$

$= \frac{2}{3}(6\underset{\sim}{c} - 6\underset{\sim}{a})$

$= 4(\underset{\sim}{c} - \underset{\sim}{a})$



$\vec{OP} = \vec{OA} + \vec{AP}$

$= 6\underset{\sim}{a} + 4(\underset{\sim}{c} - \underset{\sim}{a})$

$= 2\underset{\sim}{a} + 4\underset{\sim}{c}$

2 marks correct solution
1 mark for correct $\vec{AP}$ or $\vec{PC}$.

ii) $\vec{CM} = \frac{1}{2}\vec{CB} = \frac{1}{2}\vec{OA} = \frac{1}{2}(6\underset{\sim}{a})$

$= 3\underset{\sim}{a}$

$\vec{OM} = \vec{OC} + \vec{CM}$

$= 6\underset{\sim}{c} + 3\underset{\sim}{a}$

$= 3(2\underset{\sim}{c} + \underset{\sim}{a})$

$= 3(\underset{\sim}{a} + 2\underset{\sim}{c})$

$= \frac{3}{2}\vec{OP}$

2 marks correct proof
1 mark to find $\vec{CM}$ or $\vec{OM}$.

$\therefore \ \vec{OM} \parallel \vec{OP}$

Since they both share the common point O.

$\therefore$ OPM is a straight line.

b) Since $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are unit vectors

$$\therefore |\underset{\sim}{a}| = |\underset{\sim}{b}| = |\underset{\sim}{c}| = 1$$

$$(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c})\cdot(\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}) = 0$$

$$\therefore \underset{\sim}{a}\cdot\underset{\sim}{a} + \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{a}\cdot c + \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{c} + \underset{\sim}{c}\cdot\underset{\sim}{a} + \underset{\sim}{c}\cdot\underset{\sim}{b} + \underset{\sim}{c}\cdot\underset{\sim}{c} = 0$$

$$2(\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{c} + \underset{\sim}{c}\cdot\underset{\sim}{a}) + |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 = 0$$

$$2(\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{c} + \underset{\sim}{c}\cdot\underset{\sim}{a}) + 1 + 1 + 1 = 0$$

$$\therefore \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{c} + \underset{\sim}{c}\cdot\underset{\sim}{a} = -\frac{3}{2}$$

2 mark correct solution
1 mark attempt to prove by using unit vector with $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=0$

8 c)

Let $S(n)$ be the statement that

$$\frac{3}{1\times2\times2} + \frac{4}{2\times3\times2^2} + \cdots + \frac{n+2}{n(n+1)2^n} = 1 - \frac{1}{(n+1)2^n}$$

<u>Show $S(1)$ is true</u>

$$L.H.S = \frac{1+2}{1(1+1)2^1} = \frac{3}{1\times2\times2} = \frac{3}{4}$$

$$R.H.S = 1 - \frac{1}{(1+1)2^1} = 1 - \frac{1}{4} = \frac{3}{4}$$

Since $L.H.S = R.H.S$

$\therefore$ $S(1)$ is true.

Assume $S(K)$ is true

i.e., $\dfrac{3}{1\times2\times2} + \dfrac{4}{2\times3\times2^2} + \cdots + \dfrac{k+2}{k(k+1)2^k} = 1 - \dfrac{1}{(k+1)2^k}$

Show $S(k+1)$ is true

i.e., $\dfrac{3}{1\times2\times2} + \dfrac{4}{2\times3\times2^2} + \cdots + \dfrac{k+2}{k(k+1)2^k} + \dfrac{k+3}{(k+1)(k+2)2^{k+1}} = 1 - \dfrac{1}{(k+2)2^{k+1}}$

$$L.H.S = \underbrace{\frac{3}{1\times2\times2} + \frac{4}{2\times3\times2^2} + \cdots + \frac{k+2}{k(k+1)2^k}}_{S(k)} + \frac{k+3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+1)2^k} + \frac{k+3}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \left(\frac{2(k+2) - (k+3)}{(k+1)(k+2)2^{k+1}}\right)$$

$$= 1 - \left(\frac{2k+4-k-3}{(k+1)(k+2)2^{k+1}}\right)$$

$$= 1 - \frac{(k+1)}{(k+1)(k+2)2^{k+1}}$$

$$= 1 - \frac{1}{(k+2)2^{k+1}}$$

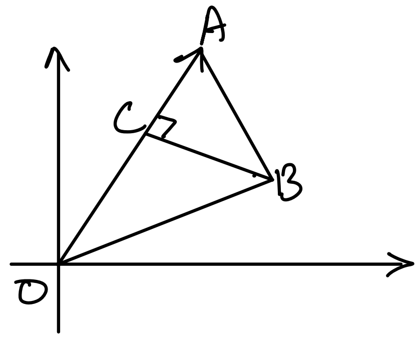
$= R.H.S$

3 marks correct induction.
2 marks to use the assumption to prove the required result.
1 mark correct base case.

$\therefore S(k+1)$ is true if $S(k)$ is assumed true.

Since $S(1)$ is true, by the principle of mathematical induction, $S(n)$ is true for all positive integers n.

8 d)



$\overrightarrow{OC}$ is the projection of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$

$$\therefore \overrightarrow{OC} = \text{proj}_{\underset{\sim}{a}} \underset{\sim}{b}$$

$$= \left(\frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}}\right) \underset{\sim}{a}$$

2 marks correct proof

1 mark to find $\overrightarrow{OC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$

$$= \left(\frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}}\right) \underset{\sim}{a} - \underset{\sim}{b}$$

$$= \left(\frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}}\right) \underset{\sim}{a} - \left(\frac{\underset{\sim}{a} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}}\right) \underset{\sim}{b}$$

$$= \frac{1}{\underset{\sim}{a} \cdot \underset{\sim}{a}} \left[(\underset{\sim}{b} \cdot \underset{\sim}{a}) \underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{a}) \underset{\sim}{b} \right]$$

= Hence shown

ii) Area of $\Delta OAB = \frac{1}{2} |\overrightarrow{OA}| \, |\overrightarrow{BC}|$

$$= \frac{1}{2} |\underset{\sim}{a}| \left| \frac{1}{\underset{\sim}{a} \cdot \underset{\sim}{a}} \left[(\underset{\sim}{b} \cdot \underset{\sim}{a}) \underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{a}) \underset{\sim}{b} \right] \right|$$

$$= \frac{|\underset{\sim}{a}|}{2|\underset{\sim}{a}|^2} \left| (\underset{\sim}{b} \cdot \underset{\sim}{a}) \underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{a}) \underset{\sim}{b} \right|$$

$$= \frac{1}{2|\underset{\sim}{a}|} \left| (\underset{\sim}{b} \cdot \underset{\sim}{a}) \underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{a}) \underset{\sim}{b} \right|$$

Hence shown.

$\boxed{\text{1 mark correct proof}}$

End of Solutions