BAULKHAM HILLS HS PERSEVERE

2023 **PRELIMINARY HALF-YEARLY EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour 30 mins
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 58

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II – 48 marks (pages 7 – 10)

- Attempt Questions 11 – 14
- Allow about 1 hour and 15 minutes for this section.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 Let $y = f(x)$ be a function. Which of the following MUST be true of the inverse of $y = f(x)$?

A. It must be one-to-one.

B. It could be one-to-one but cannot be one-to-many.

C. It must be either one-to-one or one-to-many.

D. It could be either one-to-one, many-to-one or one-to-many.

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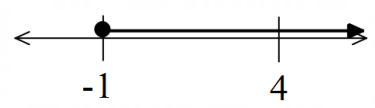
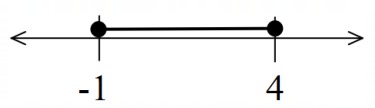
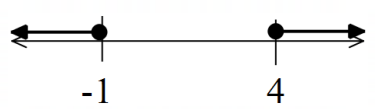
2 Which diagram best represents the set of solutions of the inequality $|2x-3| \geq 5$?

A.


B.


C.


D.


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3 A group of 4 men and 8 girls include a father and daughter. From this group, a team consisting of 2 men and 2 girls is to be chosen. How many ways can the team be chosen if at least one of the father and daughter is on the team?

A. 105

B. 136

C. 144

D. 147

4 Which of the following is a simplified expression for ${}^{n+1}C_{n-1}$?

A. $\frac{1}{2}\left(n^2 - n\right)$

B. $\frac{1}{2}\left(n^2 + n\right)$

C. $n^2 - n$

D. $n^2 + n$

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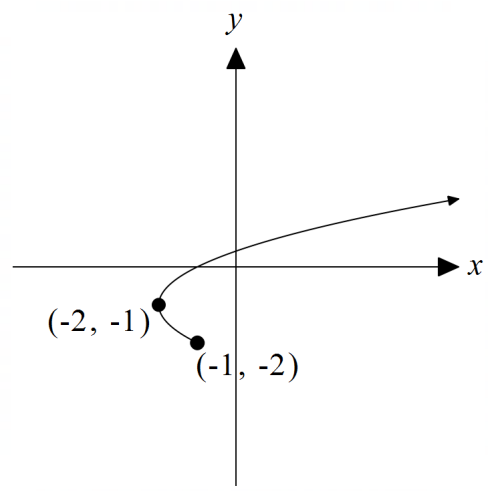
5 A section of a parabola is shown in the graph below.



Which pair of parametric equations best represents the graph?

A. $\begin{cases} x = 2 + t^2 \\ y = 1 + t \end{cases}, t \geq -1$

B. $\begin{cases} x = -2 + t^2 \\ y = -1 + t \end{cases}, t \geq -1$

C. $\begin{cases} x = 2 + t^2 \\ y = 1 + t \end{cases}, t \leq -1$

D. $\begin{cases} x = -2 + t^2 \\ y = -1 + t \end{cases}, t \leq -1$

6 A club has n boys and $n + 1$ girls for $n \geq 3$.

How many ways are there to select 3 children from the club if more girls than boys are selected?

A. ${}^{2n+1}C_3 + {}^{2n+1}C_2$

B. ${}^{2n+1}C_3$

C. ${}^{n+1}C_2 + {}^{n}C_1$

D. ${}^{n+1}C_3 \, {}^{n}C_0 + {}^{n+1}C_2 \, {}^{n}C_1$

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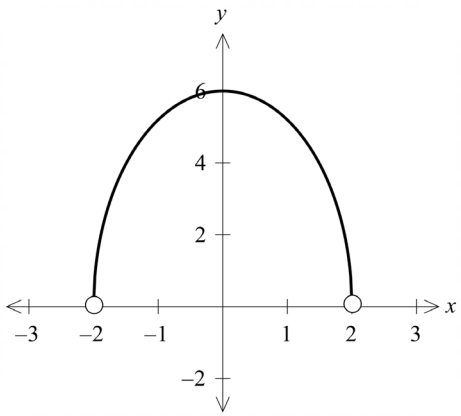
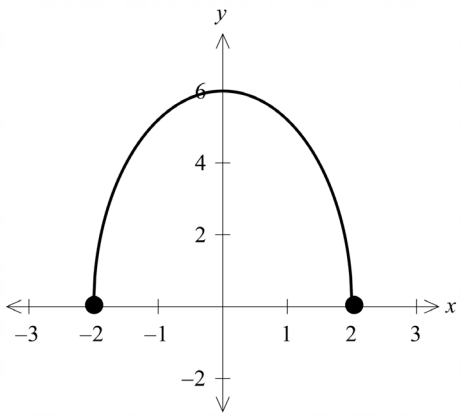
7 Which graph best represents the curve $\dfrac{y}{\sqrt{36-9x^2}} = 1$?

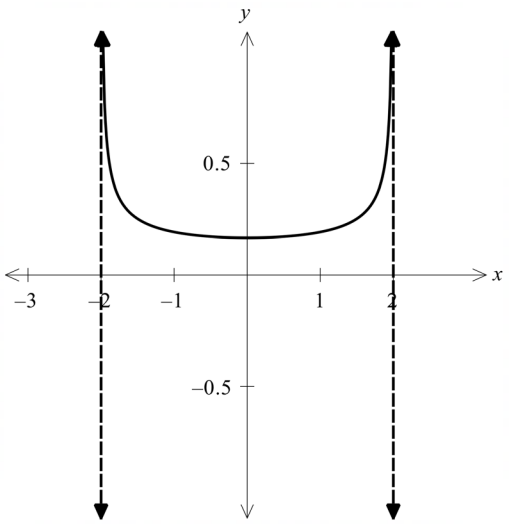
A.



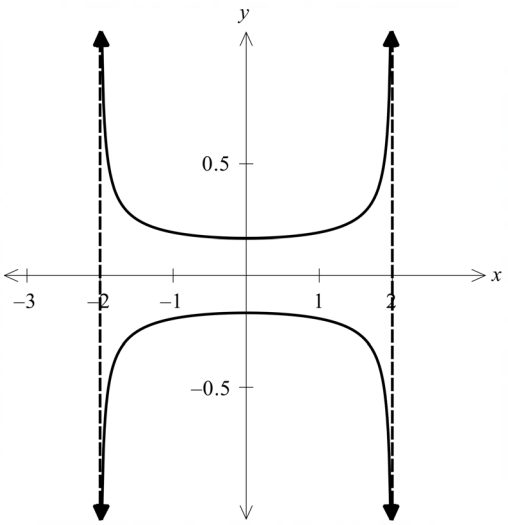
B.



C.



D.



8 The lock for a gate opens with a five-digit code. Each wheel rotates through the digits 0 to 9. What is the approximate percentage of five-digit codes that have no repeated digits?

A. 17%

B. 30%

C. 50%

D. 63%

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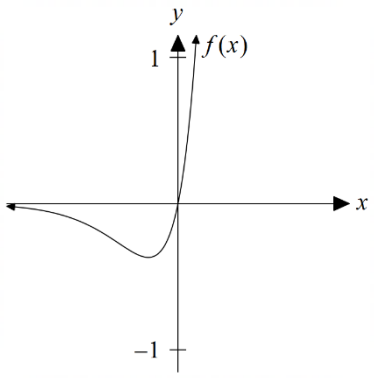
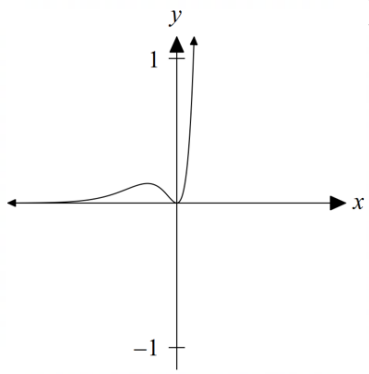
9 The graph of $f(x)$ is shown below.

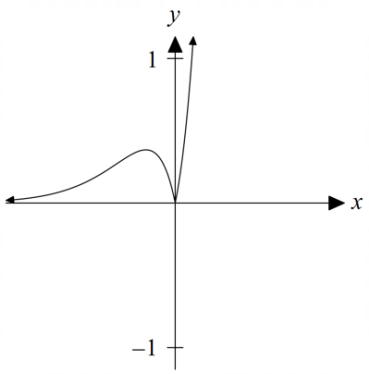


Which of the following graphs best represents $y=\left[f(x)\right]^2$?

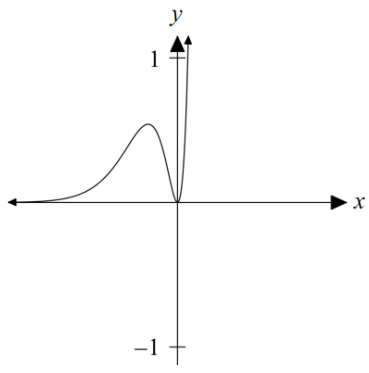
A.



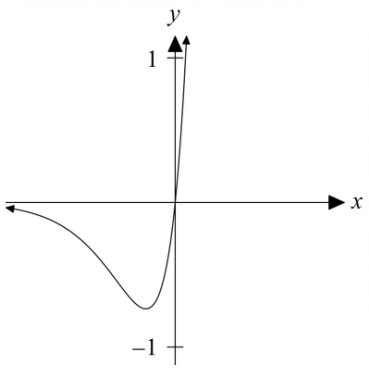
B.



C.



D.



10 Sowon has a jar of coins. The jar contains 10 coins **each** of 5 cent, 10 cent, 20 cent and 50 cent denominations. Sowon asks her friend Judy to choose eight coins from the jar. In how many ways can Judy choose the eight coins from the jar?

A. 4^8

B. ${}^{40}C_8$

C. ${}^{11}C_3$

D. ${}^{8}C_4$

End of Section I

Section II

48 marks

Attempt Questions 11-14

Allow about 1 hour and 15 minutes for this section.

Instructions

- Answer the questions in the appropriate page of your answer booklet.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your name, and your teacher's name on the paper and clearly indicate which question you are answering.

Question 11 Start on the relevant page of your answer booklet (13 marks)

(a) Simplify $\frac{(n+1)!}{n!}$. **1**

(b) Solve $\frac{4}{x+1} \leq 3$. **3**

(c) (i) How many different arrangements are possible using all the letters in the word *STATISTICS*? **2**

(ii) How many words can be formed using all the letters such that it begins and ends with *T*? **1**

(iii) One of the possible arrangements from (i) is chosen at random. How many arrangements have all three *T*'s next to each other and all three *S*'s together? **1**

(d) Two school captains, two vice-captains and three other students are sitting around a circular table.

(i) How many seating arrangements are possible? **1**

(ii) How many arrangements are possible if the two captains sit together, the two vice-captains sit together and the three other students sit together in their groups? **2**

(iii) What is the probability that the two school captains sit next to each other and the two vice captains do not sit next to each other? **2**

Question 12 Start on the relevant page of your answer booklet (11 marks)

(a) Let $y = x(x+1)(x-4)$.

(i) Without the use of calculus, sketch the graph of $y = x(x+1)(x-4)$, showing all the intercepts with the axes. **1**

(ii) Hence, or otherwise, solve the equation $\frac{x(x+1)}{x-4} \geq 0$. **2**

(b) A curve has parametric equations of $x = 3t$ and $y = 4t^2$. Find the Cartesian equation of the curve. **2**

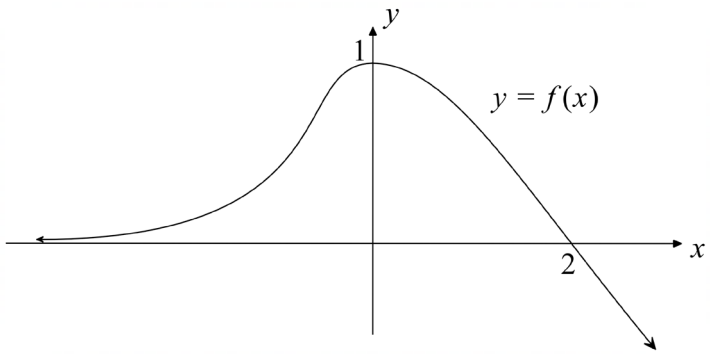
(c) From a PDHPE class of 13 students, two teams of 5 are formed, and then 2 umpires and one scorer is selected. Find the total number of ways of making the selections. **2**

(d) (i) By sketching two appropriate graphs, or otherwise, solve $x+4 > \frac{2}{x+3}$. **2**

(ii) Hence, or otherwise, solve, $|x+4| > \frac{2}{x+3}$. **2**

Question 13 Start on the relevant page of your answer booklet (13 marks)

(a) The diagram below shows the graph of the function $y = f(x)$.



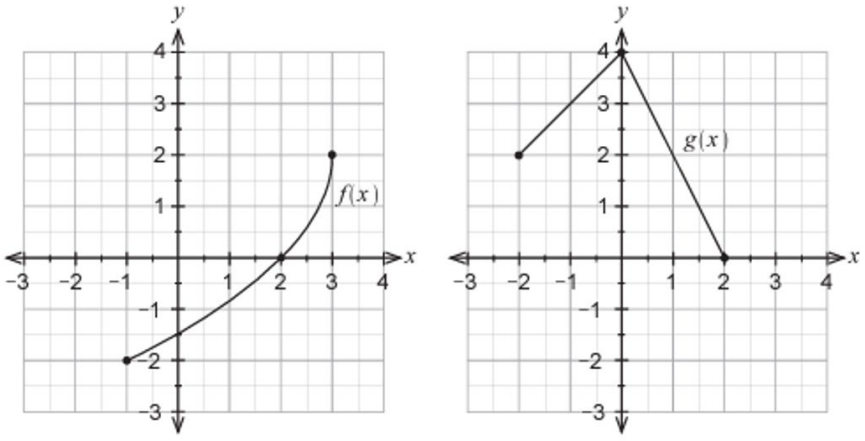
Draw a separate half-page graph for each of the following, showing all asymptotes and intercepts.

(i) $y = f\left(|x|\right)$. **1**

(ii) $y = \dfrac{1}{f(x)}$. **2**

(iii) $y^2 = f\left(x\right)$. **2**

(b) The graphs of functions $y = f\left(x\right)$ and $y = g\left(x\right)$ are shown below.



(i) Sketch the inverse function $y = f^{-1}\left(x\right)$ on the same axes used for $y = f\left(x\right)$ on the grid provided in your answer booklet. **2**

(ii) Explain why the inverse of $y = g\left(x\right)$ is not a function. **1**

The defining rule for $y = f\left(x\right)$ is $f(x) = 2 - 2\sqrt{3 - x}$ where $-1 \le x \le 3$.

(iii) Determine the rule for $y = f^{-1}\left(x\right)$ **3**

(iv) Calculate the value of $g(f^{-1}(0))$. **2**

Question 14 Start on the relevant page of your answer booklet (11 marks)

(a) Assuming that every person in Australia (26 million people) has less than 8 children, there exist at least k people that have the same number of children, first and last initials, and birthday. **2**
Calculate the value of k.

(b) Consider the word *MATHEMATICS*.

(i) How many possible arrangements using all 11 letters have all the vowels in alphabetical order? **1**

(ii) How many arrangements using all 11 letters have the vowels all separated by at least one consonant? **2**

(iii) How many arrangements using any 4 of the 11 letters are possible? **3**

(c) Solve $x|x|+4|x|-3\geq 0$. **3**

End of Paper

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BAULKHAM HILLS HIGH SCHOOL
YEAR 11 P1 HALF-YEARLY
EXAMINATION 2023
MATHEMATICS EXTENSION 1

FINAL MARK

Name:_____ **Teacher:**_____

		Outcomes		
Qn	**Parts**	**F1**	**A1**	**Total**
Multiple Choice		**1,2,5,7,9**	**3,4,6,8,10**	
		/5	/5	/10
11	a		/1	/13
	b	/3		
	c		/4	
	d		/5	
12	a	/3		/11
	b	/2		
	c		/2	
	d	/4		
13	a	/5		/13
	b	/8		
14	a		/2	/11
	b		/6	
	c	/3		
	Total	/33	/25	/58

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BAULKHAM HILLS HIGH SCHOOL

YEAR 11 P1 HALF-YEARLY EXAMINATION 2023

MATHEMATICS EXTENSION 1

NAME: _____

Teacher: _____

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○	**6.**	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○	**7.**	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○	**8.**	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○	**9.**	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○	**10.**	A ○	B ○	C ○	D ○

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PRELIMINARY MATHEMATICS EXTENSION 1

HALF-YEARLY SOLUTIONS

1. To be a function $y = f(x)$ must be one-to-one or many-to-one
∴ The inverse must be one-to-one or one to many.
∴ C

2. $|2x-3| \geq 5$

$2x-3 \leq -5$ or $2x-3 \geq 5$

$2x \leq -2$ $\quad$ $2x \geq 8$

$x \leq -1$ $\quad$ $x \geq 4$

∴ D

3. Combinations = total number of ways − no father, no daughter

$= {}^{4}C_2 \times {}^{8}C_2 - {}^{3}C_2 \times {}^{7}C_2$

$= 105$

∴ A

4. ${}^{n+1}C_{n-1} = \dfrac{(n+1)!}{(n+1-(n-1))!(n-1)!}$

$= \dfrac{(n+1)!}{2(n-1)!}$

$= \dfrac{(n+1)\,n\,(n-1)!}{2(n-1)!}$

$= \frac{1}{2}(n^2+n)$

∴ B

5. For A; $x = 2t(y-1)^2$ using $t = y-1$

$\therefore x \geq 2$ (squares can't be negative)

Not A or C

For B; If $y = -1+t$ and $t \geq -1$

$y \geq -1-1$

$y \geq -2$ which is the correct range

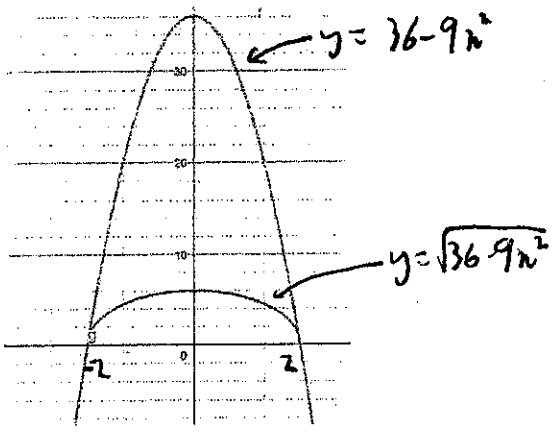
∴ B

6. More girls than boys = 2G, 1B or 3G, 0B

$= {}^{n+1}C_2 \, {}^{n}C_1 + {}^{n+1}C_3 \, {}^{n}C_0$

$\therefore D$

7. $\frac{y}{\sqrt{36-9x^2}} = 1 \Rightarrow y = \sqrt{36-9x^2}$



$\therefore A$

8. For no repeated digits: $\frac{10P_5}{10^5}$

$= 30.24\%$

$\therefore B$

9. When $-1 < f(x) < 0$

$0 < [f(x)]^2 < |f(x)| < 1$

$\therefore A$ (y values are smaller for $[f(x)]^2$, same magnitude for B and larger for C.

10. Use 3 separators to divide into 5c, 10c, 20c and 50c.

cc|cc|cccc|c

8 coins and 3 separators

No. of ways $= \frac{11!}{8!3!}$

$= {}^{11}C_3$

$\therefore C$

Section II

11a) $\frac{(n+1)!}{n!} = \frac{(n+1)n!}{n!}$ — (1) correct answer

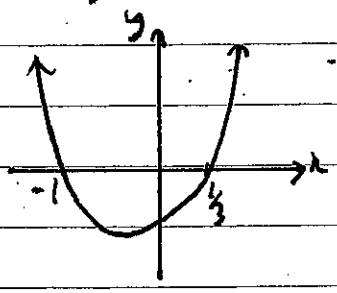
$= n+1$

11b) $\frac{4}{x+1} \leq 3 \quad x \neq -1$

$4(x+1) \leq 3(x+1)^2$

$(x+1)[3(x+1) - 4] \geq 0$

$(x+1)(3x-1) \geq 0$



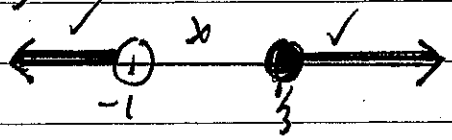
$\therefore x < -1$ or $x \geq \frac{1}{3}$

or Critical values occur when $\frac{4}{x+1} = 3$ or $x+1 = 0$

$4 = 3x + 3$ $\quad x = -1$

$3x = 1$

$x = \frac{1}{3}$



$\therefore x < -1$ or $x \geq \frac{1}{3}$

(3) correct solution
(2) identifies both critical values or obtains $(x+1)(3x-1)$ as factors
(1) identifies one critical value and tests or multiplies by $(x+1)^2$

11c) i) SSSTTTAIIC

Arrangements $= \frac{10!}{3!3!2!}$

$= 50400$

(2) Correct solution
(1) calculates 10!

ii) Arrangements SSSTAIIC

T _ _ _ _ _ _ _ T

Arrangements $= \frac{8!}{3!2!}$

$= 3360$

(1) Correct answer

iii) [TTT] [SSS] A II C

Arrangements $= \frac{6!}{2!}$

$= 360$

(1) Correct answer

11d) i) Arrangements $= (7-1)!$
$= 6!$
$= 720$

(1) correct answer

ii) $\boxed{C_1 C_2}$ $\boxed{V_1 V_2}$ $\boxed{S_1 S_2 S_3}$

(2) correct solution
(1) significant progress with explanation (eg 2 of 4 factors)

Arrangements $= (3-1)! \times 2 \times 2 \times 3!$

- $(3-1)!$: 3 groups in circle
- 2×2: arrangements of captains and vice captains
- $3!$: arrangements of students

$= 48$

iii) Probability $=$ $\frac{\text{arrangements with captains together} - \text{captains and vice captains together}}{\text{total arrangements}}$

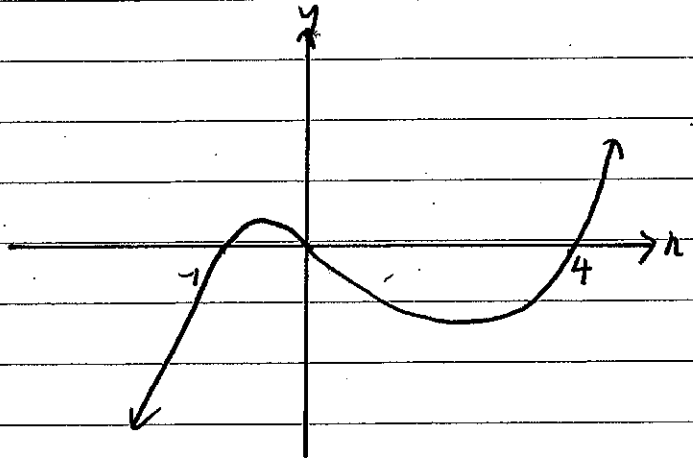
$= \frac{5! \times 2 - 4! \times 2 \times 2}{6!}$ $\boxed{C_1 C_2}$ $\boxed{V_1 V_2}$ SSS

5 groups $\rightarrow$ 4! ways
2 ways each of arranging captains and vice captains

$= \frac{1}{5}$

(2) correct solution
(1) significant progress (calculates 1 term of numerator and attempts to express as a probability)

12a) i)



(1) correct graph

ii) $\frac{x(x+1)}{x-4} \geq 0 \quad (x \neq 4)$

(2) correct solution

$x(x-4)^2 \quad x(x+1)(x-4) \geq 0 \quad (x \neq 4)$

from (i) $-1 \leq x \leq 0$ or $x > 4$

(1) significant progress (ie obtains one branch $-1 \leq x \leq 0$ or $x > 4$ or "$-1 \leq x \leq 0$ or $x \geq 4$")

12b) $x = 3t$

$t = \frac{x}{3}$

$y = 4\left(\frac{x}{3}\right)^2$

$y = \frac{4x^2}{9}$ or $9y = 4x^2$

(2) correct solution

(1) makes t the subject and attempts to solve simultaneously

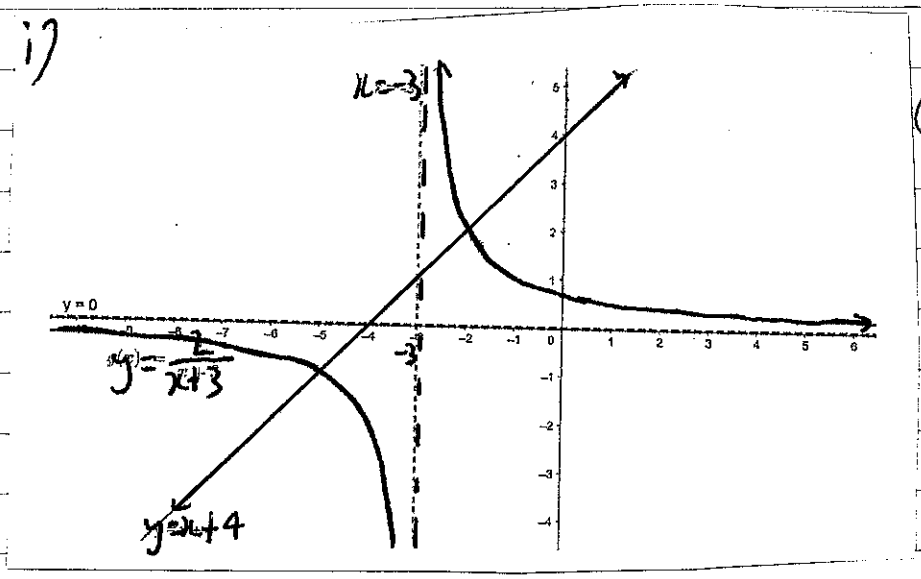
12c) no. of selections $= {}^{8}C_2 \times {}^{11}C_1 \times \frac{{}^{10}C_5 \times {}^{5}C_5}{2}$

$= 108108$

(2) correct solution

(1) makes significant progress (chooses umpires and scorers or forms 2 groups of 5 correctly)

12d) i)



(2) correct solution

(1) correct graphs or one correct point of the solution

$x + 4 = \frac{2}{x+3}$

$(x+4)(x+3) = 2$

$x^2 + 7x + 12 - 2 = 0$

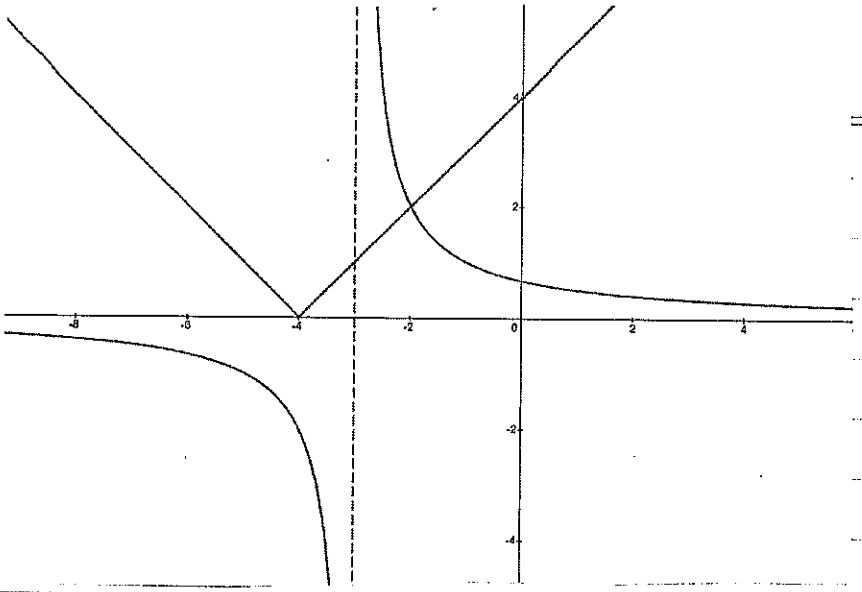
$x^2 + 7x + 10 = 0$

$(x+5)(x+2) = 0$

$x = -5$ or -2

$\therefore$ From graph $-5 < x \leq -3$ or $x > -2$

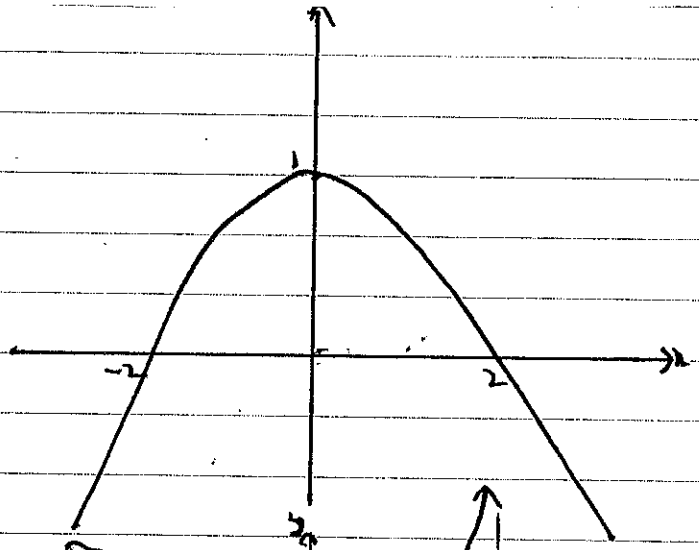
12 d ii)



(2) Correct solution
(1) obtains $x > 2$ OR $x <$ (ie one correct branch)

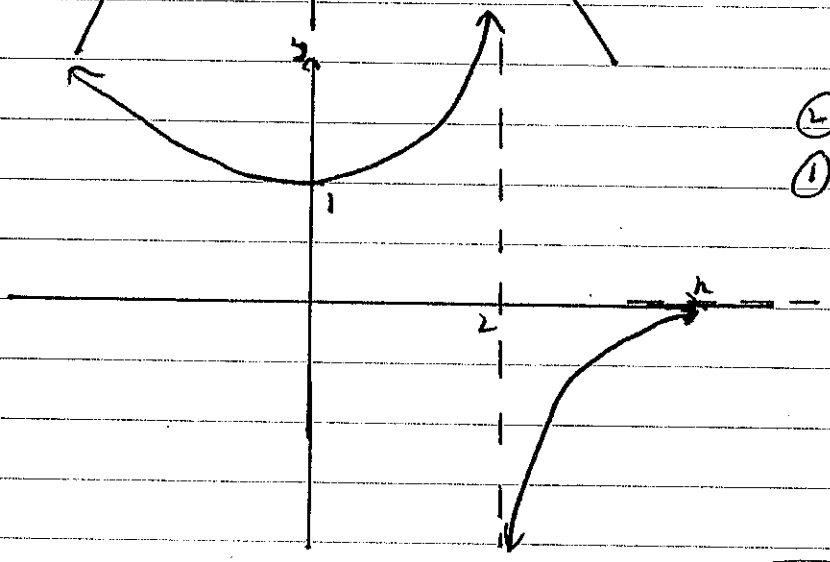
From graph, $x > -2$ or $x < -3$

13 a) i)



(1) correct graph

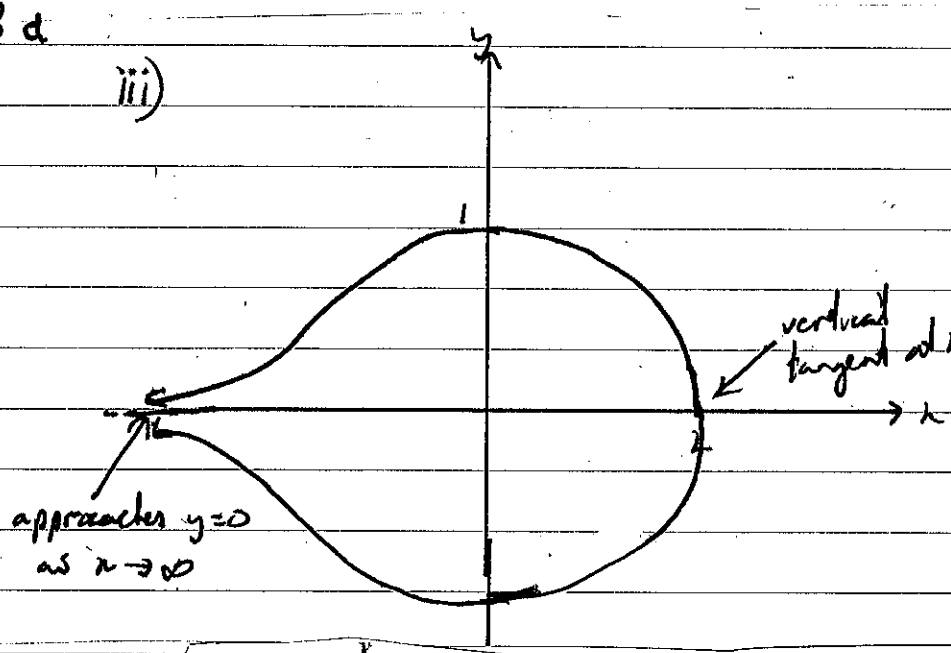
ii)



(2) correct graph
(1) asymptote and 1 branch correct

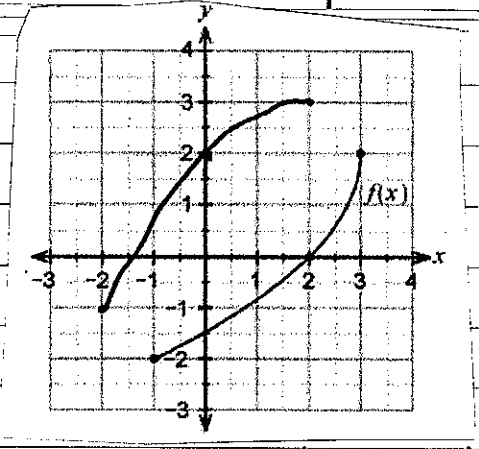
13 d

iii)



(2) correct graph
(1) significant progress (one "branch" correct or basic shape without reflecting in x axis (note: don't penalise if tangent is not vertical)

13 b) i)



(2) correct graph
(1) correct shape (one of the points (-2,-1), (0,2) or (2,3) is correctly located)

ii) · it fails horizontal line test
· g(x) is many to one, inverse is one to many and not a function.

(1) correct reason

iii) f^{-1}: $x = 2 - 2\sqrt{3-y}$ for $-1 \le y \le 3$ and $-2 \le x \le 2$

$2\sqrt{3-y} = 2-x$

$4(3-y) = (x-2)^2$

$3-y = \frac{(x-2)^2}{4}$

$y = 3 - \frac{(x-2)^2}{4}$ for $-2 \le x \le 2$

(3) correct solution
(2) significant progress (squares both sides)
(1) interchanges x and y

iv) $g(f^{-1}(0)) = g(2)$

$= 0$

(2) correct solution
(1) calculates $f^{-1}(0) = 2$

14 a) $k \geq \frac{26 \times 10^6}{8 \times 26^2 \times 366}$

$= 13.13577\ldots$

$\therefore$ There are at least 14 people that share all the same given data.

(2) correct solution
(1) significant progress and uses the pigeonhole principle to round up

14b) MMAATTHEICS

i) AAEI can be arranged in $\frac{4!}{2!}$ $= 12$ ways

$\therefore$ Number of ways $= \frac{1}{12} \times \frac{11!}{2!2!2!}$

$= 415800$

(1) correct answer

ii) _C_M_S_M_H_T_T_

Number of ways = arrangements of consonants × insertion of vowels

$= \frac{7!}{2!2!} \times {}^{8}C_2 \times {}_{6}P_2$

${}^{8}C_2$: chooses 2 slots for A's; ${}_{6}P_2$: 2 slots for E and I

$= 1058400$

(2) correct solution
(1) orders consonants and attempts to insert vowels

iii) Case 1: 2 pairs: ${}^{3}C_2 \times \frac{4!}{2!2!} = 18$

chooses which 2 pairs (M, A or T)

Case 2: 1 pair, 2 different: ${}^{3}C_1 \times {}^{7}C_2 \times \frac{4!}{2!} = 756$

(eg MMAC) ${}^{3}C_1$: which pair; ${}^{7}C_2$: chooses other 2 letters; $\frac{4!}{2!}$: arranges

Case 3: all different: MATHEICS

${}_{8}P_4 = 1680$

Total $= 18 + 756 + 1680$

$= 2454$

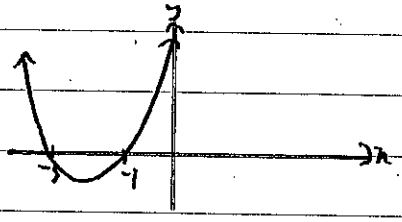
(3) correct solution
(2) calculates 2 cases correctly
(1) identifies all cases and calculates 1 case correctly

14 c) If $x < 0$

$-x^2 - 4x - 3 \geq 0$

$x^2 + 4x + 3 \leq 0$

$(x+3)(x+1) \leq 0$



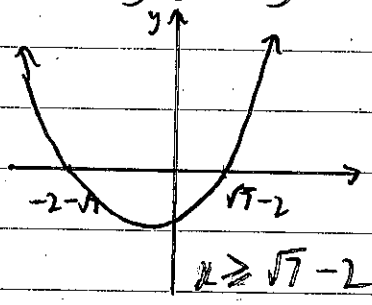
$-3 \leq x \leq -1$

If $x \geq 0$

$x^2 + 4x - 3 \geq 0$

$(x+2)^2 - 7 \geq 0$

$(x+2+\sqrt{7})(x+2-\sqrt{7}) \geq 0$



$x \geq \sqrt{7} - 2$

$\therefore -3 \leq x \leq -1$ or $x \geq \sqrt{7} - 2$

(3) correct solution

(2) identifies all four critical values or solves one of the two cases correctly

(1) identifies critical values from 1 case.