BAULKHAM HILLS

2024	**HIGHER SCHOOL CERTIFICATE TASK 2**

Mathematics Extension 1

General Instructions	• Reading time – 10 minutes • Working time – 1 hour 30 minutes • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 45	**Section I – 5 marks** (pages 2 – 3) • Attempt Questions 1 – 5 • Allow about 8 minutes for this section **Section II – 40 marks** (pages 4 – 7) • Attempt Questions 6 – 8 • Allow about 1 hour and 22 minutes for this section

Section I

5 marks
Attempt Questions 1 – 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5.

---

1 Which of the following is an expression for $\int \cos^2 4x\,dx$?

A. $\frac{x}{2}+\frac{1}{8}\sin 8x + C$

B. $\frac{x}{2}-\frac{1}{8}\sin 8x + C$

C. $\frac{x}{2}+\frac{1}{16}\sin 8x + C$

D. $\frac{x}{2}-\frac{1}{16}\sin 8x + C$

2 Which of the following is equivalent to $3\sin x - 4\cos x$, to the nearest degree?

A. $5\cos(x+143°)$

B. $5\cos(x-143°)$

C. $5\sin(x+143°)$

D. $5\sin(x-143°)$

3 What is the derivative of $\cos^{-1}\left(9x^2\right)$?

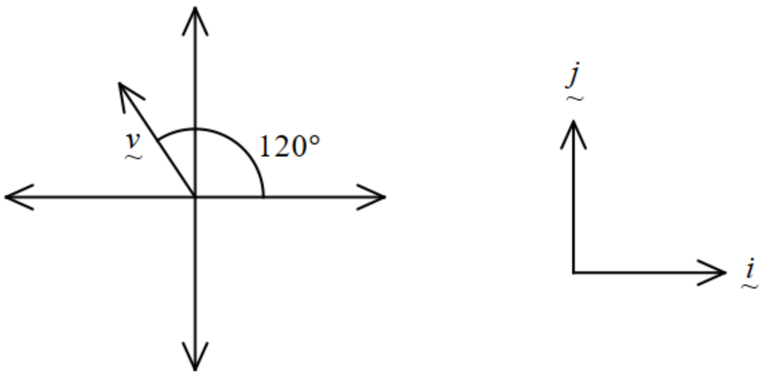
A. $\frac{-18x}{\sqrt{1-81x^4}}$

B. $\frac{-18x}{\sqrt{1-81x^2}}$

C. $\frac{-6x}{\sqrt{1-9x^4}}$

D. $\frac{-6x}{\sqrt{1-9x^2}}$

4 Consider the vector $\underset{\sim}{v}$ shown below. It is known that $|\underset{\sim}{v}| = 2$.



Which of the following vectors is perpendicular to the vector $\underset{\sim}{v}$?

A. $\underset{\sim}{a} = -\underset{\sim}{i} - \sqrt{3}\underset{\sim}{j}$

B. $\underset{\sim}{b} = -\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}$

C. $\underset{\sim}{c} = \underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$

D. $\underset{\sim}{d} = \sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}$

5 Given two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, $|\underset{\sim}{a} + \underset{\sim}{b}| = |\underset{\sim}{a} - \underset{\sim}{b}|$.
Which of the following must be true?

A. $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

B. $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$

C. $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$

D. $|\underset{\sim}{a}| + |\underset{\sim}{b}| = |\underset{\sim}{a}| - |\underset{\sim}{b}|$

Section II

40 marks
Attempt Questions 6 – 8
Allow about 1 hour and 22 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 6 – 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks) Use the Question 6 Writing Booklet.

(a) Evaluate $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{16-9x^2}}$. **3**

(b) Solve $7\sin\theta - 2\cos\theta = 5$, for $0° \le \theta \le 360°$, by using the substitution $t = \tan\frac{\theta}{2}$. **3**
Write the solutions correct to the nearest minute.

(c) Consider the vectors $\underset{\sim}{a} = 5\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$.

(i) Find $\underset{\sim}{a} \bullet \underset{\sim}{b}$. **1**

(ii) Find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

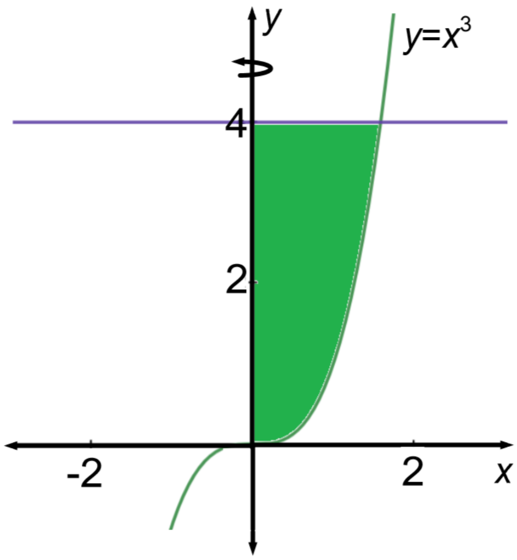
(d) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x+\alpha)$, where $A > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve the equation $\sqrt{3}\cos x - \sin x = -1$ for $-\pi \le x \le \pi$. **2**

End of Question 6

Question 7 (13 marks) Use the Question 7 Writing Booklet.

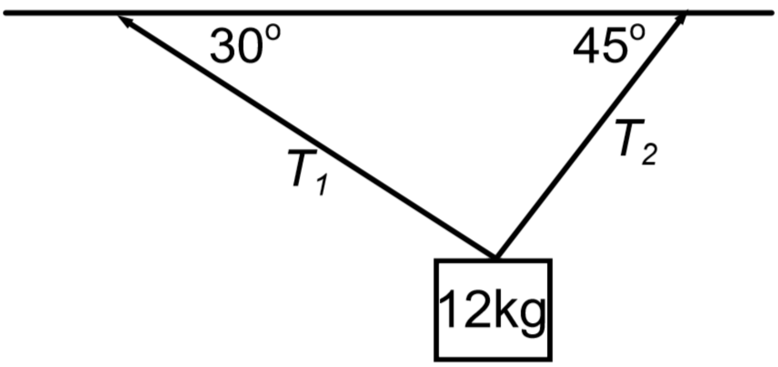
(a) In the diagram below, the shaded region bounded by the curve $y = x^3$, y-axis and the line $y = 4$, is rotated about the y-axis. **3**



Find the volume of the solid of revolution correct to 1 decimal place.

(b) Solve $2\cos^2 2x = 6\sin^2 x - 1$, for $0 \le x \le 2\pi$. **3**

(c) A particle of mass 12kg is suspended by two strings attached to two points in the same horizontal plane. The two strings make angles of 30° and 45° respectively to the horizontal. Let the tensions on the string be T_1 and T_2 newtons as shown.



(i) Show that $T_1 = \dfrac{\sqrt{6}}{3} T_2$. **1**

(ii) Hence, or otherwise, find the value of T_1 to the nearest newton. **2**
Let the acceleration due to gravity be 10m/s^2.

Question 7 continues on page 6

(d) (i) Show that $\dfrac{d}{dx} x \tan^{-1}\left(\dfrac{x}{2}\right) = \dfrac{2x}{4+x^2} + \tan^{-1}\left(\dfrac{x}{2}\right)$. **2**

(ii) Hence, find $\int \tan^{-1}\left(\dfrac{x}{2}\right) dx$. **2**

End of Question 7

Question 8 (14 marks) Use the Question 8 Writing Booklet.

(a) Find $\int \cos 4x \cos x \, dx$. **2**

(b) Use the substitution $u = x^2 + 5$ to evaluate $\int_0^2 \frac{2x^3}{\sqrt{x^2+5}} dx$, leaving your answer as an exact value in simplest form. **3**

(c) Use mathematical induction to prove that $2^n + 3^n - 5^n$ is divisible by 6 for all positive integers n. **3**

(d) (i) If $t = \tan x$, show that $\tan 3x = \frac{3t - t^3}{1 - 3t^2}$. **2**

(ii) If $\tan 3x \tan x = 1$, show that $t^4 - 6t^2 + 1 = 0$. **1**

(iii) By considering $x^4 - 6x^2 + 1 = 0$ and using the result of (ii), deduce that $\tan \frac{\pi}{8} = \sqrt{3 - 2\sqrt{2}}$. **3**

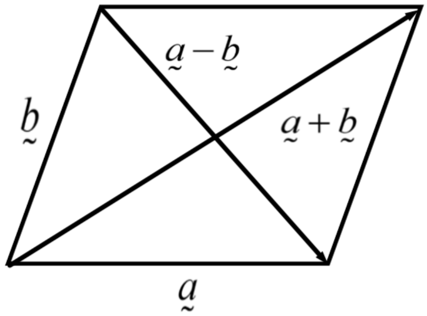
End of paper

BLANK PAGE

BAULKHAM HILLS HIGH SCHOOL

2024 YEAR 12 EXTENSION 1 TASK 2 SOLUTIONS

	Suggested Solutions	Marks
Section I		
1. C	$\int \cos^2 4x\, dx = \frac{1}{2}\int (1+\cos 8x)dx$ $= \frac{1}{2}\left(x+\frac{1}{8}\sin 8x\right)+C$ $= \frac{x}{2}+\frac{1}{16}\sin 8x + C$	**1**
2. B	$5\cos(x-143°) = 5\cos x\cos 143° + 5\sin x \sin 143°$ $= -4\cos x + 3\sin x$ $= 3\sin x - 4\cos x$	**1**
3. A	$\frac{d}{dx}\cos^{-1}\left(9x^2\right) = -\frac{18x}{\sqrt{1-(9x^2)^2}}$ $= -\frac{18x}{\sqrt{1-81x^4}}$	**1**
4.D	$\underset{\sim}{v} = 2\cos 120° \underset{\sim}{i} + 2\sin 120° \underset{\sim}{j}$ $= -\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$ The vector $a\underset{\sim}{i}+b\underset{\sim}{j}$ is perpendicular to $\underset{\sim}{v}$: $(a\underset{\sim}{i}+b\underset{\sim}{j})\bullet(-\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}) = 0$ $-a+b\sqrt{3}=0 \qquad (1)$ The vector has a magnitude of 2: $\sqrt{a^2+b^2}=2$ $a^2+b^2=4 \qquad (2)$ Sub $a=b\sqrt{3}$ into (2): $(b\sqrt{3})^2+b^2=4$ $3b^2+b^2=4$ $4b^2=4$ $b^2=1$ $b=\pm 1$ When $b=1: a=\sqrt{3}$ When $b=-1: a=-\sqrt{3}$ $\therefore$ The vector is $\sqrt{3}\underset{\sim}{i}+\underset{\sim}{j}$.	**1**

5. B	 $\underset{\sim}{a}+\underset{\sim}{b}$ and $\underset{\sim}{a}-\underset{\sim}{b}$ are the diagonals of a parallelogram. Since the diagonals are given as equal in length, the parallelogram must be a rectangle. Therefore $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$.	**1**

Section II	**Suggested Solutions**	**Mk**	**Comments**
6(a)	$\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{16-9x^2}} = \left[\frac{1}{3}\sin^{-1}\frac{3x}{4}\right]_0^{\frac{2}{3}}$ $=\frac{1}{3}\sin^{-1}\left(\frac{2}{4}\right)-0$ $=\frac{1}{3}\times\frac{\pi}{6}$ $=\frac{\pi}{18}$	**3**	3Mk: Provides correct solution. 2Mk: Correctly finds primitive. 1Mk: Obtains $\sin^{-1} f(x)$.
6(b)	$7\sin\theta - 2\cos\theta = 5$ $7\left(\frac{2t}{1+t^2}\right)-2\left(\frac{1-t^2}{1+t^2}\right)=5$ $14t-2+2t^2=5+5t^2$ $3t^2-14t+7=0$ $t=\frac{14\pm\sqrt{14^2-4(3)(7)}}{6}$ $\tan\frac{\theta}{2}=4.097167541 \text{ or } 0.569499126$ $\frac{\theta}{2}=76.28392° \text{ or } 29.661475°$ $\theta=152°34' \text{ or } 59°19'$	**3**	3Mk: Provides correct solution. 2Mk: Correctly obtains one correct value of θ. 1Mk: Obtains correct quadratic equation in terms of t.
6(c)(i)	$\underset{\sim}{a}=5\underset{\sim}{i}-3\underset{\sim}{j}$ and $\underset{\sim}{b}=3\underset{\sim}{i}+4\underset{\sim}{j}$ $\underset{\sim}{a}\bullet\underset{\sim}{b}=(5)(3)+(-3)(4)$ $=15-12$ $=3$	**1**	1Mk: Provides correct solution.

6(c)(ii)	$\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \dfrac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\lvert \underset{\sim}{a} \rvert^2} \times \underset{\sim}{a}$ $= \dfrac{3}{\left(\sqrt{5^2+3^2}\right)^2} \times \underset{\sim}{a}$ $= \dfrac{3}{34}(5\underset{\sim}{i} - 3\underset{\sim}{j})$ $= \dfrac{15}{34}\underset{\sim}{i} - \dfrac{9}{34}\underset{\sim}{j}$	**2**	2Mk: Provides correct solution. 1Mk: Obtains correct $\lvert \underset{\sim}{a} \rvert$.
6(d)(i)	$A\cos(x+\alpha) = A\cos x\cos\alpha - A\sin x\sin\alpha$ $A\cos\alpha = \sqrt{3}$ $A\sin\alpha = 1$ $A^2 = 3^2 + 1^2$ $A^2 = 4$ $A = 2$ $\tan\alpha = \dfrac{1}{\sqrt{3}}$ $\alpha = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ $= \dfrac{\pi}{6}$ $\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x + \dfrac{\pi}{6}\right)$	**2**	2Mk: Provides correct solution. 1Mk: Obtains A or α.
6(d)(ii)	$\sqrt{3}\cos x - \sin x = -1$ $2\cos\left(x+\dfrac{\pi}{6}\right) = -1$ $\cos\left(x+\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$ $x + \dfrac{\pi}{6} = \dfrac{-2\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}...$ $x = -\dfrac{5\pi}{6}, \dfrac{\pi}{2}$	**2**	2Mk: Provides correct solution. 1Mk: Obtains one correct value of $x+\dfrac{\pi}{6}$.

7(a)	$x = y^{\frac{1}{3}}$ $V = \pi \int_0^4 x^2 dy$ $= \pi \int_0^4 y^{\frac{2}{3}} dy$ $= \pi \left[\frac{3y^{\frac{5}{3}}}{5} \right]_0^4$ $= \frac{3(4)^{\frac{5}{3}}}{5}\pi - 0$ $= 18.999...$ $= 19.0 \text{ u}^3$ (Ignore rounding)	**3**	3Mk: Provides correct solution. 2Mk: Correctly integrates $y^{\frac{2}{3}}$ with a correct expression to find volume. 1Mk: Obtains the correct integrand to find volume (must contain π).
7(b)	$2\cos^2 2x = 6\sin^2 x - 1$, for $0 \le x \le 2\pi$. $2\cos^2 2x = 6\sin^2 x - 1$ $2(1 - 2\sin^2 x)^2 = 6\sin^2 x - 1$ $2(1 - 4\sin^2 x + 4\sin^4 x) = 6\sin^2 x - 1$ $2 - 8\sin^2 x + 8\sin^4 x - 6\sin^2 x + 1 = 0$ $3 - 8\sin^2 x + 8\sin^4 x - 6\sin^2 x = 0$ $8\sin^4 x - 14\sin^2 x + 3 = 0$ $8\sin^4 x - 12\sin^2 x - 2\sin^2 x + 3 = 0$ $4\sin^2 x(2\sin^2 x - 3) - 1(2\sin^2 x - 3) = 0$ $(2\sin^2 x - 3)(4\sin^2 x - 1) = 0$ $\sin^2 x = \frac{3}{2}$ $\sin x = \pm\sqrt{\frac{3}{2}}$ x has no solutions $4\sin^2 x - 1 = 0$ $\sin^2 x = \frac{1}{4}$ $\sin x = \pm\sqrt{\frac{1}{4}}$ $\sin x = \pm\frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	**3**	3Mk: Provides correct solution. 2Mk: Obtains a value for sin$x$. 1Mk: Obtains a quadratic equation in $\sin^2 x$, $\cos^2 x$ or equivalent.

7(c)(i)	Resolving horizontal forces: $T_1 \cos 30 = T_2 \cos 45$ $T_1 = \dfrac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2}\right)} T_2$ $= \dfrac{2}{\sqrt{6}} T_2$ $= \dfrac{\sqrt{6}}{3} T_2$	**1**	1Mk: Provides correct proof.
7(c)(ii)	Resolving vertical forces: $T_1 \sin 30 + T_2 \sin 45 = 12g$ $\dfrac{\sqrt{6}}{3} T_2 \sin 30 + T_2 \sin 45 = 12(10)$ $\dfrac{\sqrt{6}}{3} T_2 \times \dfrac{1}{2} + T_2 \times \dfrac{1}{\sqrt{2}} = 120$ $T_2\left(\dfrac{\sqrt{6}}{6} + \dfrac{1}{\sqrt{2}}\right) = 120$ $T_2 = 120 \div \left(\dfrac{\sqrt{6}}{6} + \dfrac{1}{\sqrt{2}}\right)$ $= 107.589...$ $T_1 = \dfrac{\sqrt{6}}{3} \times 107.589...$ $= 87.846...$ $= 88\text{N}$ (to nearest newton)	**2**	2Mk: Provides correct solution. 1Mk: Obtains a correct equation in terms of T_2.

7(d)(i)	$\frac{d}{dx}x\tan^{-1}\left(\frac{x}{2}\right)=x\left(\frac{\left(\frac{1}{2}\right)}{1+\left(\frac{x}{2}\right)^2}\right)+\tan^{-1}\left(\frac{x}{2}\right)$ $=\frac{x}{2}\div\left(1+\frac{x^2}{4}\right)+\tan^{-1}\left(\frac{x}{2}\right)$ $=\frac{x}{2}\div\left(\frac{4+x^2}{4}\right)+\tan^{-1}\left(\frac{x}{2}\right)$ $=\frac{x}{2}\times\left(\frac{4}{4+x^2}\right)+\tan^{-1}\left(\frac{x}{2}\right)$ $=\frac{4x}{8+2x^2}+\tan^{-1}\left(\frac{x}{2}\right)$ $=\frac{2x}{4+x^2}+\tan^{-1}\left(\frac{x}{2}\right)$	**2**	2Mk: Provides correct solution. 1Mk: Correctly differentiates $\tan^{-1}\left(\frac{x}{2}\right)$.
7(d)(ii)	$\frac{d}{dx}x\tan^{-1}\left(\frac{x}{2}\right)=\frac{2x}{4+x^2}+\tan^{-1}\left(\frac{x}{2}\right)$ $\int\frac{d}{dx}x\tan^{-1}\left(\frac{x}{2}\right)dx=\int\frac{2x}{4+x^2}+\tan^{-1}\left(\frac{x}{2}\right)dx$ $\int\tan^{-1}\left(\frac{x}{2}\right)dx=\int\frac{d}{dx}x\tan^{-1}\left(\frac{x}{2}\right)dx-\int\frac{2x}{4+x^2}dx$ $=x\tan^{-1}\left(\frac{x}{2}\right)-\ln(4+x^2)+C$	**2**	2Mk: Provides correct solution. 1Mk: Makes a valid attempt to use part (i) to write an integrand for $\tan^{-1}\left(\frac{x}{2}\right)$.

8(a)	$\int \cos 4x \cos x \, dx$ $= \int \frac{1}{2}[\cos(4x - x)\cos(4x + x)dx$ $= \int \frac{1}{2}[\cos(3x) + \cos(5x)dx$ $= \frac{\sin 3x}{6} + \frac{\sin 5x}{10} + C$	**2**	2Mk: Provides correct solution. 1Mk: Correctly applies the product to sum formula.
8(b)	$u = x^2 + 5$ $du = 2xdx$ and $x^2 = u - 5$ when $x = 2,\ u = 9$ and $x = 0,\ u = 5$ $\int_0^2 \frac{2x^3}{\sqrt{x^2+5}} dx = \int_0^2 \frac{2x(x^2)}{\sqrt{x^2+5}} dx$ $= \int_5^9 \frac{u-5}{\sqrt{u}} du$ $= \int_5^9 u^{\frac{1}{2}} - 5u^{-\frac{1}{2}} du$ $= \left[\frac{2u^{\frac{3}{2}}}{3} - 10u^{\frac{1}{2}} \right]_5^9$ $= \left[\frac{2}{3}\left(9^{\frac{3}{2}}\right) - 10\left(9^{\frac{1}{2}}\right) \right] - \left[\frac{2}{3}\left(5^{\frac{3}{2}}\right) - 10\left(5^{\frac{1}{2}}\right) \right]$ $= [18 - 30] - \left[\frac{10\sqrt{5}}{3} - 10\sqrt{5} \right]$ $= \frac{20\sqrt{5}}{3} - 12$ or $\frac{20\sqrt{5} - 36}{3}$	**3**	3Mk: Provides correct solution. 2Mk: Correctly integrates the expression with correct limits. 1Mk: Correctly obtains the integrand in *u*.
8(c)	Prove $2^n + 3^n - 5^n = 6M$, where $M \in \mathbb{Z}$ Prove true for $n = 1$: $\text{LHS} = 2^1 + 3^1 - 5^1$ $= 0$ which is divisible by 6 The statement is true for $n = 1$. Assume true for $n = k$: where $k \in \mathbb{Z}^+$ *i.e.* $2^k + 3^k - 5^k = 6P$, where $P \in \mathbb{Z}$ *i.e.* $5^k = 2^k + 3^k - 6P$	**3**	3Mk: Provides correct solution. 2Mk: Successfully does 3 of 4 key parts below. 1Mk: Successfully does 2 of 4 key parts below.

	Prove true for $n=k+1$: i.e. $2^{k+1}+3^{k+1}-5^{k+1}=6Q$, where $Q\in\mathbb{Z}$ $LHS=2^{k+1}+3^{k+1}-5^{k+1}$ $=2^{k+1}+3^{k+1}-5(2^k+3^k-6P)$ $=2(2^k)+3(3^k)-5(2^k)-5(3^k)+30P$ $=-3(2^k)-2(3^k)+30P$ $=-6(2^{k-1})-6(3^{k-1})+30P$ $=6(5P-2^{k-1}-3^{k-1})$ $=6Q$, where $Q=5P-2^{k-1}-3^{k-1}$ is clearly an integer since k is a positive integer. The statement is true for $n=k+1$ if true for $n=k$. Hence if it is true for $n=1$, then it is true for $n=2,3$ and so on. $\therefore$ By the principle of mathematical induction, the statement is true for all positive integers *n*.		Key parts: 1. Proves true for $n=1$. 2. Clearly states assumption and what must be proven. 3. Correctly uses assumption. 4. Correctly proves required statement.
8(d)(i)	$\tan 3x=\tan(2x+x)$ $=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ $=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$ $=\dfrac{\left(\dfrac{2t+t-t^3}{1-t^2}\right)}{\left(\dfrac{1-t^2-2t^2}{1-t^2}\right)}$ $=\dfrac{3t-t^3}{1-3\mathrm{t}^2}$	**2**	2 Mk: Provides correct proof. 1Mk: Uses compound angle formula to express $\tan 3x$ in terms of $\tan 2x$ and $\tan x$.
8(d)(ii)	$\tan 3x\tan x=1$ $\left(\dfrac{3t-t^3}{1-3\mathrm{t}^2}\right)t=1$ $3t^2-t^4=1-3\mathrm{t}^2$ $t^4-6\mathrm{t}^2+1=0$	**1**	1Mk: Provides correct proof.

8(d)(iii)	$\tan 3x \tan x = \dfrac{\sin 3x \sin x}{\cos 3x \cos x}$ $= \dfrac{\frac{1}{2}\left[\cos(3x-x)-\cos(3x+x)\right]}{\frac{1}{2}\left[\cos(3x-x)+\cos(3x+x)\right]}$ $= \dfrac{\cos 2x - \cos 4x}{\cos 2x + \cos 4x}$ $\tan 3x \tan x = 1$ $\dfrac{\cos 2x - \cos 4x}{\cos 2x + \cos 4x} = 1$ $\cos 2x - \cos 4x = \cos 2x + \cos 4x$ $2\cos 4x = 0$ $4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$ $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \ldots$ $\therefore$ The solutions to $x^4 - 6x^2 + 1 = 0$ are $\tan\frac{\pi}{8}, \tan\frac{3\pi}{8}, \tan\frac{5\pi}{8}, \tan\frac{7\pi}{8} \ldots$ Also the solutions to $x^4 - 6x^2 + 1 = 0$ are $x^2 = \dfrac{6 \pm \sqrt{36-4(1)}}{2}$ $= \dfrac{6 \pm \sqrt{32}}{2}$ $= \dfrac{6 \pm 4\sqrt{2}}{2}$ $= 3 \pm 2\sqrt{2}$ $\therefore x = \pm\sqrt{3 \pm 2\sqrt{2}}$ Equating the smallest positive solution: $\tan\frac{\pi}{8} = \sqrt{3 - 2\sqrt{2}}$	**3**	3Mk: Provides correct solution. 2Mk: Correctly solves for $\tan 3x \tan x = 1$. 1Mk: Correctly applies a product to sum formula or obtains $x = \pm\sqrt{3 \pm 2\sqrt{2}}$.
	End of Paper		