BAULKHAM HILLS HS PERSEVERE

2024 **PRELIMINARY HALF-YEARLY EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour 30 mins
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 58

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II – 49 marks (pages 6 – 9)

- Attempt Questions 11 – 14
- Allow about 1 hour and 15 minutes for this section.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section.

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 A committee of 6 is to be obtained from a group of 10 men and 5 women. How many committees are possible if there are 4 men and 2 women in each committee?

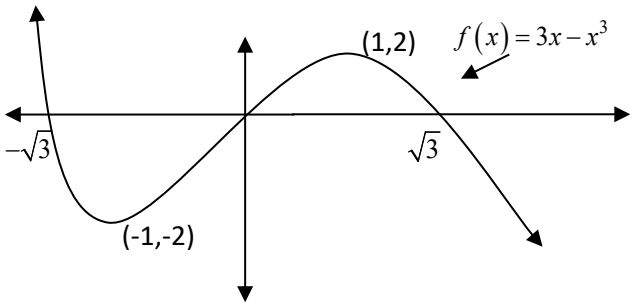
A. 136

B. 2300

C. 60

D. 2100

2 The curve $f(x) = 3x - x^3$ is shown.



What is the largest domain containing the origin for which $f(x)$ has an inverse function?

A. $[-1,1]$

B. $\left[-\sqrt{3},\sqrt{3}\right]$

C. $[-2,2]$

D. $[0,2]$

3 In how many different ways can three girls and three boys be seated on a bench for a photograph if the girls must all sit apart?

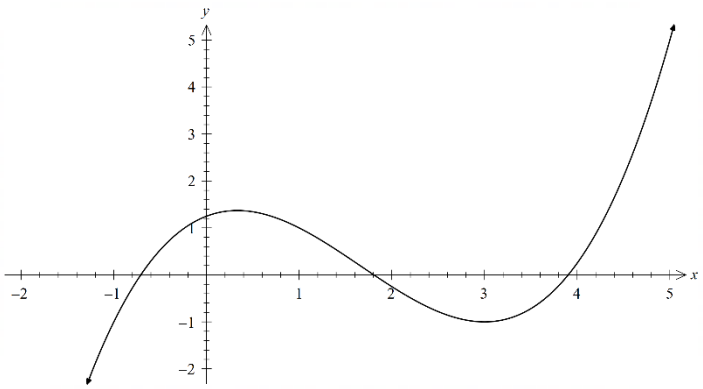
A. 72

B. 120

C. 144

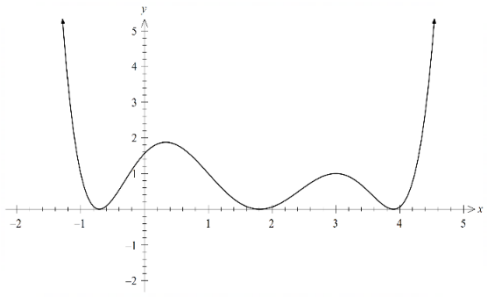
D. 576

4 The graph of the function $y = f(x)$ is drawn below:

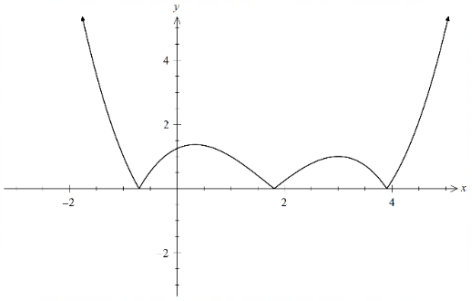


Which of the following represents the graph of $y = \left| f\left(\left| x \right| \right) \right|$?

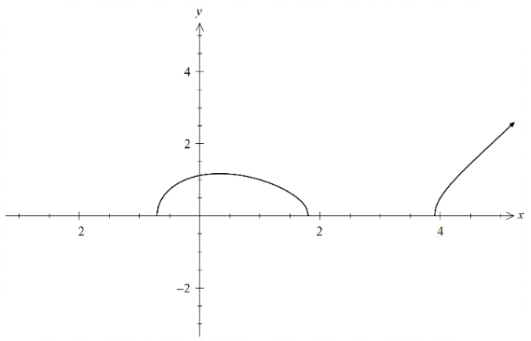
A.



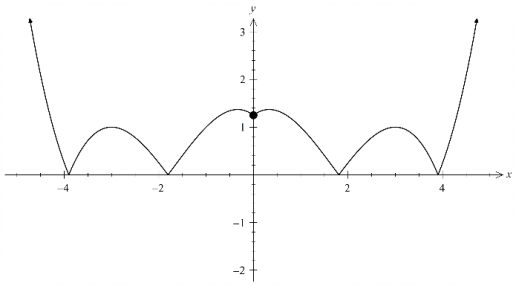
B.



C.



D.



5 Consider the function $f(x)=\dfrac{2x}{x+1}$ and its inverse $f^{-1}(x)$. What is the value of $f^{-1}(3)$?

A. -3

B. $\dfrac{2}{3}$

C. $\dfrac{3}{2}$

D. 3

6 What is the solution to $\dfrac{4}{x+3}\geq 1$?

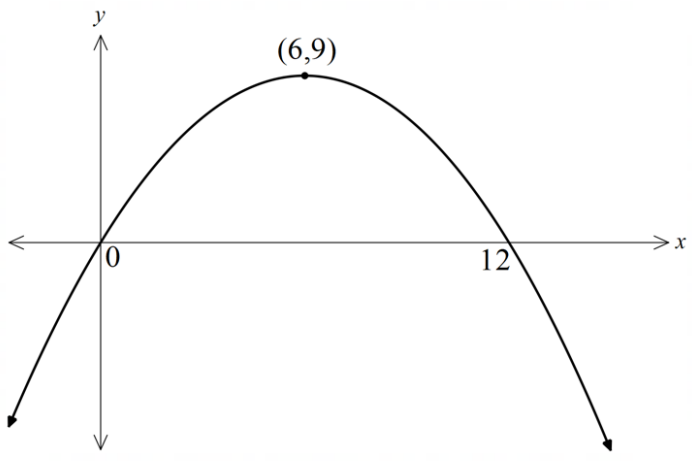
A. $(-3,1]$

B. $[-3,1]$

C. $(-3,10]$

D. $[-3,10)$

7 Which of the parametric equations below represents the parabola above?



A. $x=12t,\ y=2t$ B. $x=12t,\ y=9-t^2$

C. $x=6-2t,\ y=9-t^2$ D. $x=2t-6,\ y=9t-t^2$

8 Which of the following is equivalent to ${}^{n+2}C_{k+1} - {}^{n+1}C_{k+1}$?

A. ${}^{n+2}C_{k-1}$

B. ${}^{n+2}C_{k}$

C. ${}^{n+1}C_{k}$

D. ${}^{n+1}C_{k-1}$

9 In a barrel there are 50 marbles of various colours. Of these, 5 are green, 17 are blue, 12 are yellow, 12 are purple and 4 are orange. What is the least number of marbles that must be selected from the barrel to ensure that 7 of the selected marbles are of the same colour?

A. 27

B. 28

C. 30

D. 31

10 What are the solutions to $|x+5||x-3| \geq 9$?

A. $x \leq -6$ or $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$ or $x \geq 4$

B. $x \leq -6$ or $x \geq 4$

C. $x \leq -6$ or $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$

D. $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$ or $x \geq 4$

End of Section I

Section II

48 marks

Attempt Questions 11-14

Allow about 1 hour and 15 minutes for this section.

Instructions

- Answer the questions in the appropriate page of your answer booklet.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your name, and your teacher's name on the paper and clearly indicate which question you are answering.

Question 11 Start on the relevant page of your answer booklet (13 marks)

(a) Solve for n: $^{2n}P_n = 6 \times {}^{2n-1}P_{n-1}$. **2**

(b) Solve the inequality $\dfrac{x-5}{x-2} \leq -2$. **3**

(c) Ten men and nine women (all of different ages) are called for selection for a trial. A jury of twelve people is to be chosen and the lawyers agree that there should be an equal number of women and men on the jury. How many juries containing the youngest woman and the youngest man can be formed? **2**

(d) To pass an examination, a student needs to choose to attempt 3 of the available 5 sections and pass each of them.

(i) How many options of 3 sections could the students choose? **1**

(ii) If 122 students passed the examination, use the pigeonhole principle to explain why at least 13 students passed exactly the same 3 sections. **2**

(e) Consider the function $f(x) = 5 + \dfrac{10}{x-2}$.

i. Find the inverse function $f^{-1}(x)$. **1**

ii. State the domain and range of $f^{-1}(x)$. **2**

Question 12 Start on the relevant page of your answer booklet (12 marks)

(a) Find the Cartesian equation for the parametric equations **2**

$$x = \frac{1}{t+1}, \quad y = t^2 - 1.$$

(b) At a circular table there are 4 boys and 8 girls.

(i) How many ways can 12 people be seated at random? **1**

(ii) If 4 particular girls wish to be seated in a group, in how many ways can this be accommodated? **1**

(iii) A particular girl, Linda, does not want to be the only person sitting between Ryan and Bob. In how many ways can this seating arrangement be accommodated? **2**

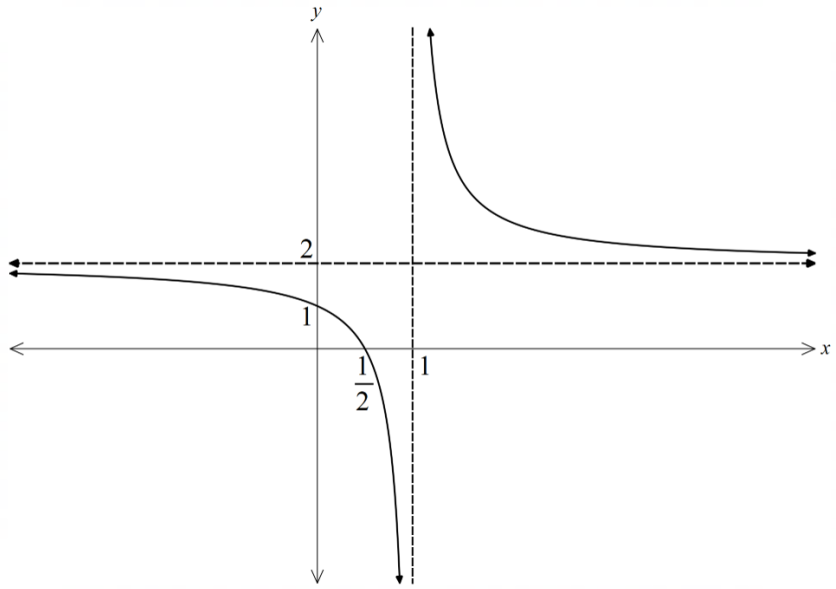
(c) (i) Sketch graphs of $y = |2x+1|$ and $y = |x-4|$ on the same set of axes, showing all intercepts. **2**

(ii) Hence, or otherwise, solve $|2x+1| \le |x-4|$. **1**

(d) Sketch $y = f(x)$ given $f(x) = \dfrac{x^2}{x^2-4}$, showing all asymptotes and intercepts. **3**

Question 13 Start on the relevant page of your answer booklet (15 marks)

(a) The diagram below shows the graph of the function $y = f(x)$.



On a given diagram in your answer booklet sketch each of the following, showing all asymptotes and intercepts.

(i) $y = \sqrt{f(x)}$. **2**

(ii) $y = \dfrac{1}{f(x)}$. **2**

(iii) $|y| = f(x)$. **2**

(iv) $y = x + f(x)$. **2**

(b) Consider the function $f(x) = x^2 + 6x + 4, \quad x \le -3$

(i) Find the inverse function $f^{-1}(x)$. **2**

(ii) Show that $f(f^{-1}(x)) = x$. **1**

(iii) Find the intersection point where $f(x) = f^{-1}(x)$. **2**

(iv) Sketch both $f(x)$ and $f^{-1}(x)$ on the same set of axes, showing clearly the restrictions placed upon them. **2**

Question 14 Start on the relevant page of your answer booklet (9 marks)

(a) A plane has 72 rows of seats. Each row has 6 seats, 3 on either side of the aisle. Seats are allocated at random.

(i) Find the number of seating arrangements possible for 3 friends if all three friends must sit in the same row on the same side of the aisle. **1**

(ii) Find the number of seating arrangements possible for 3 friends if no more than two of friends can sit together in the same row on the same side of the aisle. **2**

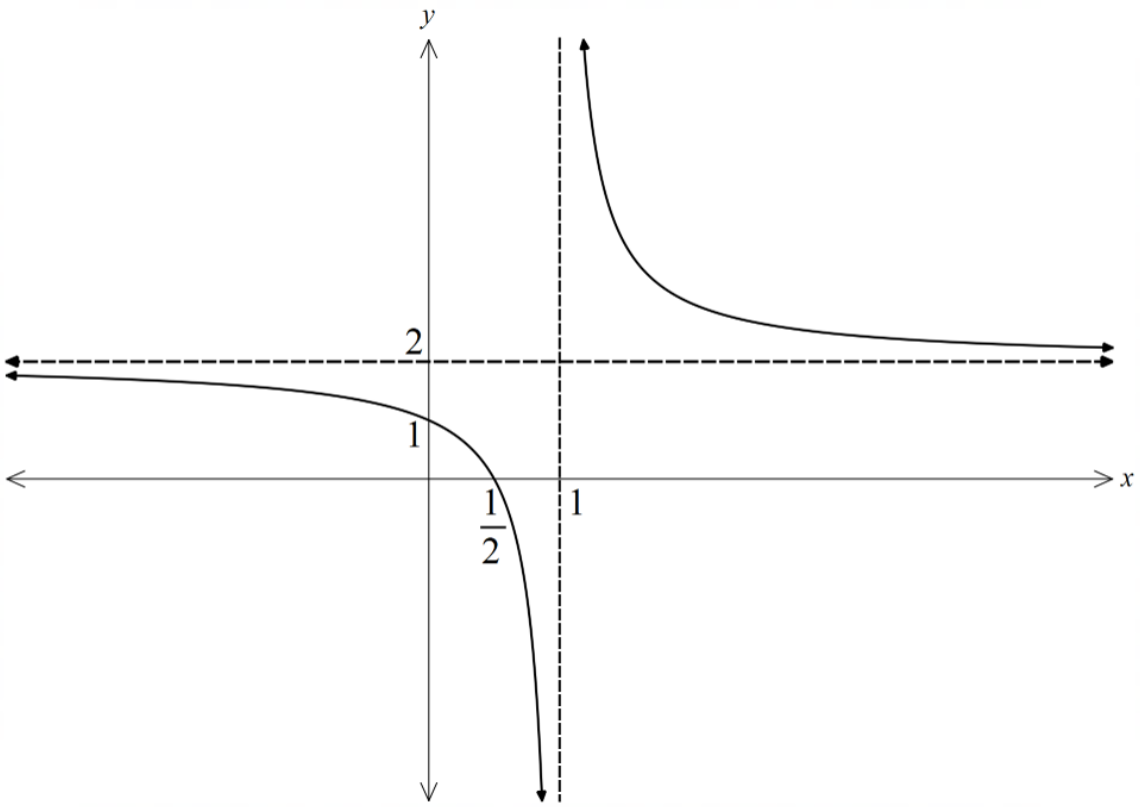
(b) How many eight-letter words can be formed from the letters of the word, GEOMETRY, if the G is always somewhere to the left of both Es and the T is somewhere between the Es? **3**

(c) Solve $|3x+2|-|x-2|<\frac{1}{2}(x+2)^2$. **3**

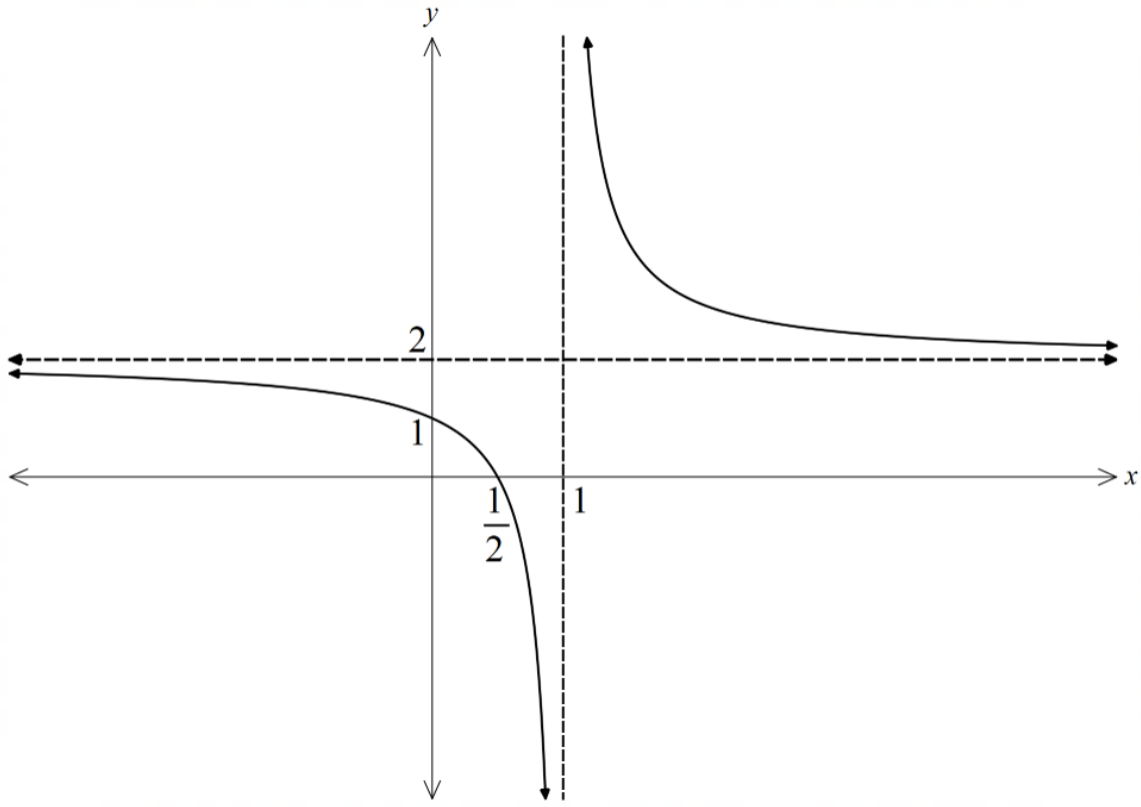
End of Paper

Question 13 Graphs for question 13 part (a) for $y = f(x)$.

(a) (i)

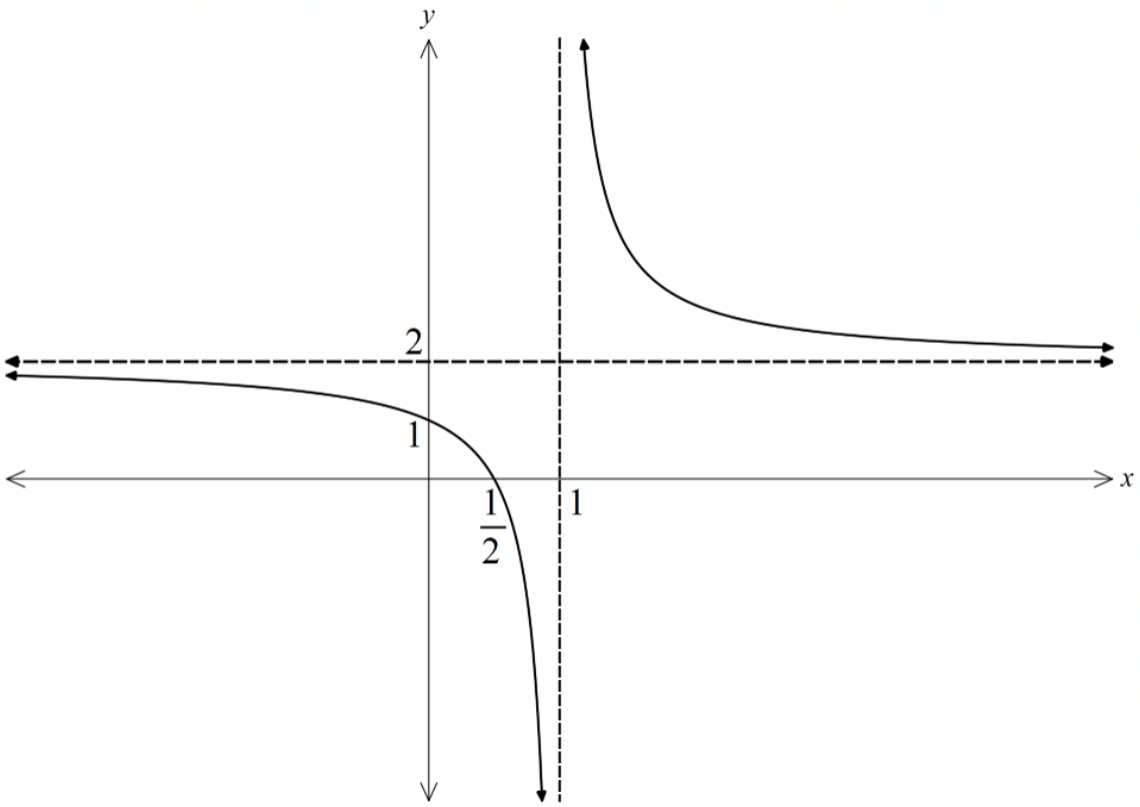


(a) (ii)

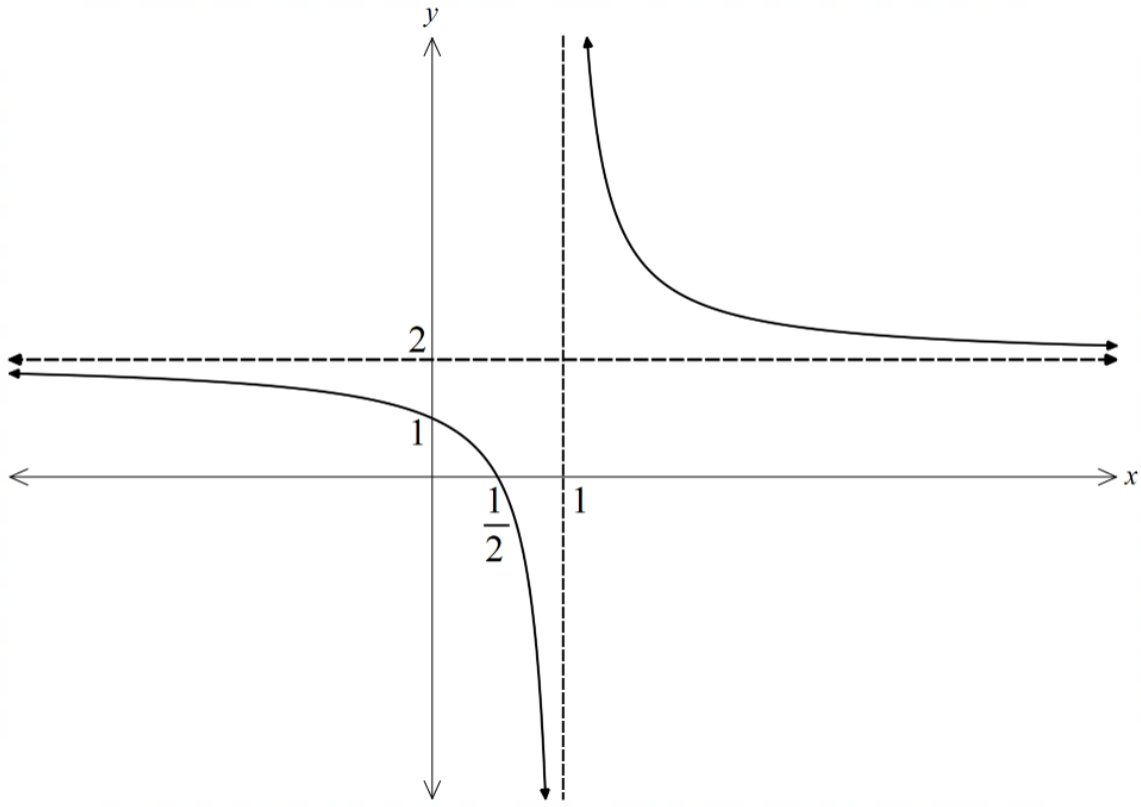


Question 13 Graphs for question 13 part (a) for $y = f(x)$.

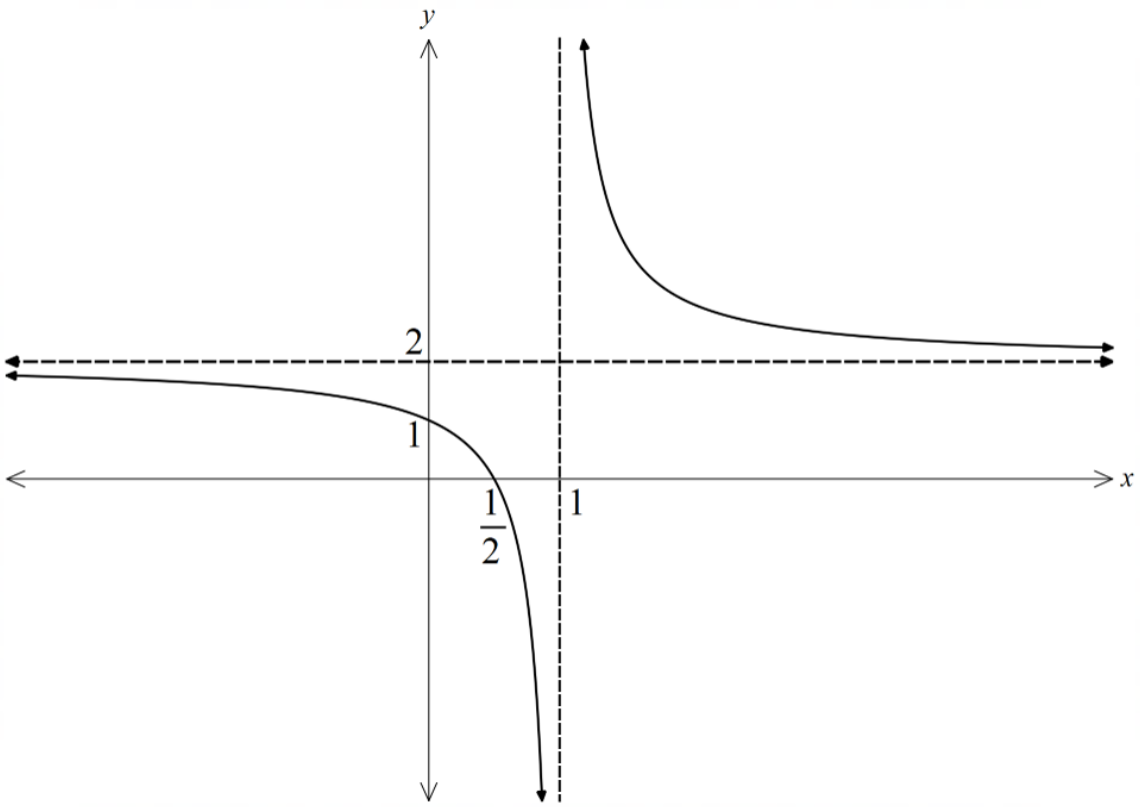
(a) (iii)

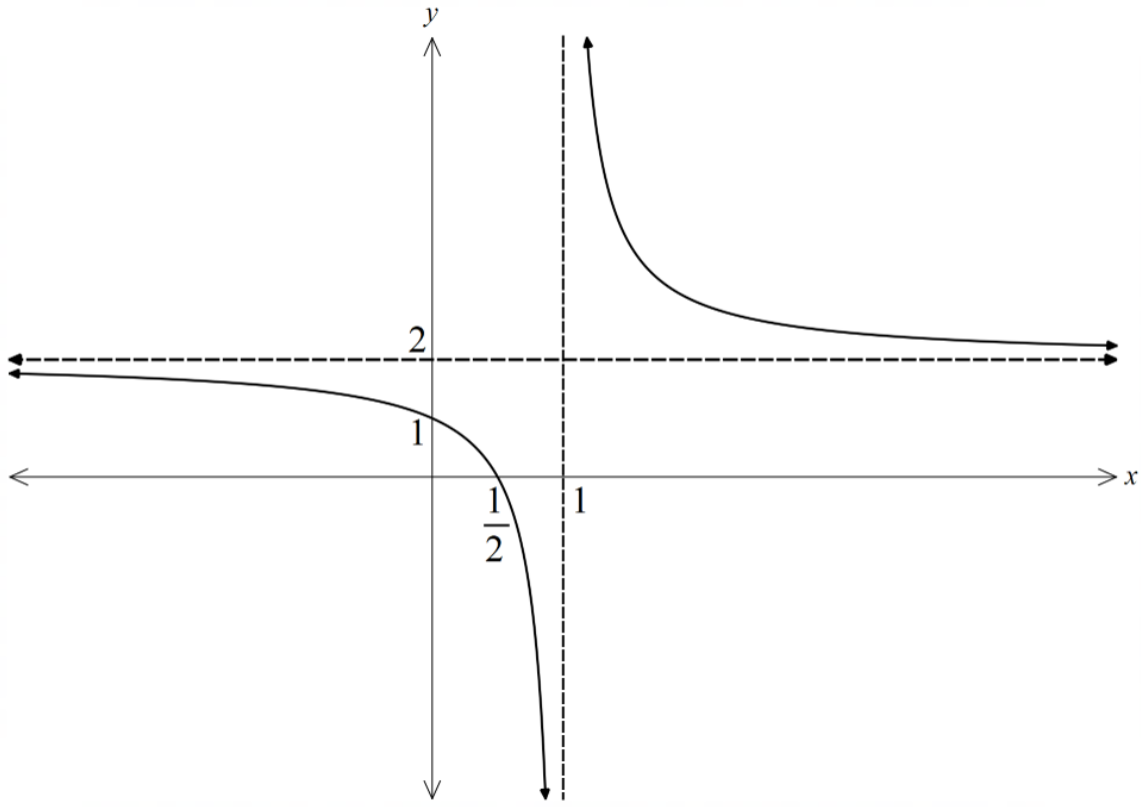


(a) (iv)



Question 13 **EXTRA** Graphs for question 13 part (a) for $y = f(x)$.





1 Committees of 6 are to be obtained from a group of 10 men and 5 women. How many committees are possible if there are 4 men and 2 women in each committee?

A. 136

B. 2300

C. 60

D. 2100

$^{10}C_4 \times {}^5C_2 = 2100$

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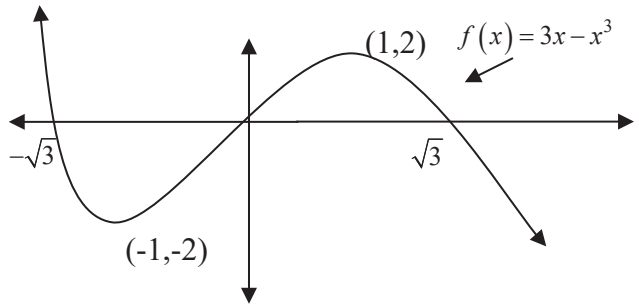

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B. $\left[-\sqrt{3},\sqrt{3}\right]$

C. $[-2,2]$

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3 In how many different ways can three girls and three boys be seated on a bench for a photograph if the girls must all sit apart?

A. 72

B. 120

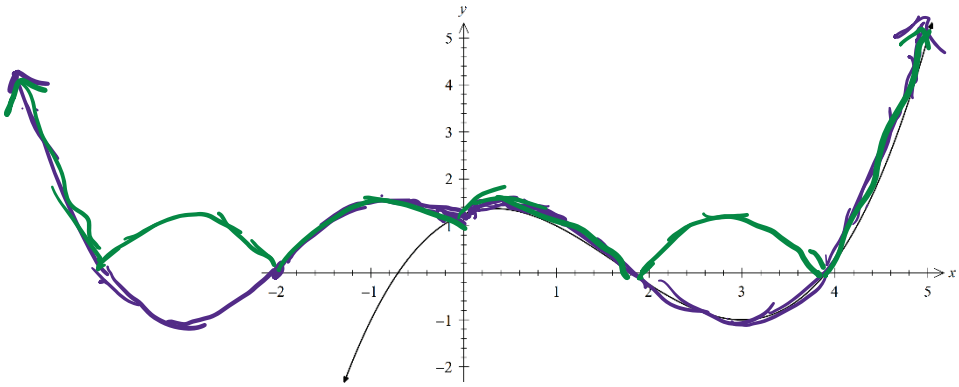
C. 144

D. 576

B _ B _ B _

$^4P_3 \times 3! = 144$

4 The graph of the function $y = f(x)$ is drawn below

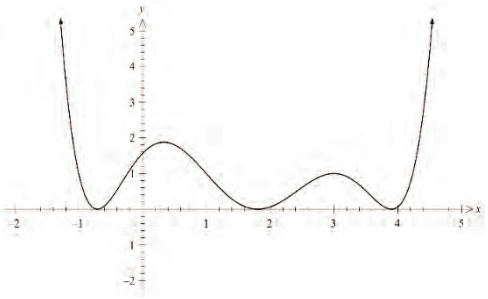


$y = f|x|$

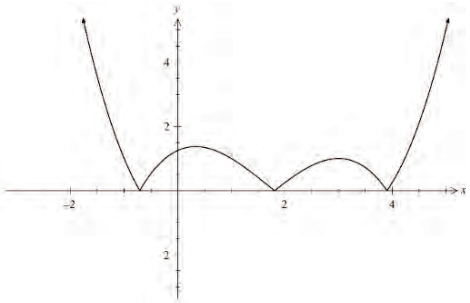
$y = |f|x||$

Which of the following represents the graph of $y = \left| f\left(|x|\right) \right|$?

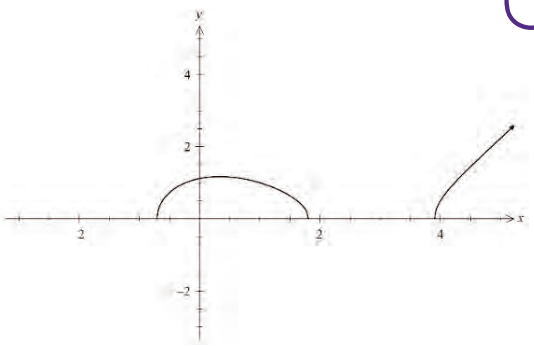
A.



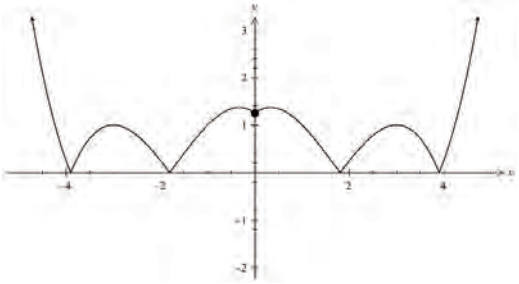
B.



C.



D.



5 Consider the function $f(x) = \dfrac{2x}{x+1}$ and its inverse $f^{-1}(x)$. Evaluate $f^{-1}(3)$.

A. -3

B. $\dfrac{2}{3}$

C. $\dfrac{3}{2}$

D. 3

$x = \dfrac{2y}{y+1}$

$xy + x = 2y$

$y = \dfrac{x}{2-x}$

$f^{-1}(3) = \dfrac{3}{2-3} = -3$

6 The solution to $\frac{4}{x+3} \geq 1$ is

A. $(-3,1]$

B. $[-3,1]$

C. $(-3,10]$

D. $[-3,10)$

$\left(\frac{4}{x+3} \geq 1\right)(x+3)^2,\ x \neq -3$

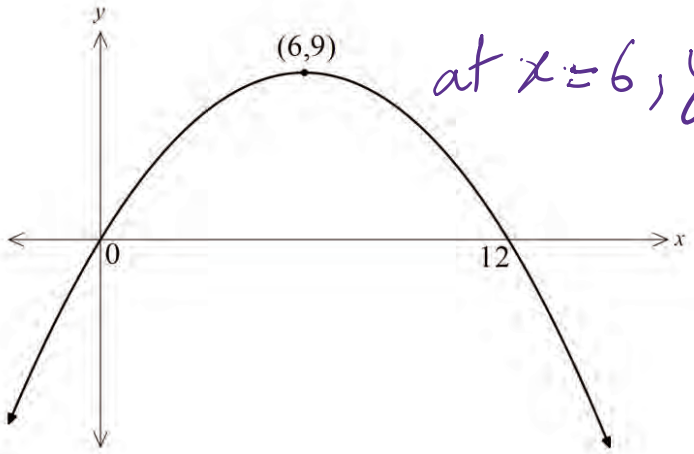
$4(x+3) \geq (x+3)^2$

$(x+3)[x+3-4] \leq 0$

$(x+3)(x-1) \leq 0$

$-3 < x \leq 1,\ x \neq -3$

7



at $x=6, y=9$

A ×

B ×

C ✓

D ×

Which of the parametric equations below represents the parabola above?

A. $x = 12t,\ y = 2t$

B. $x = 12t,\ y = 9 - t^2$

C. $x = 6 - 2t,\ y = 9 - t^2$

D. $x = 2t - 6,\ y = 9t - t^2$

8 Which of the following is equivalent to ${}^{n+2}C_{k+1} - {}^{n+1}C_{k+1}$?

A. ${}^{n+2}C_{k-1}$

B. ${}^{n+2}C_{k}$

C. ${}^{n+1}C_{k}$

D. ${}^{n+1}C_{k-1}$

${}^{n+1}C_k + {}^{n+1}C_{k+1} = {}^{n+2}C_{k+1}$

$\Rightarrow {}^{n+2}C_{k+1} - {}^{n+1}C_{k+1} = {}^{n+1}C_k$

9 In a barrel there are 50 marbles of various colours. Of these, 5 are green, 17 are blue, 12 are yellow, 12 are purple and 4 are orange. What is the least number of marbles that can be selected from the barrel to ensure that 7 of the selected marbles are of the same colour?

A. 27

B. 28

C. 30

D. 31

We assume the worst case, select 5 green, 6 blue, 6 yellow, 6 purple and 4 orange. Then select any one ball of the remaining 3 colours (blue, yellow or purple) and would mean 7 of that particular colour. The number is $5+6+6+6+4+1=28$

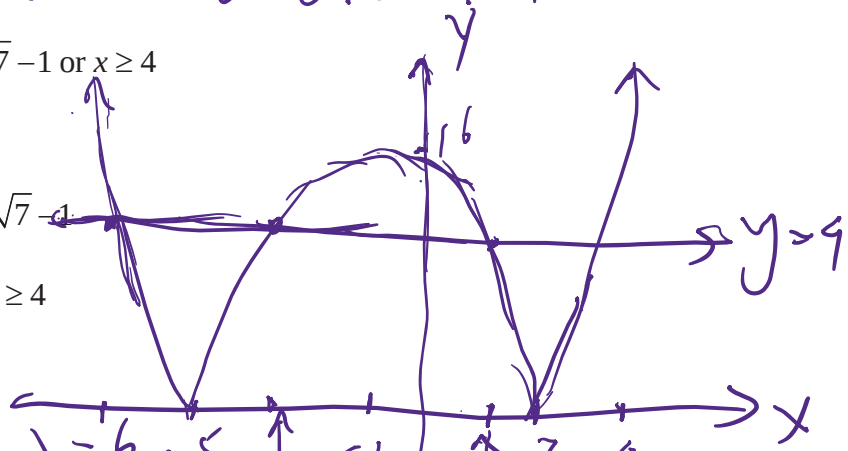
10 The solutions to $|x+5||x-3| \geq 9$ are:

A. $x \leq -6$ or $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$ or $x \geq 4$

B. $x \leq -6$ or $x \geq 4$

C. $x \leq -6$ or $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$

D. $-\sqrt{7}-1 \leq x \leq \sqrt{7}-1$ or $x \geq 4$



$(x+5)(x-3)=9$

$x^2+2x-24=0$

$(x-6)(x+4)=0$

$x=6, x=-4$

$(x+5)(x-3)=-9$

$x^2+2x-6=0$

$(x+1)^2=7 \Rightarrow x=-1\pm\sqrt{7}$

$-\sqrt{7}-1$ $\sqrt{7}-1$

$x \leq -6, -\sqrt{7}-1 \leq x \leq \sqrt{7}-1, x \geq 4$

Q11 a) ${}^{2n}P_n = 6 \times {}^{2n-1}P_{n-1}$

$\frac{(2n)!}{(2n-n)!} = \frac{6 \times (2n-1)!}{(2n-1-(n-1))!}$

$\frac{(2n)!}{n!} = 6 \times \frac{(2n-1)!}{n!}$

$(2n)(2n-1)! = 6(2n-1)!$

$2n = 6$

$n = 3$

2/ correct solution
1/ correctly simplifies ${}^{2n}P_n$ and ${}^{2n-1}P_{n-1}$ expressions OR correct answer from incorrect working

b) $\left(\frac{x-5}{x-2} \leq -2\right)(x-2)^2, \; x \neq 2$

$(x-5)(x-2) \leq -2(x-2)^2$

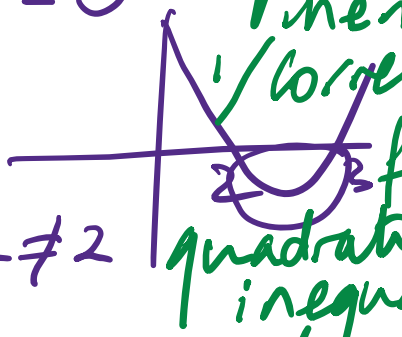
$2(x-2)^2 + (x-5)(x-2) \geq 0$

$(x-2)[2(x-2)+x-5] \geq 0$

$(x-2)(3x-9) \geq 0$

$(x-2)(x-3) \geq 0$

$2 < x \leq 3$, since $x \neq 2$



3/ correct solution
2/ Gives $[2, 3]$ as solution or equivalent ineit
1/ correctly forms quadratic inequation or equivalent ineit

c) 10M, 9W

Select the youngest woman and the youngest man $\Rightarrow$ select another 5M and 5W

$\therefore \binom{9}{5}\binom{8}{5} = 126 \times 56$

$= 7056$ ways

2/ correct solution
1/ correctly finds $\binom{9}{5}$ or $\binom{8}{5}$

Q11 d) i) ${}^5C_3 = 10$ options 1/ correct solution

ii) $\frac{122}{10} = 12.2$ 2/ correct solution

120 students equally distributed would put 12 in each group of 3 sections.

∴ the extra two students must do one of these sections, so one section must have 13 or more 1/ correctly applies PHP or equivalent merit

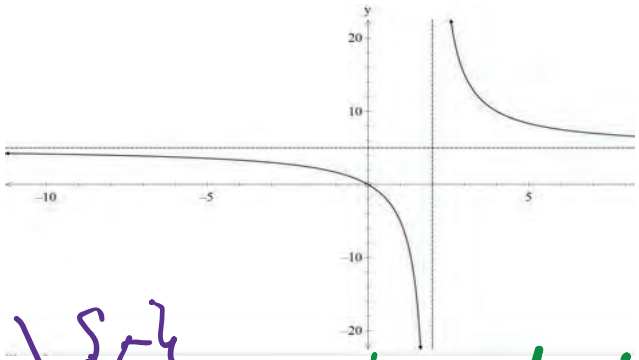
e) i) let $y = 5 + \frac{10}{x-2}$

Interchanging x and y, 1/ correct solution

$$x = 5 + \frac{10}{y-2}$$

$$y - 2 = \frac{10}{x-5} \Rightarrow f^{-1}(x) = \frac{10}{x-5} + 2 = \frac{2x}{x-5}$$

ii) $f(x) = 5 + \frac{10}{x-2}$



Domain $f(x)$: $\mathbb{R} \setminus \{2\}$

Range $f(x)$: $\mathbb{R} \setminus \{5\}$

$\Rightarrow$ Domain $f^{-1}(x)$: $\mathbb{R} \setminus \{5\}$

Range $f^{-1}(x)$: $\mathbb{R} \setminus \{2\}$

2/ correct solution

1/ correctly finds domain or range of $f^{-1}(x)$

(Q12a) $x = \frac{1}{t+1}$, $y = t^2 - 1$

$t + 1 = \frac{1}{x}$

$t = \frac{1}{x} - 1$

$y = \left(\frac{1}{x} - 1\right)^2 - 1$

$= \frac{1}{x^2} - \frac{2}{x} + 1 - 1$

$= \frac{1}{x^2} - \frac{2}{x}$

OR $y = \frac{1 - 2x}{x^2}$

2/ correct solution
1/ correctly finds t in terms of x or equivalent merit

b) i) $11!$ ways

1/ correct solution

ii) As a group $= 8!$

within the group $= 4!$

$\therefore 8! \times 4! = 967680$

2/ correct solution
1/ Gives $8!$ or $4!$ as part of the answer or equivalent merit

iii) Assume Linda sits between Ryan and Bob $\Rightarrow 9!2!$

(RLB) + 9

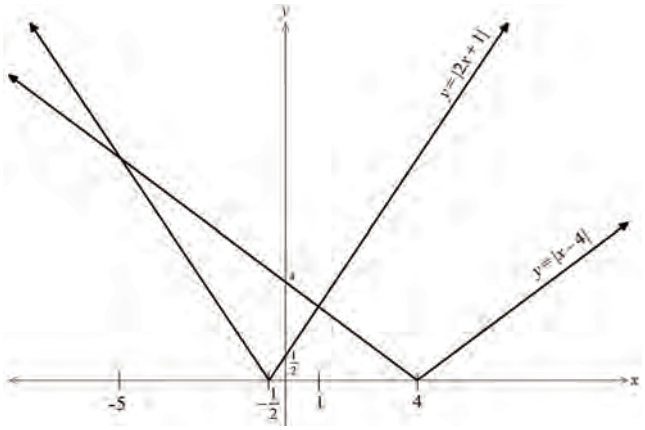
Total $= 11!$

$\therefore 11! - 9!2! = 39191040$

2/ correct solution
1/ Gives $9!2!$ as part of the answer or equivalent merit

(Q12c)

i)



2/ correct solution

1/ correctly draws $y=|x-4|$ or $y=|2x+1|$

$2x+1=-(x-4)$ $\quad -(2x+1)=-(x-4)$

$3x=3$ $\quad x=-5$

$x=1$

ii) $|2x+1| \leq |x-4|$

$-5 \leq x \leq 1$

2/ correct solution

1/ correctly forms quadratic inequation or equivalent merit

Otherwise $|2x+1| \leq |x-4|$

$(2x+1)^2 \leq (x-4)^2$

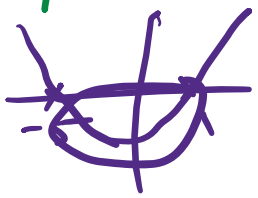
$4x^2+4x+1 \leq x^2-8x+16$

$3x^2+12x-15 \leq 0$

$x^2+4x-5 \leq 0$

$(x+5)(x-1) \leq 0$

$-5 \leq x \leq 1$



d) $f(x)=\dfrac{x^2}{x^2-4}=\dfrac{x^2-4+4}{x^2-4}$

$=1+\dfrac{4}{(x-2)(x+2)}$

3/ correct graph

2/ correct asymptotes and shape or equivalent merit

Asymptotes: $x=\pm 2$ (vertical)

as $x \to \infty$, $y \to 1^+$ $\quad \therefore y=1$ (horizontal)

$x \to -\infty$, $y \to 1^-$

1/ correctly finds vertical or horizontal asymptotes or correct shape

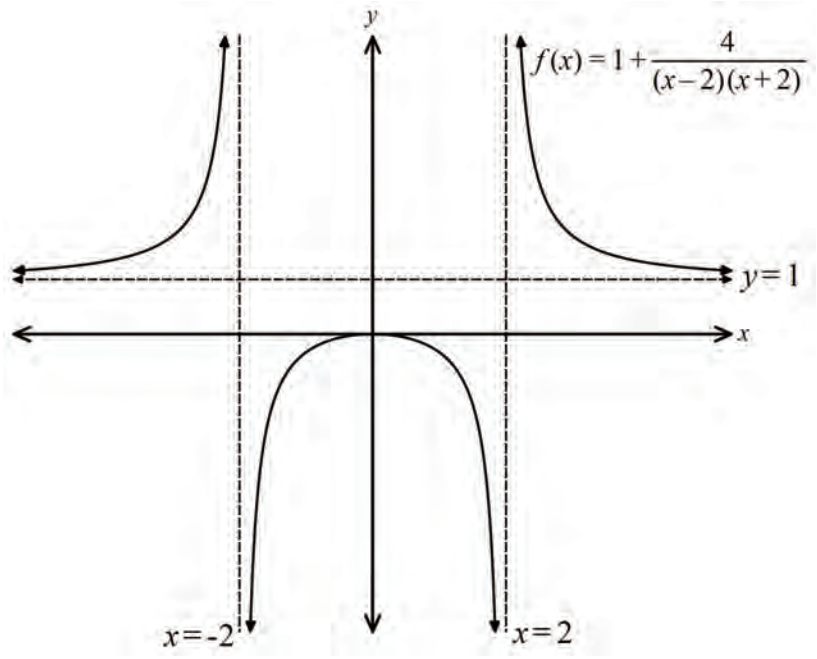
y-int $(x=0)$: $y=0$

$f(-x)=f(x) \Rightarrow f(x)$ is even

Testing: as $x \to 2^+$, $y \to \infty$

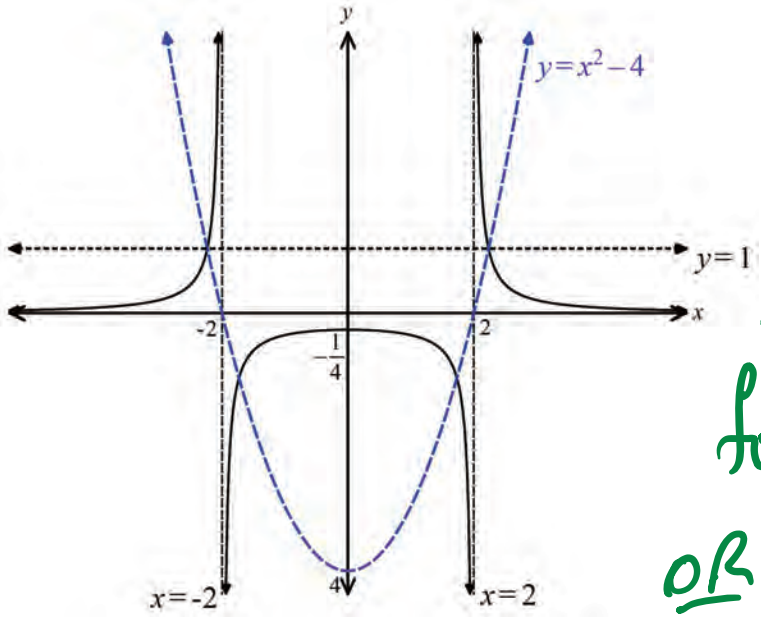
$x \to 2^-$, $y \to -\infty$

Q12d) cont'



OR Graph $y = x^2 - 4$

and $y = \frac{1}{x^2-4}$



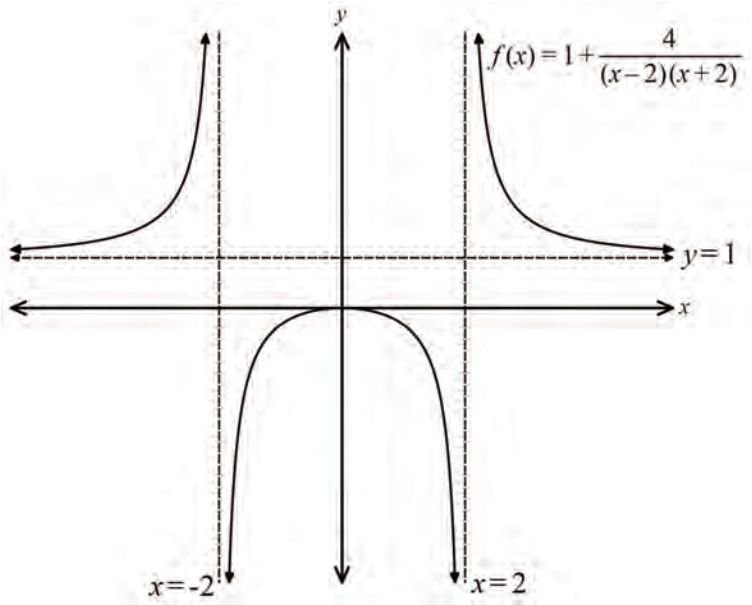
3/ correct graph

2/ correct graph for $y = \frac{4}{x^2-4}$

OR $y = \frac{1}{x^2-4}$ +

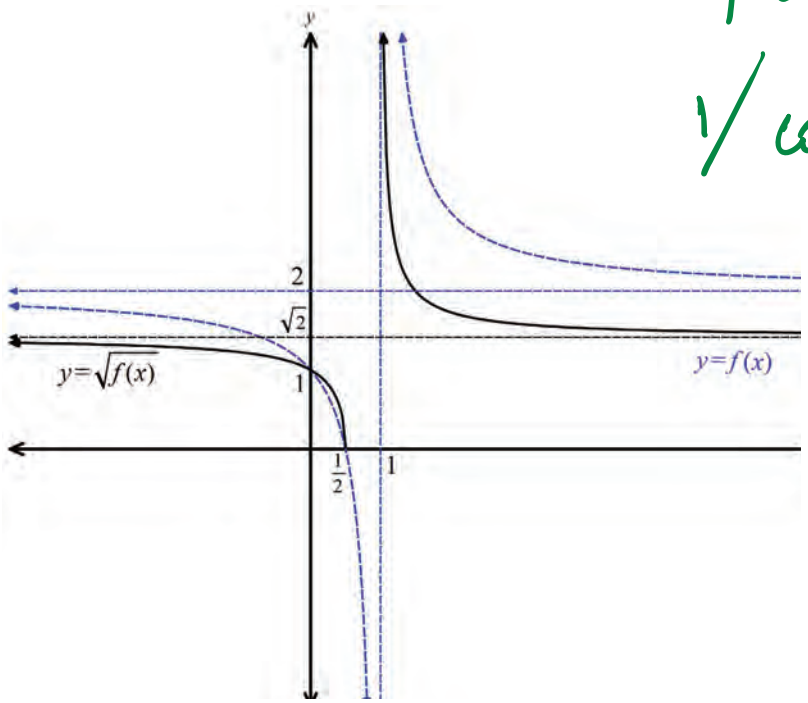
1/ correct graph for $y = \frac{1}{x^2-4}$

Then graph $f(x) = 1 + \frac{4}{(x-2)(x+2)}$



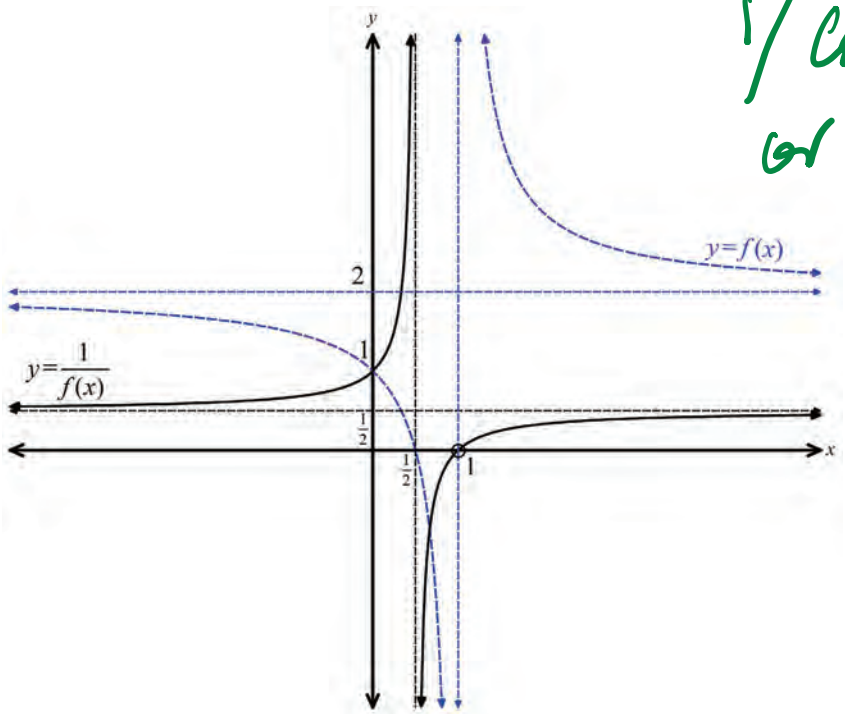
Q13 a) i) $y = \sqrt{f(x)}$

2/ correct graph
1/ correct features



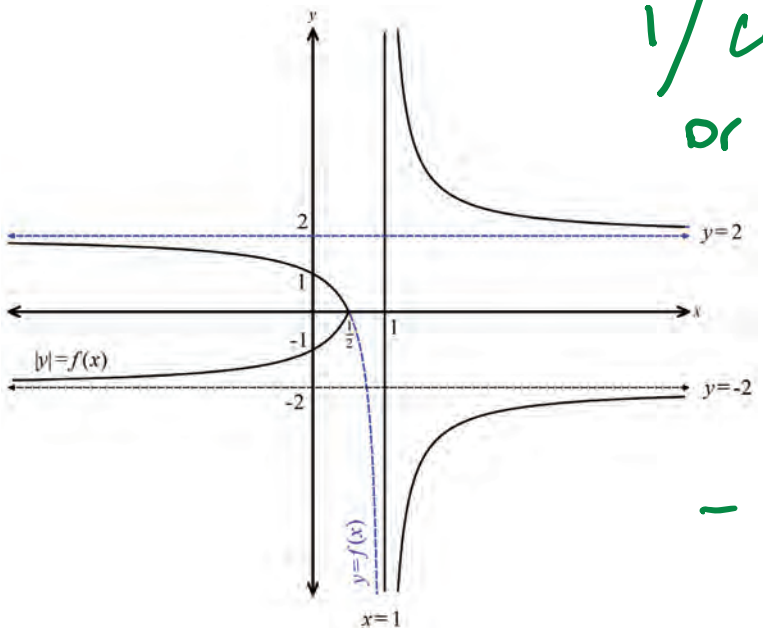
a) ii) $y = \frac{1}{f(x)}$

2/ correct graph
1/ correct shape
or correct graph for $x < \frac{1}{2}$ or $x > \frac{1}{2}$



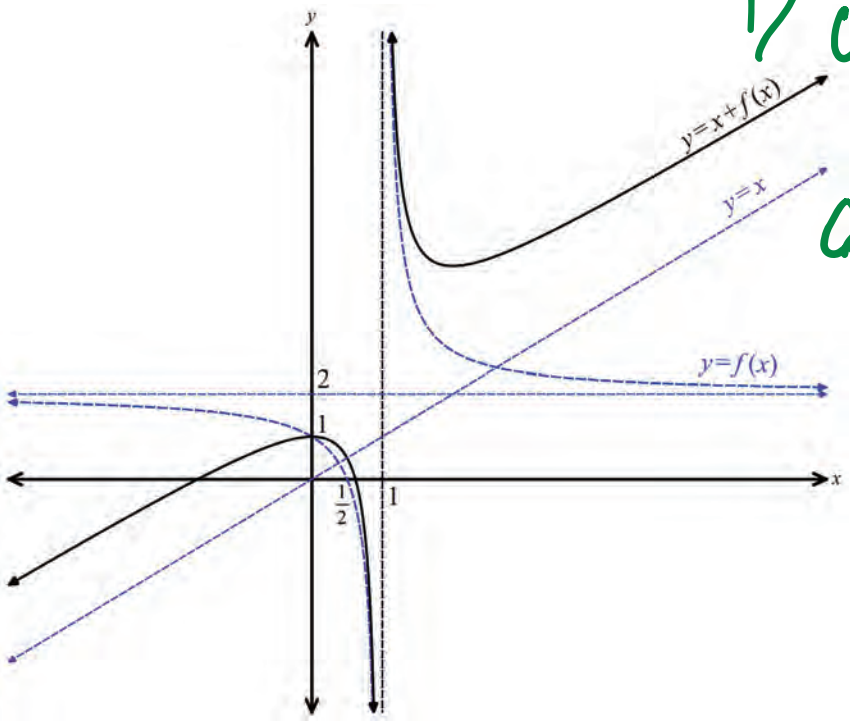
Q13 a) iii) $|y| = f(x)$

2/ correct graph
1/ correct shape
or correct graph for $y \geq 0$
or $y \leq 0$
- draws $y = f(|x|)$



iv) $y = x + f(x)$

2/ correct graph
1/ correct shape
or
correct graph for $x > 1$
or $x < 1$



Q13b) i) $f(x) = x^2 + 6x + 4, \; x \leq -3$

$= (x+3)^2 - 5$

Let $y = (x+3)^2 - 5$

Interchanging x and y,

$x = (y+3)^2 - 5$

$(y+3)^2 = x + 5$

$y = -3 \pm \sqrt{x+5}$

$\therefore f^{-1}(x) = -3 - \sqrt{x+5}$ since range $f^{-1}(x): y \leq -3$

2/ correct solution
1/ Makes substantial progress

ii)

$f(f^{-1}(x)) = (-3 - \sqrt{x+5})^2 + 6(-3 - \sqrt{x+5}) + 4$

$= 9 + x + 5 + 6\sqrt{x+5} - 18 - 6\sqrt{x+5} + 4$

$= x$

1/ correct proof

iii) $f(x) = f^{-1}(x) = x$

$x^2 + 6x + 4 = x$

$x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

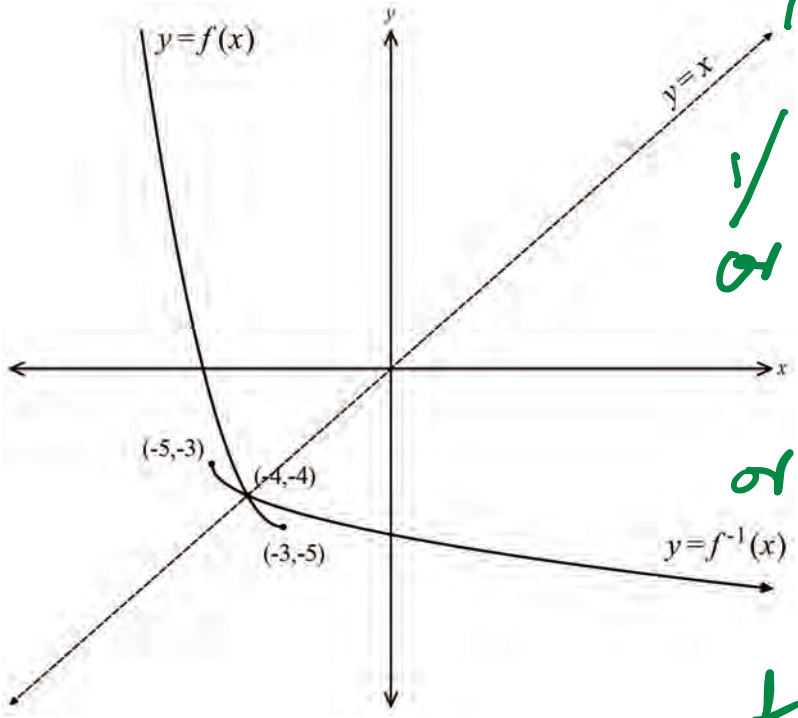
$\therefore x = -4$ since $x \leq -3$

$x = -4, \; y = -4$

Hence $(-4, -4)$ is the intersection point where $f(x) = f^{-1}(x)$

2/ correct answer
1/ correctly finds x or equivalent merit
or bald answer from graph

Q13b) iv)



2/ correct solution
1/ correct shape or correct graph for $f^{-1}(x)$ or shows understanding of symmetry of $f(x)$ and $f^{-1}(x)$

Q14 a) i) $72 \times 2 \times 3! = 864$ ways 1/correct solution

ii) Total − all 3 friends must sit together 2/correct solution

$= {}^{432}P_3 - 72 \times 2 \times 3!$ 1/correctly finds the total.

$= 80,062,560 - 864$ OR finds all 3 sitting apart

$= 80,061,696$ OR 2 together and 1 apart

b) Method 1 Arranging GEOMETRY in $\frac{8!}{2!}$ ways 3/correct solution

Arranging GETE in $\frac{4!}{2!}$ ways 2/correctly finds both arrangements

Number of ways $= \frac{8!}{2!} \div \frac{4!}{2!}$ 1/correctly finds one arrangement

$= 1680$ ways

Method 2 _G_E_T_E_ 3/correct solution

5 spaces for 4 remaining letters

Eg _G_R_E_T_E_ 2/calculates arrangements

6 spaces for 3 remaining letters of at least half the cases

Eg _G_R_O_E_T_E_

7 spaces for 2 remaining letters

and 8 spaces for 1 remaining letter 1/identifies all possible cases

Hence $5 \times 6 \times 7 \times 8 = 1680$ ways

Q14 c) $|3x+2| - |x-2| < \frac{1}{2}(x+2)^2$

3/ correct solution
2/ correct graphs OR identifies critical values $x=-6, -2$ and 2
1/ correctly finds at least 2 intersection points OR equivalent merit

$y = |3x+2| - |x-2|$

$$= \begin{cases} 2x+4, & x \geq 2 \\ 4x, & -\frac{2}{3} \leq x < 2 \\ -2x-4, & x < -\frac{2}{3} \end{cases}$$

$\frac{1}{2}(x+2)^2 = -2x-4$

$x^2+4x+4 = -4x-8$

$x^2+8x+12 = 0$

$(x+6)(x+2) = 0$

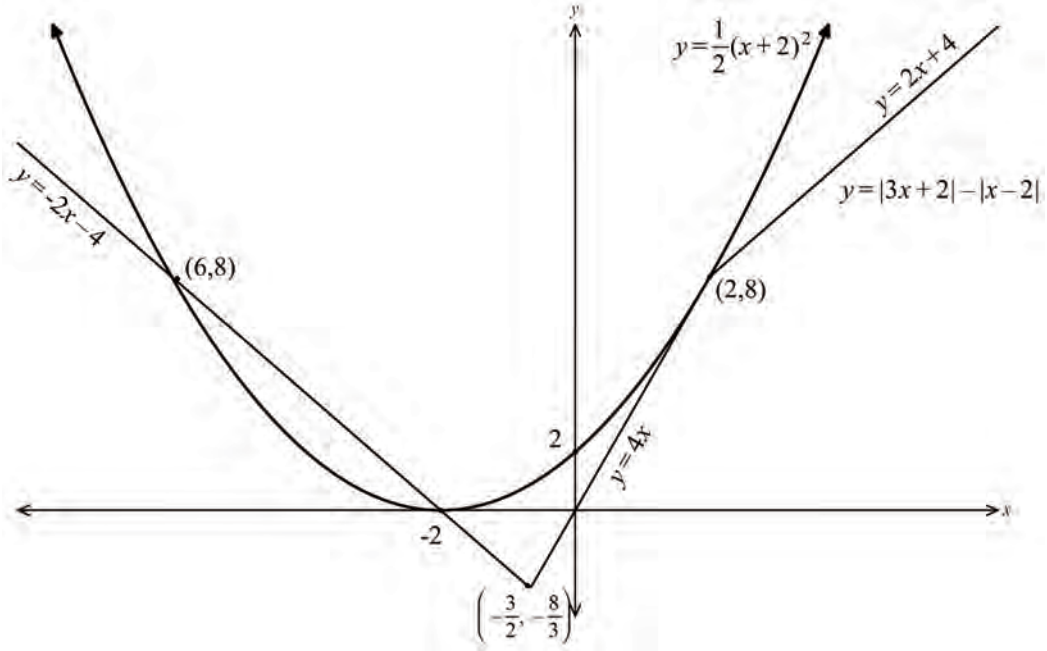
$x = -6, x = -2$

$\frac{1}{2}(x+2)^2 = 2x+4$

$x^2+4x+4 = 4x+8$

$x^2 = 4$

$x = \pm 2$



$\therefore x < -6, -2 < x < 2, x > 2$