BAULKHAM HILLS

PERSEVERE

2025 YEAR 12 ASSESSMENT TASK 3

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black pen.
- Calculators approved by NESA may be used.
- Answer all questions in the booklet provided.
- In Questions 7 – 9, show relevant mathematical reasoning and/or calculations.
- Relevant formulas are provided in the answer booklet

Total marks: 35

Section I – 6 marks (pages 2 – 4)

- Attempt Questions 1 – 6
- Allow about 10 minutes for this section.

Section II – 29 marks (pages 5 – 8)

- Attempt Questions 7 – 9
- Allow about 50 minutes for this section.

Section I
6 marks
Attempt questions 1 - 6
Allow about 10 minutes for this section

Use the multiple-choice answer page in your booklet to answer Questions 1 – 6.

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1) Which one of the following equations in **not** satisfied by $x = e^{-4t}$?

A) $\left(\dfrac{dx}{dt}\right)^2 - 16x^2 = 0$

B) $\dfrac{d^2x}{dt^2} - 16x = 0$

C) $\dfrac{dx}{dt} + 4x = 0$

D) $\dfrac{d^2x}{dt^2} + 16x = 0$

2) A particle is projected so that at time t, its position is given by $\underset{\sim}{r}(t) = 40t\underset{\sim}{i} + \left(30t - \dfrac{1}{2}gt^2\right)\underset{\sim}{j}$ where distances are in metres and time is in seconds. If α is the angle of projection and $|\underset{\sim}{v}(0)|$ is the initial speed, which of the following statements is correct?

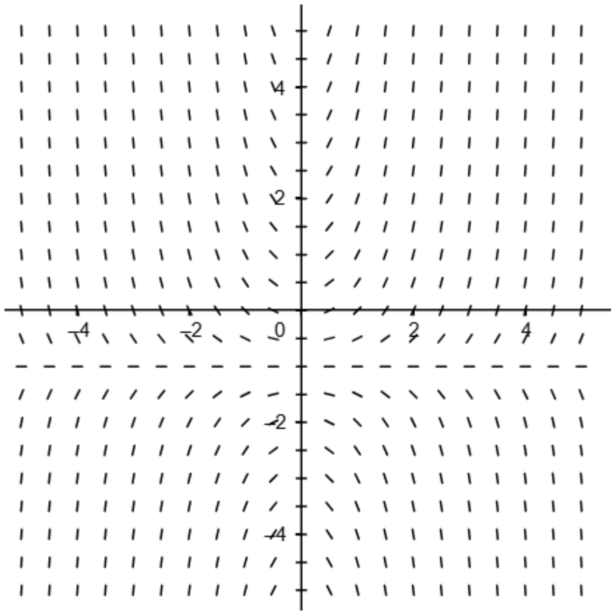
A) $|\underset{\sim}{v}(0)| = 40\text{ m/s}$ and $\alpha = \sin^{-1}\left(\dfrac{3}{5}\right)$

B) $|\underset{\sim}{v}(0)| = 30\text{ m/s}$ and $\alpha = \cos^{-1}\left(\dfrac{4}{5}\right)$

C) $|\underset{\sim}{v}(0)| = 50\text{ m/s}$ and $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$

D) $|\underset{\sim}{v}(0)| = 50\text{ m/s}$ and $\alpha = \tan^{-1}\left(\dfrac{4}{3}\right)$

3) The slope field of a first-order differential equation is shown below:



Which of the following could be the differential equation represented?

A) $\dfrac{dy}{dx} = \dfrac{y-1}{2}$

B) $\dfrac{dy}{dx} = xy + x$

C) $\dfrac{dy}{dx} = \dfrac{y}{x+2}$

D) $\dfrac{dy}{dx} = \dfrac{x-1}{y^2}$

4) The gradient of the normal to a curve at any point $P(x, y)$ is three times the gradient of the line joining P and the point $Q(2, 3)$. Which of the following differential equations describe the coordinates of the points on the curve?

A) $\dfrac{dy}{dx} + \dfrac{x-2}{3(y-3)} = 0$

B) $\dfrac{dy}{dx} - \dfrac{x-2}{3(y-3)} = 0$

C) $\dfrac{dy}{dx} - \dfrac{3(y-3)}{x-2} = 0$

D) $\dfrac{dy}{dx} + \dfrac{3(y-3)}{x-2} = 0$

5) The function P satisfies the logistic differential equation $\frac{dP}{dt}=\frac{P}{20}-\frac{P^2}{34000}$, where $P(0)=210$. Which of the following statements is **false**?

A) $\lim_{t\to\infty} P(t)=1700$

B) $\frac{dP}{dt}$ has a maximum value when $P=210$

C) $\frac{d^2P}{dt^2}=0$ when $P=850$

D) When $P>850$, $\frac{dP}{dt}>0$, $\frac{d^2P}{dt^2}<0$

6) A ball is thrown from the top of a cliff of height H metres with an initial speed u m/s at an angle θ above the horizontal. At the exact same moment, a cart starts moving horizontally from the base of the cliff at a constant speed v m/s. The ball is meant to land in the cart.
Assume there is no air resistance and that the cart starts directly below the thrower.
What should the cart's speed be for the ball to land in it?

A) $v=\frac{2u\sin\theta}{g}$

B) $v=\frac{u^2\sin 2\theta}{g}$

C) $v=u\cos\theta$

D) $v=\sqrt{u^2-2gH}$

End of Section I

Section II (29 marks)
Attempt Questions 7 - 9
Allow about 50 minutes for this section

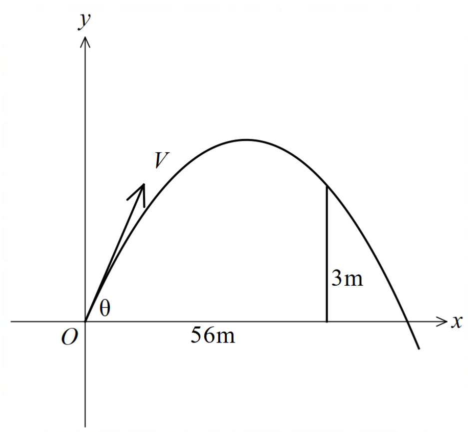
Answer each question on the appropriate pages of your answer booklet. Extra paper is available.
Your responses should include relevant calculations and/or mathematical reasoning.

Question 7 (9 marks)

a) Consider the differential equation $\frac{1}{1+x^2}\frac{dy}{dx}=\frac{x}{y}$. Find the particular solution to the differential equation, satisfying the initial condition $x=0$ and $y=-1$. **3**

b) Find the value of a, so that $y=ax-x^{-1}$ is the solution of the differential equation $xy'+y=2x$. **2**

c) In a game of baseball, a ball is hit from point O with velocity 28 m/s at an angle θ to the horizontal. The ball is hit towards a 3 metre high fence which is 56 metres away, as shown in the diagram.



Neglecting the effects of air resistance, the equations describing the motion of the ball are:

$$x=28t\cos\theta$$
$$y=28t\sin\theta-5t^2$$

(**DO NOT** derive these equations)

where t is the time in seconds, after the ball is hit and g is the acceleration due to gravity in m/s^2.

i) Show that the ball just clears the fence when $20\tan^2\theta-56\tan\theta+23=0$. **2**

ii) In what range must θ lie for the ball to go over the fence? **2**

End of Question 7

Question 8 (9 marks)

a) Solve the differential equation $\frac{dy}{dx}+1=3\cos^2\left(\frac{x}{2}\right)$. **3**

b) A rumour spreads through a community at the rate $\frac{dy}{dt}=2y(0.7-y)$, where y is the proportion of the population that has heard the rumour at time t.

It is given that $\frac{1}{y(0.7-y)}=\frac{1}{0.7y}+\frac{1}{0.7(0.7-y)}$.

i) If at $t=0$, 20% of the people have heard the rumour, find y as a function of t in the form of $y=\frac{P}{A+Be^{-Pkt}}$, where P, A, B and k are constants. **3**

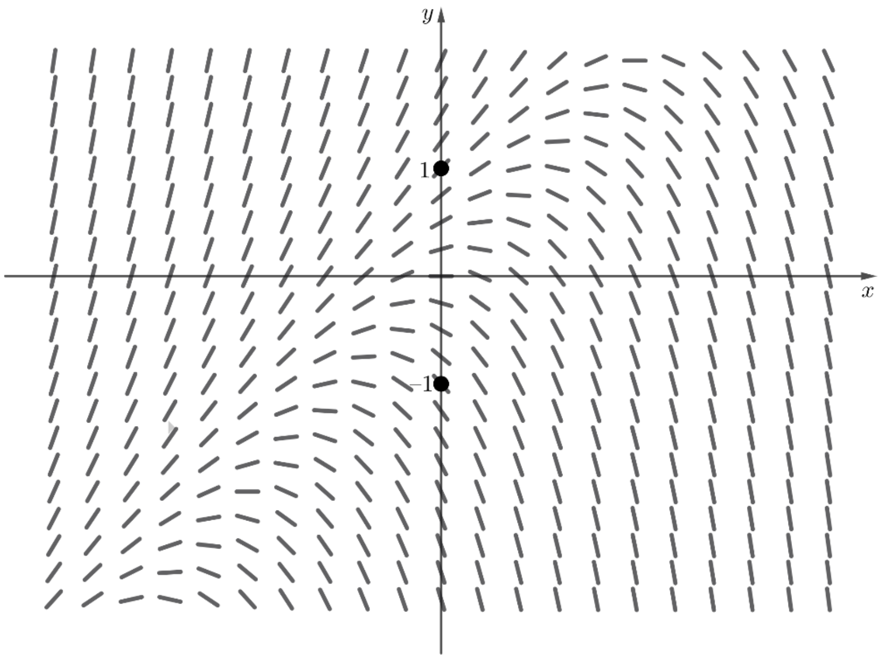
ii) What proportion of the population have heard the rumour when it is spreading the fastest? **1**

iii) At what time t is the rumour spreading the fastest? Give your answer in exact form. **2**

End of Question 8

Question 9 (11 marks)

a) The slope field of the differential equation $\frac{dy}{dx} = 2y - 4x$ is shown below:



i) It is known the solution curve passing through (0, 1) is linear. State the gradient of this solution curve. **1**

ii) On the diagram given in your answer booklet, sketch the solution curve that passes through the point (0, -1). **1**

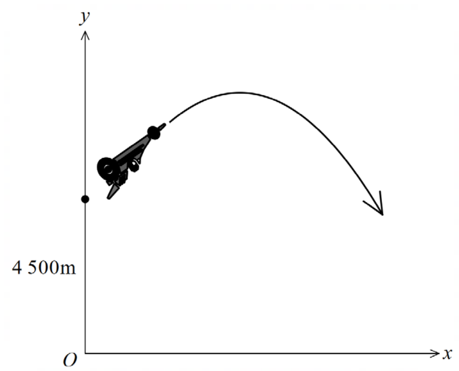
iii) The curve $y = g(x)$ satisfies the given differential equation and the initial condition $g(0) = 0$. **2**
The slope field suggests that $y = g(x)$ has a local maximum at the point (0, 0). Show with the use of calculus that this solution curve has a local maximum at (0, 0).

Question 9 continues on next page

b) An experimental spacecraft is launched from a height of 4500 m with an initial velocity of $180\sqrt{2}$ m/s at an angle of 45° to the horizontal. Immediately after launch, the experimental spacecraft undergoes a projectile motion under the influence of gravity ($g = 9.8$ m/s²).
The equations of motion are:

$$x = 180t$$
$$y = -4.9t^2 + 180t + 4500$$

where t is measured in seconds. (You are **NOT** required to derive these.)



i) Determine the maximum height reached by the spacecraft and the time taken to reach this height. **2**

ii) The pilot can only activate the parachute when the spacecraft is descending at an angle between 45° and 60° to the horizontal. Find the time interval during which the pilot can activate the parachute. Give your answer correct to one decimal place. **3**

iii) For a safe landing, the parachute can only be deployed when the spacecraft's speed is at most 325 m/s. Find the latest time (correct to one decimal place) at which the parachute can be deployed safely. **2**

End of Exam

Year-12 Assessment Task-3, 2025 Mathematics Extension-1

Marking Guidelines

Section - I

Multiple Choice (1 - 6)

1) D 2) C 3) B 4) A 5) B 6) C

Sample Solutions:

1) $x = e^{-4t}$

$$\frac{dx}{dt} = -4e^{-4t} \Rightarrow \frac{dx}{dt} = -4x \Rightarrow \frac{dx}{dt} + 4x = 0$$

$$\frac{d^2x}{dt^2} = 16e^{-4t} \Rightarrow \frac{d^2x}{dt^2} = 16x \Rightarrow \frac{d^2x}{dt^2} - 16x = 0,$$ $\therefore$ D is not correct.

2) $\underset{\sim}{v}(t) = 40\underset{\sim}{i} + (30 - gt)\underset{\sim}{j}$

$\underset{\sim}{v}(0) = 40\underset{\sim}{i} + 30\underset{\sim}{j}$

$\therefore |\underset{\sim}{v}(0)| = \sqrt{40^2 + 30^2} = 50$

$\tan\alpha = \frac{30}{40} \Rightarrow \alpha = \tan^{-1}\frac{3}{4}$ $\therefore$ C

3) $\frac{dy}{dx} = x(y+1)$

$y = -1 \Rightarrow \frac{dy}{dx} = 0$ $\therefore$ B.

4) $m_{PQ} = \frac{y-3}{x-2}$

Gradient of normal $= 3 \times m_{PQ}$

$$-\frac{1}{\frac{dy}{dx}} = \frac{3(y-3)}{x-2}$$

$$\frac{dy}{dx} = -\frac{(x-2)}{3(y-3)}$$

$$\frac{dy}{dx} + \frac{(x-2)}{3(y-3)} = 0$$ $\therefore$ A

5)

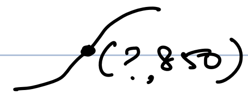
$$\frac{dP}{dt} = \frac{P}{20} - \frac{P^2}{34000}$$

$$= \frac{P}{20}\left(1 - \frac{P}{1700}\right)$$

$\therefore$ A is true as $\lim_{t\to\infty} P(t) = 1700$

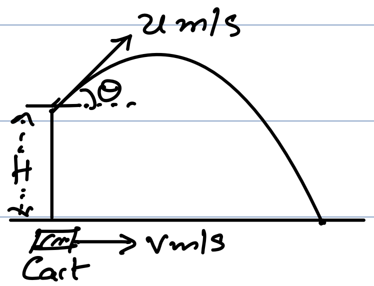
C is true as $\frac{1700}{2} = 850$

D is true $\because$ when $P > 850$, $\frac{dP}{dt} > 0$, $\frac{d^2P}{dt^2} < 0$



B is false as $\frac{dP}{dt}$ has maximum value at $P = 850$. $\therefore$ B

6)



Total horizontal distance covers by a ball at time T is

$$x_{ball} = uT\cos\theta$$

Total horizontal distance covers by the cart is

$$x_{cart} = v \cdot T$$

Since the cart starts from the same spot directly below the thrower, for it to intercept the ball, it must travel the same horizontal distance in the same time T.

$$\therefore \quad x_{ball} = x_{cart}$$

$$uT\cos\theta = vT$$

or $\quad v = u\cos\theta \qquad \therefore$ C

END OF SECTION I.

Section II

Question-7

a) $$\frac{1}{1+x^2}\frac{dy}{dx} = \frac{x}{y}$$

$$y\,dy = x(1+x^2)dx$$ 3 marks Correct solution

$$\int y\,dy = \int (x + x^3)dx$$ 2 marks Solve the differential

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{x^4}{4} + c$$ equation with initial

$$y^2 = x^2 + \frac{x^4}{2} + c$$ Condition $y(0)=-1$

Initial condition $x=0,\ y=-1$ 1 mark Rearrange the differential

$$(-1)^2 = 0+0+c$$ equation to solve.

$$\therefore c = 1$$

$$y^2 = x^2 + \frac{x^4}{2} + 1$$

$$y = -\sqrt{x^2 + \frac{x^4}{2} + 1}$$

(Reject $y>0$ because of initial condition)

b) $$y = ax - x^{-1}$$

$$y' = a + x^{-2}$$

Since y is the solution of the differential equation

$$\therefore xy' + y = 2x$$ 2 marks Correct Solution

$$x(a + x^{-2}) + ax - x^{-1} = 2x$$ 1 mark Correctly find y'

$$ax + x^{-1} + ax - x^{-1} = 2x$$ and substitute in

$$2ax = 2x$$ the given DE.

$$\therefore a = 1$$

c) i) Equations of motion are:

$x = 28t\cos\theta$ —— ① & $y = 28t\sin\theta - 5t^2$ —— ②

At $x = 56$, $y = 3$.

Sub in ① & ②

$56 = 28t\cos\theta \Rightarrow t = \frac{2}{\cos\theta}$

2 marks correct solution

$3 = 28t\sin\theta - 5t^2$

1 mark Eliminate t and Substitute $x = 56$ or $y = 3$

$3 = 28\left(\frac{2}{\cos\theta}\right)\sin\theta - 5\left(\frac{2}{\cos\theta}\right)^2$

$3 = 56\tan\theta - 20\sec^2\theta$

$3 = 56\tan\theta - 20(1+\tan^2\theta)$ —— (*)

$20(1+\tan^2\theta) - 56\tan\theta + 3 = 0$

$20\tan^2\theta - 56\tan\theta + 23 = 0$

Hence shown.

ii) For a "six" to be scored

$56\tan\theta - 20(1+\tan^2\theta) \geq 3$ (from ii) (*))

or $20\tan^2\theta - 56\tan\theta + 23 \leq 0$

Since it is quad. in $\tan\theta$

$\therefore \frac{56 - \sqrt{(56)^2 - 4\times 20\times 23}}{2\times 20} \leq \tan\theta \leq \frac{56 + \sqrt{(56)^2 - 4\times 20\times 23}}{2\times 20}$

$0.5 \leq \tan\theta \leq 2.3$

2 marks Correct solution

$27^\circ \leq \theta \leq 66^\circ$

1 mark Correctly solving the equation from part i) to find angle θ.

Question: 8

a) $\frac{dy}{dx} = 3\cos^2\frac{x}{2} - 1$

$\frac{dy}{dx} = \frac{3}{2}(1+\cos x) - 1$ $\qquad \left(\because \cos^2 x = \frac{1+\cos 2x}{2}\right)$

$\frac{dy}{dx} = \frac{3 + 3\cos x - 2}{2}$

3 marks Correct Solution

$\frac{dy}{dx} = \frac{1}{2}(1 + 3\cos x)$

2 marks Correctly expressing $\cos^2\frac{x}{2}$ in terms of $\cos x$ and progress towards solving differential eq.

$dy = \left(\frac{1}{2} + \frac{3}{2}\cos x\right)dx$

Integrate both sides

$y = \frac{x}{2} + \frac{3}{2}\sin x + C$

1 mark Correctly expressing $\cos^2\frac{x}{2}$ in terms of $\cos x$.

b) If $t = 0$; $y = 20\% = 0.2$

$\frac{dy}{dt} = 2y(0.7 - y)$; $y = 0, 0.7$ are trivial solution.

$\frac{dt}{dy} = \frac{1}{2y(0.7-y)}$

$2\,dt = \frac{dy}{y(0.7-y)}$

$2\int_0^t dt = \int_{0.2}^{y} \frac{1}{y(0.7-y)}\,dy$

$2t = \int_{0.2}^{y}\left(\frac{1}{0.7y} + \frac{1}{0.7(0.7-y)}\right)dy$

$= \frac{1}{0.7}\int_{0.2}^{y}\left(\frac{1}{y} + \frac{1}{0.7-y}\right)dy$

$$2t = \frac{1}{0.7}\left(\ln|y| - \ln|0.7-y|\right]_{0.2}^{y}$$

$$= \frac{1}{0.7}\ln\left|\frac{y}{0.7-y}\right|_{0.2}^{y}$$

$$1.4t = \ln\left|\frac{y}{0.7-y}\right| - \ln\frac{0.2}{0.5}$$

$$1.4t = \ln\left|\frac{0.5y}{0.2(0.7-y)}\right| \quad \text{——— } 1$$

3 marks Correct solution

2 marks Correctly integrate the differental eq. and attempt to express with y as the subject.

1 mark correctly separate the variables to find the integral or equivalent.

$$e^{1.4t} = \frac{5y}{1.4-2y}$$

$$e^{1.4t}(1.4-2y) = 5y$$

$$1.4e^{1.4t} = 5y + 2ye^{1.4t}$$

$$1.4e^{1.4t} = y(5+2e^{1.4t})$$

$$y = \frac{1.4e^{1.4t}}{5+2e^{1.4t}}$$

$$= \frac{1.4}{2+5e^{-1.4t}}$$

(Divide num & den. by $e^{1.4t}$)

ii) $\frac{dy}{dt} = 0$ at $y = 0$ & 0.7 1 mark correct solution.

and curve is concave down parabola

$\therefore$ Maximum at 0.35

$\therefore$ 0.35 proportion of the population has heard the rumor when it is spreading the fastest.

iii) Sub $y = 0.35$ in the eq. ① found in part i)

$$1.4t = \ln\left|\frac{0.5y}{0.2(0.7-y)}\right|$$

$$1.4t = \ln\left(\frac{0.5 \times 0.35}{0.2 \times 0.35}\right)$$

$$1.4t = \ln\frac{5}{2}$$

$$t = \frac{1}{1.4}\ln\frac{5}{2}$$

$$= \frac{5}{7}\ln\frac{5}{2}$$

2 marks Correct solution

1 mark sub $y = 0.35$ in equation found in part i)

Question-9

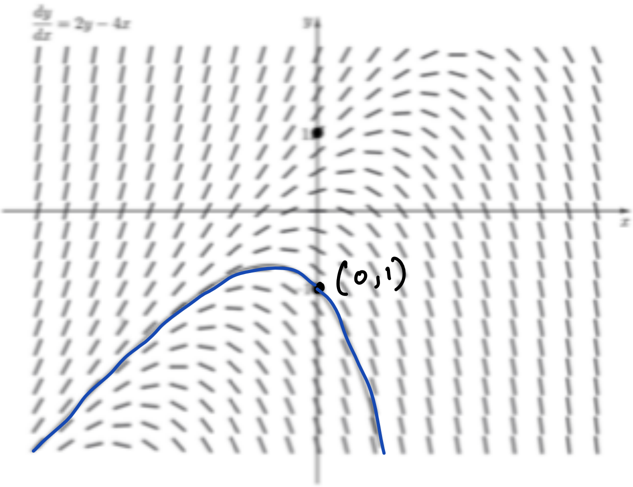
a) i) At $(0,1)$

1 mark - Correct solution

$$\frac{dy}{dx} = 2y - 4x$$
$$= 2(1) - 4(0)$$
$$= 2$$

$\therefore$ gradient of the linear solution curve is 2.

ii)



1 mark Correct sketch of Solution curve passes through $(0,1)$

Note: Curve cannot cross the asymptote $y = x + 1$

iii) At $(0,0)$

2 marks - Correct solution

1 mark - Show $\frac{dy}{dx} = 0$ and $(0,0)$ is a stationary point of solution curve $y = g(x)$ or show $\frac{d^2y}{dx^2} = -4$ and $(0,0)$ is local maximum.

$$\frac{dy}{dx} = 2y - 4x$$
$$= 2(0) - 4(0) = 0$$

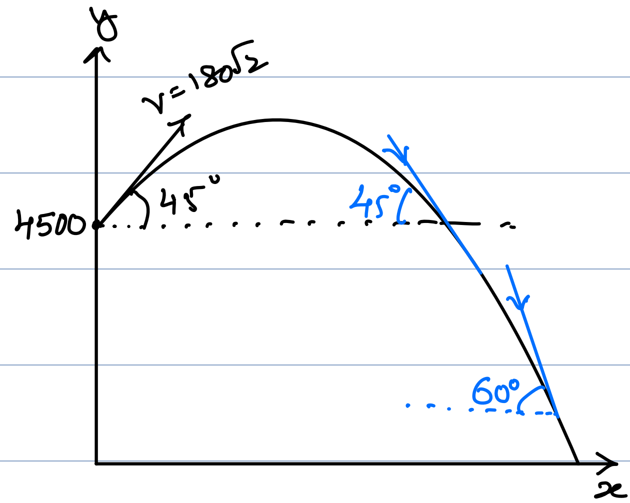
$\therefore$ $(0,0)$ is stationary point of $g(x)$

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4$$

At $(0,0)$; $\frac{d^2y}{dx^2} = 2(0) - 4 = -4 < 0$

$\therefore$ $g(x)$ has local maximum at $(0,0)$

b) i)



$y = -4.9t^2 + 180t + 4500$

$\dot{y} = -9.8t + 180$

For Maximum height

$\dot{y} = 0$

$-9.8t + 180 = 0$

$t = \frac{180}{9.8}$

$= 18.36734\ldots \approx 18.4$ seconds

Maximum height: $y = -4.9t^2 + 180t + 4500$

$= -4.9\left(\frac{180}{9.8}\right)^2 + 180\left(\frac{180}{9.8}\right) + 4500$

$= 6153.061224 \approx 6153$ metres.

2 marks Correct Solution

1 mark Finding correct time by equating $\dot{y} = 0$.

ii) Since the path is parabolic and symmetrical, the spacecraft will be descending at an angle of 45° when

$$t = 2 \times \frac{180}{9.8}$$

$$= 36.73469\ldots \approx 36.7 \text{ seconds}$$

$\therefore$ Earliest time pilot can eject is 36.7 seconds.

The latest time for ejection is when

$$\tan\theta = \frac{\dot{y}}{\dot{x}}$$

$$\theta = -60°, \quad \dot{y} = -9.8t + 180, \quad \dot{x} = 180$$

$$\therefore \tan(-60) = \frac{-9.8t + 180}{180}$$

$$-\sqrt{3} = \frac{-9.8t + 180}{180}$$

$$-180\sqrt{3} = -9.8t + 180$$

$$9.8t = 180 + 180\sqrt{3}$$

$$t = \frac{180 + 180\sqrt{3}}{9.8}$$

$$= 50.18052\ldots \approx 50.2 \text{ seconds}.$$

$\therefore$ The latest time the pilot can eject is approx. 50.2 seconds.

$\therefore$ Pilot can eject between 36.7 seconds and 50.2 seconds.

or $36.7 < t < 50.2$ (seconds)

3 marks Correct solution

2 marks Correctly find the time for parachute for 45° and correct approch to find time for 60°.

1 mark Correctly find the time for parachute for 45° or correct approach to find time for 60°.

iii) $v = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\dot{x} = 180$, $\dot{y} = -9.8t + 180$

If $v = 325$ then

$$325 = \sqrt{(180)^2 + (180 - 9.8t)^2}$$

$$325^2 = 180^2 + (180 - 9.8t)^2$$

$$(180 - 9.8t)^2 = 325^2 - 180^2$$

$$180 - 9.8t = \pm\sqrt{73225}$$

$$-9.8t = -180 \pm \sqrt{73225}$$

$$t = \frac{-180 \pm \sqrt{73225}}{-9.8}$$

$$= 45.979712\ldots \quad (t > 0) \approx 46.0 \text{ (1dp)}$$

$\therefore$ The latest time to eject is 46.0 seconds.

2 marks Correct solution

1 mark Correctly using the velocity equation by substituting $v = 325$ or equivalent approach.

END OF EXAM