Caringbah High School

Year 12 Mathematics Extension 1
Assessment Task 2
Informal Examination
2021

General Instructions

- Reading time – 5 minutes
- Working time – 75 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided separately
- In Questions 6-9, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 60

Section I **5 marks**
Attempt Questions 1-5
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **55 marks**
Attempt Questions 6-9
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: ____ **Class:** ____________

Section I					Total	
Q 1-5	Q6	Q7	Q8	Q9		
/5	/13	/14	/14	/14	/60	%

Section I -5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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1 If $\underset{\sim}{a} = -7\underset{\sim}{i} + 2\underset{\sim}{j}$, what does $|2\underset{\sim}{a}|$ equal?

A) $2\sqrt{53}$

B) $3\sqrt{41}$

C) $-14\underset{\sim}{i} + 4\underset{\sim}{j}$

D) $14\underset{\sim}{i} - 4\underset{\sim}{j}$

2 If $y = \tan^{-1}(\sin x)$, then $\dfrac{dy}{dx} = ?$

A) $\dfrac{1}{1+\sin^2 x}$

B) $\sec^2 x$

C) $\dfrac{\cos x}{1+\sin^2 x}$

D) $\sec x$

3 Using the substitution $u = \ln x$ or otherwise, which of the following is the primitive of $\dfrac{(\ln x)^2}{x}$?

A) $\dfrac{x(\ln x)^3}{3} + C$

B) $\dfrac{(\ln x)^3}{3x} + C$

C) $(\ln x)^3 + C$

D) $\dfrac{(\ln x)^3}{3} + C$

4 Consider the vector $\underset{\sim}{a} = \begin{bmatrix} 2x-1 \\ 1-2x \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} 2x+1 \\ 1+2x \end{bmatrix}$. Are these vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ perpendicular?

A) No

B) Yes

C) Not enough information

D) Vectors are parallel

5 What is the integral $\int_1^2 x\sqrt{2-x}\ dx$ transformed to by the substitution $u = \sqrt{2-x}$?

A) $\int_1^2 (2u - u^3)\ du$

B) $\int_1^0 (2u - u^3)\ du$

C) $\int_1^2 (2u^4 - 4u^2)\ du$

D) $\int_1^0 (2u^4 - 4u^2)\ du$

End of Section I

Section II

55 marks
Attempt Questions 6–9
Allow about 67 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 Marks) **Marks**

a) Differentiate

i) $y = \tan^{-1} x^3$ **1**

ii) $y = 2x^2 \sin^{-1}(2x-1)$ **2**

iii) $y = \cos^{-1}(\log_e x)$ **1**

b) Let $\underset{\sim}{u} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$, find

i) $\underset{\sim}{u} - \underset{\sim}{v}$ **1**

ii) $\underset{\sim}{u}.\underset{\sim}{v}$ **1**

iii) $\hat{\underset{\sim}{u}}$ **1**

c) The curve $y = \dfrac{1}{\sqrt{x^2+9}}$ is rotated about the $x-$axis. Find the volume of the solid of revolution formed between $x = 0$ and $x = 3\sqrt{3}$. **3**

d) Find λ and μ such that $\lambda\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \mu\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. **3**

End of Question 6

Question 7 (14 Marks)

a) If $f(x)=\sin^{-1}x+\cos^{-1}x$ and $-1\le x\le 1$,

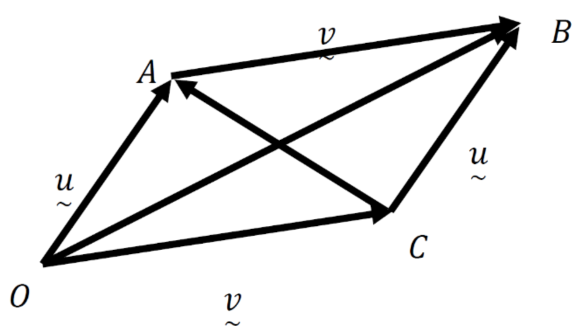
i) By using $f'(x)$ show that $f(x)=\dfrac{\pi}{2}$ for all x in the given domain. **2**

ii) Hence evaluate $\int_{-1}^{1} f(x)\,dx$ **1**

b) Find the vector projection of $\underset{\sim}{u}=2\underset{\sim}{i}-5\underset{\sim}{j}$ onto $\underset{\sim}{v}=\underset{\sim}{i}-\underset{\sim}{j}$ **2**

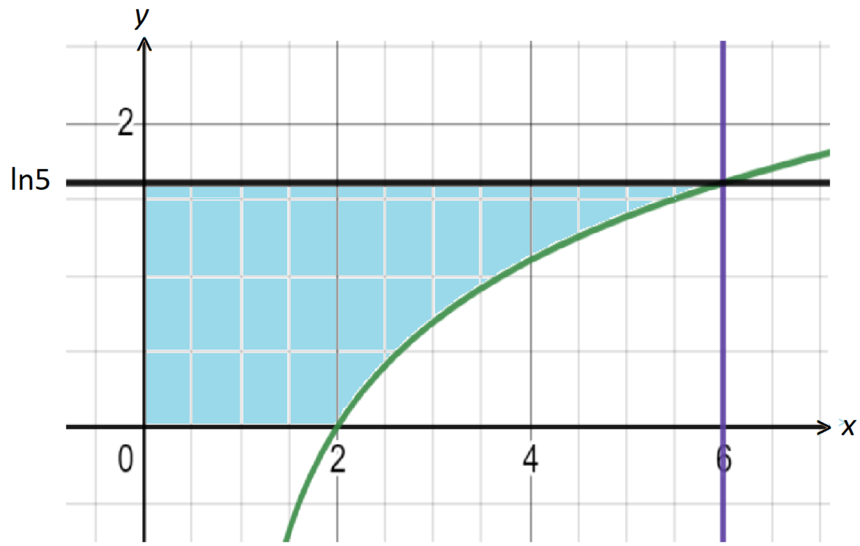
c) Evaluate $\displaystyle\int_0^{\frac{1}{\sqrt{2}}}\sqrt{1-x^2}\,dx$ using the substitution $x=\sin u$ **3**

d) In the parallelogram $OABC$, $\overrightarrow{OA}=\overrightarrow{CB}=\underset{\sim}{u}$ and $\overrightarrow{OC}=\overrightarrow{AB}=\underset{\sim}{v}$. Prove that the diagonals of a parallelogram intersect at right angles if and only if it is a rhombus. **2**



Question 7 continued on page 6

e) In the diagram, the shaded region is bounded by $y = \log_e(x-1)$, the coordinate axes and the line $y = \ln 5$. This region is then rotated about the $y-$axis.



i) Show that the volume of the solid of revolution of the shaded region formed is given by **2**

$$V = \pi \int_0^{\log_e 5} (1 + 2e^y + e^{2y})\ dy$$

ii) Evaluate, in exact form, the volume of the solid. **2**

End of Question 7

Question 8 (14 Marks)

a) Find

i) $\int \frac{e^{2x}}{\sqrt{64-e^{4x}}}\,dx$ **2**

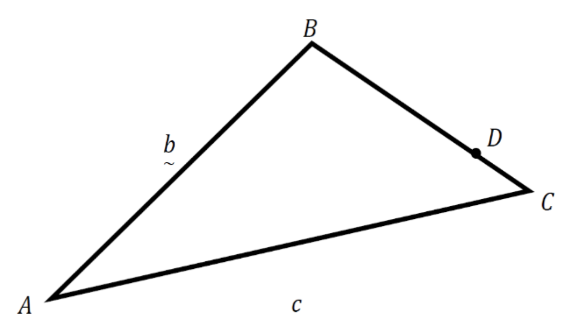
ii) $\int \frac{dx}{\sqrt{4-9x^2}}$ **2**

b) Find the angle, to the nearest degree, between $\underset{\sim}{u} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - \underset{\sim}{j}$ **2**

c) i) Show that $x^2 + 2x + 5 = (x+1)^2 + 4$ **1**

ii) Hence find $\int_{-1}^{1} \frac{1}{x^2+2x+5}\,dx$ **2**

d) In the diagram, $\overrightarrow{AB} = \underset{\sim}{b}$ and $\overrightarrow{AC} = \underset{\sim}{c}$. D divides BC in the ration 5:1. Find an expression for $\overrightarrow{AD}$ in terms of $\underset{\sim}{b}$ and $\underset{\sim}{c}$. **3**



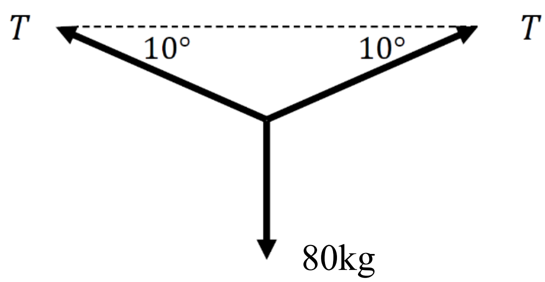
e) Find the vector $\underset{\sim}{v}$ parallel to $\underset{\sim}{u} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$ that has a magnitude of 4. **2**

End of Question 8

Question 9 (14 Marks) **Marks**

a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x \, dx$ **2**

b) A hammock of negligible mass is attached to two trees, each at an angle of $10°$ to the horizontal, with the tension in the ropes at both sides equal. A person of mass 80 kg is lying in the hammock. What is the tension *T*, assuming $g = 9.8 \text{ ms}^{-2}$. **3**



c) The gradient at the point (x, y) of the function $y = f(x)$ is given by $\frac{dy}{dx} = \sin^2 x$. The curve passes through the point $(0, -2)$. Find y when $x = \frac{\pi}{4}$. **3**

d) Evaluate $\int_0^1 \frac{(6\sqrt{x}-1)^2}{\sqrt{x}} \, dx$, using the substitution $u = 6\sqrt{x} - 1$ **3**

e) If $y = x\cos^{-1} 2x - \frac{1}{2}\sqrt{1-4x^2}$, find $\frac{dy}{dx}$, simplified fully. **3**

End of Question 9

END OF EXAMINATION!

Name: ____________________

Mathematics Extension 1

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample **2 + 4 = ?** **(A) 2 (B) 6 (C) 8 (D) 9**

A ○ **B** ● **C** ○ **D** ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● **B** ~~●~~ **C** ○ **D** ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ **B** ~~●~~ **C** ○ **D** ○

Start Here ➔ **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

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Mult. Choice

Q1 $\underset{\sim}{a} = -7\underset{\sim}{i} + 2\underset{\sim}{j}$

$|\underset{\sim}{a}| = \sqrt{(-7)^2 + 2^2}$

$= \sqrt{53}$

$\therefore |2\underset{\sim}{a}| = 2\sqrt{53}$ (A)

Q2 $y = \tan^{-1}(\sin x)$

$\frac{dy}{dx} = \frac{\cos x}{1+\sin^2 x}$ (C)

Q3 $\int \frac{(\ln x)^2}{x}\,dx$ $\qquad u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $du = \frac{1}{x}dx$

$= \int u^2\,du$

$= \frac{u^3}{3} + c$

$= \frac{(\ln x)^3}{3} + c$ (D)

Q4 $\underset{\sim}{a} = \begin{bmatrix} 2x-1 \\ 1-2x \end{bmatrix}$, $\underset{\sim}{b} = \begin{bmatrix} 2x+1 \\ 1+2x \end{bmatrix}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = (2x-1)(2x+1) + (1-2x)(1+2x)$

$= 4x^2 - 1 + 1 - 4x^2$

$= 0$

$\therefore \underset{\sim}{a} \perp \underset{\sim}{b}$ (B)

Q5 $\int_1^2 x\sqrt{2-x}\,dx$ $\qquad u = \sqrt{2-x}$, $u^2 = 2-x$, $x = 2-u^2$, $\frac{dx}{du} = -2u$, $dx = -2u\,du$

$= \int_1^0 (2-u^2)(u)(-2u)\,du$

$= \int_1^0 (2-u^2)(-2u^2)\,du$

$= \int_1^0 (-4u^2 + 2u^4)\,du$ $\qquad$ when $x=2, u=0$; " $x=1, u=1$

$= \int_1^0 (2u^4 - 4u^2)\,du$ (D)

Q6

(a)(i) $y = \tan^{-1} x^3$

$\frac{dy}{dx} = \frac{3x^2}{1+(x^3)^2}$

$= \frac{3x^2}{1+x^6}$ ①

(ii) $y = 2x^2\sin^{-1}(2x-1)$

$\frac{dy}{dx} = 2x^2 \times \frac{2}{\sqrt{1-(2x-1)^2}} + \sin^{-1}(2x-1)(4x)$ ①

$= \frac{4x^2}{\sqrt{1-(2x-1)^2}} + 4x\sin^{-1}(2x-1)$

$= \frac{4x}{\sqrt{4x-4x^2}} + 4x\sin^{-1}(2x-1)$ ① or

$= \frac{2x}{\sqrt{x-x^2}} + 4x\sin^{-1}(2x-1)$

(iii) $y = \cos^{-1}(\log_e x)$

$\frac{dy}{dx} = -\frac{\frac{1}{x}}{\sqrt{1-(\log_e x)^2}}$

$= -\frac{1}{x\sqrt{1-(\log_e x)^2}}$ ①

Q6 cont

(b) $\underset{\sim}{u} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$, $\underset{\sim}{v} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$

(i) $\underset{\sim}{u} - \underset{\sim}{v} = (4\underset{\sim}{i} - 2\underset{\sim}{j}) - (2\underset{\sim}{i} - 5\underset{\sim}{j})$

$= 2\underset{\sim}{i} + 3\underset{\sim}{j}$ (1)

(ii) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 4(2) + (-2)(-5)$

$= 8 + 10$

$= 18$ (1)

(iii) $\hat{\underset{\sim}{u}} = \sqrt{4^2 + (-2)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$ $\Rightarrow \therefore \hat{\underset{\sim}{u}} = \frac{2}{\sqrt{5}}\underset{\sim}{i} - \frac{1}{\sqrt{5}}\underset{\sim}{j}$ (1)

(c) $y = \frac{1}{\sqrt{x^2+9}}$, rotated about x-axis, $x=0$, $x = 3\sqrt{3}$.

$y^2 = \frac{1}{x^2+9}$

$V = \pi \int_0^{3\sqrt{3}} \frac{1}{x^2+9}\,dx$ (1)

$= \pi \left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^{3\sqrt{3}}$ (1)

$= \frac{\pi}{3}\left[\tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) - \tan^{-1}(0)\right]$

$= \frac{\pi}{3}\left[\tan^{-1}(\sqrt{3}) - \tan^{-1}(0)\right]$

$= \frac{\pi}{3}\left[\frac{\pi}{3} - 0\right]$

$= \frac{\pi^2}{9}\,u^3$ (1)

(d) $\lambda \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$2\lambda + 4\mu = 0 \quad \ldots (1) \quad \times 3$ (1)

$3\lambda + 5\mu = 1 \quad \ldots (2) \quad \times 2$

$6\lambda + 12\mu = 0$

$6\lambda + 10\mu = 2$ −

$2\mu = -2$

$\mu = -1$ (1)

sub $\mu = -1$ into (1)

$2\lambda - 4 = 0$

$2\lambda = 4$

$\lambda = 2$ (1)

$\therefore \lambda = 2, \mu = -1$

Q7

(a) $f(x) = \sin^{-1}x + \cos^{-1}x$, $-1 \le x \le 1$

(i) $f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$ (1)

$= 0$

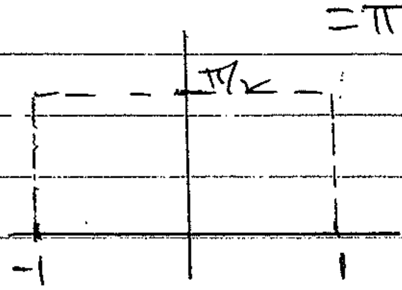
Let $x = 0$, $f(0) = \sin^{-1}0 + \cos^{-1}0$

$= \frac{\pi}{2}$ (1)

$\therefore f(x) = \frac{\pi}{2}$ for all x, $-1 \le x \le 1$

(ii) $\int_{-1}^{1} f(x)\,dx = 2 \times \frac{\pi}{2}$

$= \pi$



(1)

Q7 cont.

(b) $\underset{\sim}{u} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$, $\underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j}$

$$\text{proj}_{\underset{\sim}{v}} \underset{\sim}{u} = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\underset{\sim}{v} \cdot \underset{\sim}{v}} \cdot \underset{\sim}{v}$$

$$= \frac{2(1) + (-5)(-1)}{1^2 + (-1)^2} (\underset{\sim}{i} - \underset{\sim}{j}) \quad (1)$$

$$= \frac{7}{2}(\underset{\sim}{i} - \underset{\sim}{j})$$

$$= \frac{7}{2}\underset{\sim}{i} - \frac{7}{2}\underset{\sim}{j} \quad (1)$$

(c) $\int_0^{\frac{1}{\sqrt{2}}} \sqrt{1-x^2}\, dx$

$x = \sin u$

$\frac{dx}{du} = \cos u$

$dx = \cos u\, du$

when $x = \frac{1}{\sqrt{2}}, u = \frac{\pi}{4}$

" $x = 0$, $u = 0$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 - \sin^2 u} \times \cos u\, du \quad (1)$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\cos^2 u} \times \cos u\, du$$

$$= \int_0^{\frac{\pi}{4}} (\cos u \times \cos u)\, du$$

$$= \int_0^{\frac{\pi}{4}} \cos^2 u\, du \quad (1)$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2} + \frac{1}{2}\cos 2u\, du$$

$$= \left[\frac{1}{2}u + \frac{1}{4}\sin 2u\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{4}\sin\frac{\pi}{2} - 0$$

$$= \frac{\pi}{8} + \frac{1}{4}(1)$$

$$= \frac{\pi}{8} + \frac{1}{4} = \frac{\pi + 2}{8} \quad (1)$$

(d) $\overrightarrow{OB} = \underset{\sim}{u} + \underset{\sim}{v}$ & $\overrightarrow{CA} = \underset{\sim}{u} - \underset{\sim}{v}$

If diagonals are $\perp$, $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$$(\underset{\sim}{u} + \underset{\sim}{v}) \cdot (\underset{\sim}{u} - \underset{\sim}{v}) = 0. \quad (1)$$

$$\underset{\sim}{u}^2 - \underset{\sim}{v}^2 = 0$$

$$|\underset{\sim}{u}| - |\underset{\sim}{v}| = 0.$$

$$|\underset{\sim}{u}| = |\underset{\sim}{v}|$$

$$\therefore \underset{\sim}{u} = \underset{\sim}{v} \quad (1)$$

$\therefore$ the sides are = & the parallelogram must be a rhombus if the diagonals meet at $90°$.

(e) $V = \pi \int_a^b x^2\, dy$ $\quad y = \log_e(x-1)$

(i)

$x - 1 = e^y$

$x = e^y + 1$

$x^2 = (e^y + 1)^2 \quad (1)$

$= e^{2y} + 2e^y + 1$

$$V = \pi \int_0^{\ln 5} (e^{2y} + 2e^y + 1)\, dy \quad (1)$$

(ii) $$V = \pi \left[\frac{1}{2}e^{2y} + 2e^y + y\right]_0^{\ln 5} \quad (1)$$

$$= \pi \left[\left(\frac{1}{2}e^{2\ln 5} + 2e^{\ln 5} + \ln 5\right) - \left(\frac{1}{2}e^0 + 2e^0 + 0\right)\right]$$

$$= \pi \left[\frac{1}{2}(25) + 2(5) + \ln 5 - \frac{1}{2} - 2\right]$$

$$= \pi \left[\frac{25}{2} + 10 + \ln 5 - \frac{1}{2} - 2\right]$$

$$= \pi (20 + \ln 5)\, u^3 \quad (1)$$

Q8

(a)(i) $\displaystyle\int \frac{e^{2x}}{\sqrt{64-e^{4x}}}\,dx$

$\displaystyle = \frac{1}{2}\int \frac{2e^{2x}}{\sqrt{8^2-(e^{2x})^2}}\,dx$ (1)

$\displaystyle = \frac{1}{2}\sin^{-1}\left(\frac{e^{2x}}{8}\right)+c$ (1)

(ii) $\displaystyle\int \frac{dx}{\sqrt{4-9x^2}}$

$\displaystyle = \frac{1}{3}\int \frac{dx}{\sqrt{2^2-3^2x^2}}$ (1)

$\displaystyle = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+c$ (1)

(b) $\underset{\sim}{u} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$, $\underset{\sim}{v} = 2\underset{\sim}{i} - \underset{\sim}{j}$

$\displaystyle \cos\theta = \frac{2(2)+(-3)(-1)}{\sqrt{2^2+(-3)^2}\times\sqrt{2^2+(-1)^2}}$ (1)

$\displaystyle = \frac{4+3}{\sqrt{13}\times\sqrt{5}}$

$\displaystyle = \frac{7}{\sqrt{65}}$

$\displaystyle \therefore \theta = \cos^{-1}\left(\frac{7}{\sqrt{65}}\right)$

$\doteqdot 30^\circ$ (1)

(c)(i) $x^2+2x+5 = (x+1)^2+4$

LHS:

$x^2+2x+5 = x^2+2x+1+4$

$= (x+1)^2+4$

$= \text{RHS}$. (1)

$\therefore$ LHS = RHS

(c)(ii) $\displaystyle\int_{-1}^{1} \frac{1}{x^2+2x+5}\,dx$

$\displaystyle = \int_{-1}^{1} \frac{1}{(x+1)^2+4}\,dx$

$\displaystyle = \int_{-1}^{1} \frac{1}{4+(x+1)^2}\,dx$

$\displaystyle = \frac{1}{2}\left[\tan^{-1}\left(\frac{x+1}{2}\right)\right]_{-1}^{1}$ (1)

$\displaystyle = \frac{1}{2}\left[\tan^{-1}\left(\frac{2}{2}\right)-\tan^{-1}(0)\right]$

$\displaystyle = \frac{1}{2}\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$

$\displaystyle = \frac{1}{2}\left[\frac{\pi}{4}-0\right]$

$\displaystyle = \frac{\pi}{8}$ (1)

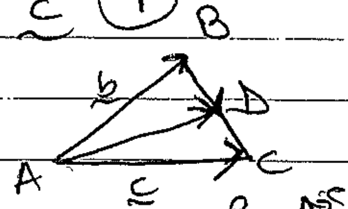
(d) $\overrightarrow{AD} = \overrightarrow{AB}+\overrightarrow{BD}$

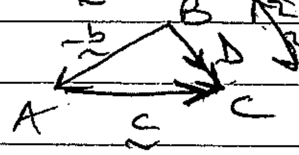
$\displaystyle = \overrightarrow{AB}+\frac{5}{6}\overrightarrow{BC}$ (1)

$\displaystyle = \underset{\sim}{b}+\frac{5}{6}(\underset{\sim}{c}-\underset{\sim}{b})$ (1)

$\displaystyle = \underset{\sim}{b}+\frac{5}{6}\underset{\sim}{c}-\frac{5}{6}\underset{\sim}{b}$

$\displaystyle = \frac{1}{6}\underset{\sim}{b}+\frac{5}{6}\underset{\sim}{c}$ (1)





(e) $\underset{\sim}{u} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$, magnitude $= 4$

$|\underset{\sim}{u}| = \sqrt{(-2)^2 + 5^2}$

$= \sqrt{29}$

$\therefore \hat{\underset{\sim}{u}} = \frac{-2}{\sqrt{29}}\underset{\sim}{i} + \frac{5}{\sqrt{29}}\underset{\sim}{j}$ (1)

$\therefore \underset{\sim}{v} = \hat{\underset{\sim}{u}} \times (\pm 4)$

$\underset{\sim}{v} = \hat{\underset{\sim}{u}} \times 4$

$= 4\left(\frac{-2}{\sqrt{29}}\underset{\sim}{i} + \frac{5}{\sqrt{29}}\underset{\sim}{i}\right)$

$= \frac{-8}{\sqrt{29}}\underset{\sim}{i} + \frac{20}{\sqrt{29}}\underset{\sim}{j}$

OR

$\underset{\sim}{v} = \hat{\underset{\sim}{u}} \times (-4)$ (1)

$= -4\left(\frac{-2}{\sqrt{29}}\underset{\sim}{i} + \frac{5}{\sqrt{29}}\underset{\sim}{j}\right)$

$= \frac{8}{\sqrt{29}}\underset{\sim}{i} - \frac{20}{\sqrt{29}}\underset{\sim}{j}$

Q9 $\int_0^{\pi/2} \sin^3 x \cos x \, dx$

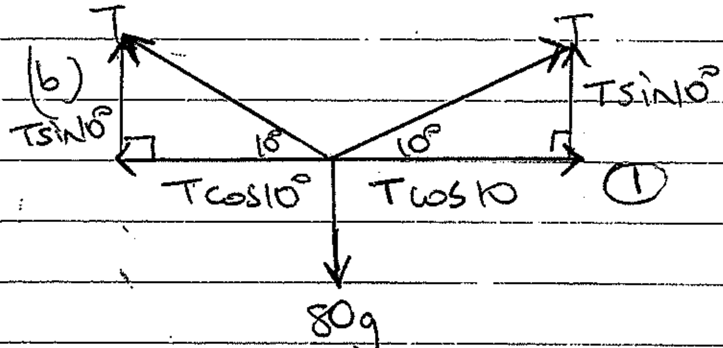
$= \left[\frac{\sin^4 x}{4}\right]_0^{\pi/2}$ (1)

$= \frac{1}{4}\left[\sin^4\left(\frac{\pi}{2}\right) - \sin^0(0)\right]$

$= \frac{1}{4}(1 - 0)$

$= \frac{1}{4}$ (1)

(b)



$2T\sin 10° = 80 \times 9.8$

$T = \frac{40 \times 9.8}{\sin 10°}$ (1)

$T = 2257\text{N}$ (1)

(c) $\frac{dy}{dx} = \sin^2 x$, $(0, -2)$

when $x = \frac{\pi}{4}$, $y = ?$

$y = \int \sin^2 x \, dx$

$y = \int\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)dx$ (1)

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$.

$-2 = 0 - \frac{1}{4}\sin 0 + c$.

$c = -2$.

Q9(c) cont.

$\therefore y = \frac{1}{2}x - \frac{1}{4}\sin 2x - 2$ (1)

when $x = \frac{\pi}{4}$,

$y = \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\sin\frac{\pi}{2} - 2$

$y = \frac{\pi}{8} - \frac{1}{4} - 2$

$= \frac{\pi}{8} - \frac{9}{4}$

$y = \frac{\pi - 18}{8}$ (1)

(d) $\int_0^1 \frac{(6\sqrt{x}-1)^2}{\sqrt{x}}\,dx$, $u = 6\sqrt{x} - 1$

$\frac{du}{dx} = \frac{3}{\sqrt{x}}$ (1)

$\frac{1}{3}\int_0^1 \frac{3(6\sqrt{x}-1)^2}{\sqrt{x}}\,dx$ $du = \frac{3}{\sqrt{x}}\,dx$

when $x=1$, $u=5$

$= \frac{1}{3}\int_{-1}^{5} u^2\,du$ (1) " $x=0$, $u=-1$

$= \frac{1}{3}\left[\frac{u^3}{3}\right]_{-1}^{5}$

$= \frac{1}{9}\left(5^3 - (-1)^3\right)$

$= \frac{1}{9}(126)$

$= 14$ (1)

(e) $y = x\cos^{-1}2x - \frac{1}{2}\sqrt{1-4x^2}$

$y = x\cos^{-1}2x - \frac{1}{2}(1-4x^2)^{1/2}$

$\frac{dy}{dx} = x\left(\frac{-2}{\sqrt{1-4x^2}}\right) + \cos^{-1}(2x)(1)$ (1)

$- \frac{1}{4}(1-4x^2)^{-1/2}(-8x)$

$= \frac{-2x}{\sqrt{1-4x^2}} + \cos^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$ (1)

$= \cos^{-1}(2x)$ (1)