Caringbah High School

Year 12 Mathematics Extension 1
Assessment Task 2
2022

General Instructions
- Reading time – 5 minutes
- Working time – 75 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A reference sheet is provided separately
- In Questions 6-10, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 60

Section I **5 marks**
Attempt Questions 1-5
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **55 marks**
Attempt Questions 6-10
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: ________________________________ **Class:** ____________

Section I	**Section II**					**Total**	
Q1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**		
/5	/11	/11	/11	/11	/11	/60	%

Section I -5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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1 If $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$, the unit vector in the direction of $\underset{\sim}{u}$ is:

A) $\frac{1}{13}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

B) $\frac{1}{5}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

C) $\frac{1}{\sqrt{13}}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

D) $\sqrt{13}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

2 What is the derivate of $\tan^{-1}\left(\frac{x}{2}\right)$?

A) $\frac{1}{2\left(4+x^2\right)}$

B) $\frac{1}{4+x^2}$

C) $\frac{2}{4+x^2}$

D) $\frac{4}{4+x^2}$

3 Which expression is equal to $\int \cos^2 4x \, dx$?

A) $\frac{-\sin^3 4x}{12} + C$

B) $\frac{\sin^3 4x}{12} + C$

C) $\frac{1}{2}\left(x + \frac{1}{8}\sin 8x\right) + C$

D) $\frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + C$

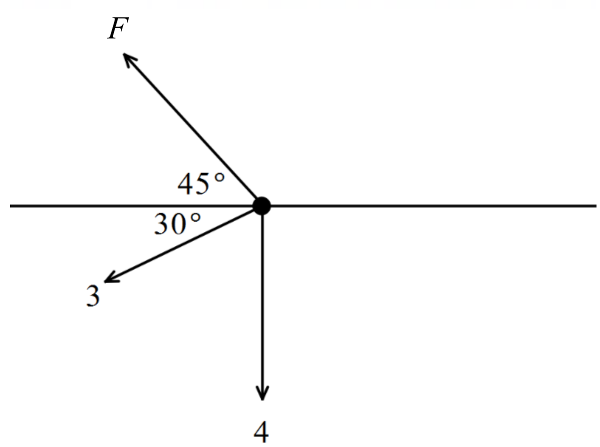
Section I continues on page 3

4 Which of the following is a primitive of $2x\left(3x^2-1\right)^4$?

A) $\dfrac{1}{15}\left(3x^2-1\right)^5$ B) $\dfrac{3}{5}\left(3x^2-1\right)^5$

C) $\dfrac{2x}{5}\left(3x^2-1\right)^5$ D) $\dfrac{2x}{15}\left(3x^2-1\right)^5$

5 The diagram below shows a mass being acted upon by a number of forces whose magnitudes are labelled. All forces are in newtons, and the system is in equilibrium.



The value of F is:

A) $\dfrac{\sqrt{2}}{2}\left(8+3\sqrt{3}\right)$ B) $\dfrac{11\sqrt{2}}{2}$

C) $\dfrac{3\sqrt{2}}{2}$ D) 7

End of Section I

Section II

55 marks
Attempt Questions 6–10
Allow about 67 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks) **Marks**

a) Given the vectors $\underset{\sim}{a} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, find:

i) $\underset{\sim}{b} - 2\underset{\sim}{a}$ **1**

ii) $|3\underset{\sim}{b}|$ **1**

iii) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

iv) the angle between the two vectors, correct to the nearest minute **2**

v) a vector that is perpendicular to $\underset{\sim}{a}$ **1**

b) Differentiate, expressing in simplest form:

i) $\sin^{-1}(4x)$ **1**

ii) $\cos^{-1}(\ln x)$ **2**

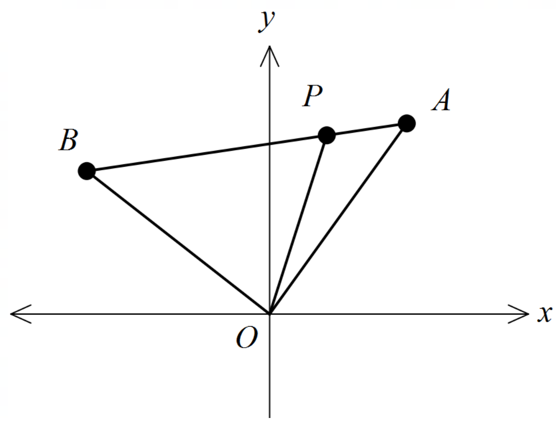
iii) $e^x \tan^{-1} x$ **2**

End of Question 6

Question 7 (11 marks) **Marks**

a) Consider two position vectors $\overrightarrow{OA} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\overrightarrow{OB} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$.

Let P be a point on the line passing through A and B such that $\overrightarrow{AP} = m\overrightarrow{AB}$.



NOT TO SCALE

i) Write an expression for $\overrightarrow{AB}$ **1**

ii) Show that the position vector $\overrightarrow{OP} = \begin{bmatrix} 3-7m \\ 4-m \end{bmatrix}$ **2**

iii) Find the value of m if P lies on the line $y = 2x$ **2**

b) Find:

i) $\int \frac{dx}{2+x^2}$ **1**

ii) $\int \frac{1}{\sqrt{9-25x^2}}\, dx$ **2**

c) Find the area bounded by the curve $y = \sin^2 x$ and the x-axis for $0 \le x \le \pi$ **3**

End of Question 7

Question 8 (11 marks) **Marks**

a) Evaluate $\int \tan^2 x \, dx$ **1**

b) Given $f(x) = \ln(\sec x + \tan x)$

(i) Show that $f'(x) = \sec x$ **2**

(ii) Hence, show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec x \, dx = \ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$ **2**

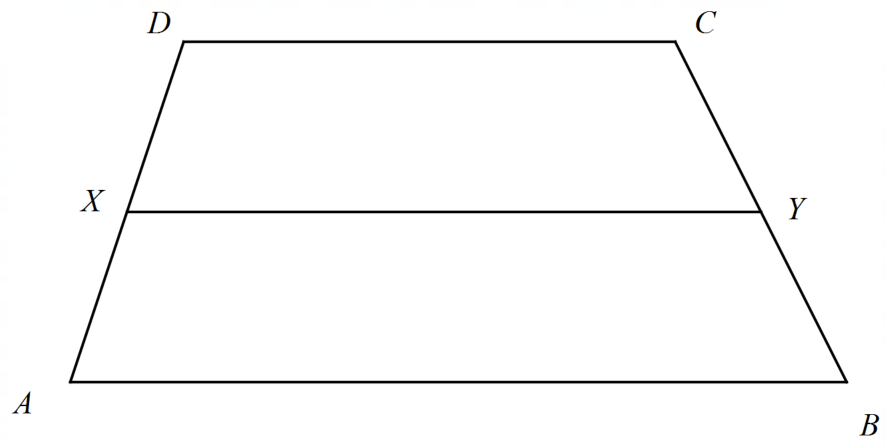
c) Evaluate $\int_{-1}^{0} x\sqrt{1+x} \, dx$ using the substitution $u = 1 + x$ **3**

d) Find the projection of $\overrightarrow{AB}$ onto $\underset{\sim}{v} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ given $A(-1,3)$ and $B(-2,-6)$. **3**

End of Question 8

Question 9 (11 marks) **Marks**

a) $ABCD$ is a trapezium with $AB \| DC$. X and Y are midpoints of AD and BC respectively.



NOT TO SCALE

i) By letting $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{DC}$, $\underset{\sim}{c} = \overrightarrow{CB}$, and $\underset{\sim}{d} = \overrightarrow{AD}$, show that $\overrightarrow{XY} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$ **3**

ii) Hence, show that $XY \parallel AB$ **2**

b) Consider the function $f(x) = \sin^{-1}\left(x^2\right)$

i) State the domain of $f(x)$ **1**

ii) Find $f'(x)$ **1**

iii) Hence, find the minimum value of $y = f(x)$ **2**

iv) Sketch the graph of $y = f(x)$, labelling all key features **2**

End of Question 9

Question 10 (11 marks) **Marks**

a) Find $\int \sin 7x \cos 4x \, dx$ **2**

b) **i)** Show that $6x - x^2 - 5 = 4 - (x-3)^2$ **1**

ii) Hence, find $\int \frac{-dx}{\sqrt{6x - x^2 - 5}}$ **2**

c) Using the substitution $x = \cos u$, find $\int_{\frac{1}{2}}^{1} \frac{1}{x^2\sqrt{1-x^2}} \, dx$ **3**

d) The sum of two unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is a unit vector (i.e. $\underset{\sim}{a} + \underset{\sim}{b}$ is also a unit vector). **3**

Prove using vector properties that

$$\left|\underset{\sim}{a} - \underset{\sim}{b}\right| = \sqrt{3}$$

End of paper

Name: ____________________

Mathematics Extension 1

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample **2 + 4 = ?** **(A) 2 (B) 6 (C) 8 (D) 9**

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here ➔ **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

Yr 12 Maths Ext 1 AT2 Solutions

Section 1

1. C
2. C
3. C
4. A
5. B

1. $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$$|\underset{\sim}{u}| = \sqrt{2^2+3^2} = \sqrt{13}$$

$$\hat{\underset{\sim}{u}} = \frac{\underset{\sim}{u}}{|\underset{\sim}{u}|} = \frac{1}{\sqrt{13}}(2\underset{\sim}{i}+3\underset{\sim}{j})$$

2. $$\frac{d}{dx}\tan^{-1}\left(\frac{x}{2}\right) = \frac{1}{1+(x/2)^2}\cdot\left(\frac{x}{2}\right)' = \frac{1}{1+x^2/4}\cdot\frac{1}{2} = \frac{4}{4+x^2}\cdot\frac{1}{2} = \frac{2}{4+x^2}$$

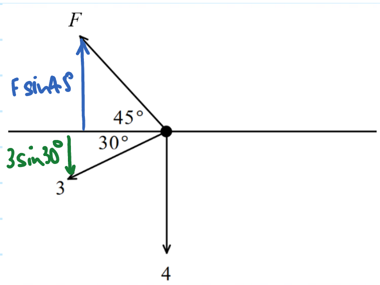
3. $$\int \cos^2 4x\,dx = \int \frac{1}{2}(1+\cos 8x)\,dx = \frac{1}{2}\left(x+\frac{\sin 8x}{8}\right)+C$$

4. $$\int 2x(3x^2-1)^4\,dx = \frac{1}{3}\int 6x(3x^2-1)^4\,dx = \frac{1}{3}\cdot\frac{(3x^2-1)^5}{5}+C = \frac{(3x^2-1)^5}{15}+C$$

5. $$F\sin 45° = 3\sin 30° + 4$$

$$\frac{F}{\sqrt{2}} = \frac{3}{2}+4$$

$$F = \frac{11\sqrt{2}}{2}$$



Section 2

Question 6

(a) $\underset{\sim}{a} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $\quad \underset{\sim}{b} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(i) $\underset{\sim}{b} - 2\underset{\sim}{a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -10 \\ -1 \end{bmatrix}$ ✓ Correct answer

(ii) $|3\underset{\sim}{b}| = 3|\underset{\sim}{b}|$

$= 3\sqrt{(-2)^2 + 3^2}$

$= 3\sqrt{13}$ or $\sqrt{117}$ ✓ Correct answer (should be simplified but do not penalise)

(iii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = 4 \times -2 + 1 \times 3$

$= -5$ ✓ Correct answer

(iv) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$-5 = \sqrt{17} \times \sqrt{13} \times \cos\theta$ ✓ Substitution into dot product formula

$\cos\theta = \dfrac{-5}{\sqrt{221}}$

$\theta = 109° 39'$ (nrst min) ✓ Correct answer

(v) We want $\underset{\sim}{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $\underset{\sim}{u} \perp \underset{\sim}{a}$

i.e. $\underset{\sim}{u} \cdot \underset{\sim}{a} = 0$

$4x + y = 0$

$y = -4x$

$\therefore$ any vector $\underset{\sim}{u} = \lambda \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ where λ is a scalar ✓ Correct answer, presumably $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(b) (i) $\dfrac{d}{dx}\left[\sin^{-1}(4x)\right] = \dfrac{1}{\sqrt{1-(4x)^2}} \cdot (4x)'$

$= \dfrac{4}{\sqrt{1-16x^2}}$ ✓ Correct answer

(ii) $\dfrac{d}{dx}\left[\cos^{-1}(\ln x)\right] = \dfrac{-1}{\sqrt{1-(\ln x)^2}} \cdot (\ln x)'$ ✓ Correct progress

$= \dfrac{-1}{x\sqrt{1-(\ln x)^2}}$ ✓ Correct answer as single fraction

(iii) $\dfrac{d}{dx}\left[e^x \tan^{-1} x\right] = (e^x)' \tan^{-1} x + e^x (\tan^{-1} x)'$

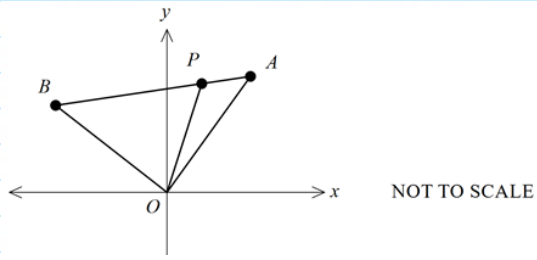
$= e^x \tan^{-1} x + e^x \dfrac{1}{1+x^2}$ ✓ Correct use of product rule

$= e^x \tan^{-1} x + \dfrac{e^x}{1+x^2}$ ✓ Correct answer (no need to factorise)

$= e^x \left(\tan^{-1} x + \dfrac{1}{1+x^2}\right)$

Question 7

(a) $\vec{OA} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\vec{OB} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$ $\vec{AP} = m\vec{AB}$



(i) $\vec{AB} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -7 \\ -1 \end{bmatrix}$ ✓ correct answer

(ii) $\vec{OP} = \vec{OA} + \vec{AP}$

$= \vec{OA} + m\vec{AB}$ ✓ Correct addition of vectors

$= \begin{bmatrix} 3 \\ 4 \end{bmatrix} + m\begin{bmatrix} -7 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 3-7m \\ 4-m \end{bmatrix}$ ✓ correct 'show'

(iii) Sub $\vec{OP}$ in $y=2x$

$4-m = 2(3-7m)$ ✓ Correct substitution & linear equation

$= 6-14m$

$13m = 2$

$\therefore m = \frac{2}{13}$ ✓ Correct answer

(b) (i) $\int \frac{dx}{2+x^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$ ✓ Correct answer

(ii) $\int \frac{1}{\sqrt{9-25x^2}}\,dx = \frac{1}{5}\int \frac{5}{\sqrt{3^2-(5x)^2}}\,dx$ ✓ Correct progress

$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$ ✓ Correct answer

(c) Area $= \int_0^{\pi} \sin^2 x\,dx$

$= \int_0^{\pi} \frac{1}{2}(1-\cos 2x)\,dx$ ✓ Correct expression for $\sin^2 x$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$ ✓ Correct integration

$= \frac{1}{2}\left(\pi - \frac{1}{2}\sin 2\pi - \left(0 - \frac{1}{2}\sin 0\right)\right)$

$= \frac{\pi}{2}$ ✓ correct answer

Question 8

(a) $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx$

$= \tan x - x + C$ ✓ correct answer

(b) (i) $f(x) = \ln(\sec x + \tan x)$

$f'(x) = \frac{1}{\sec x + \tan x} \cdot (\underbrace{\sec x}_{\downarrow} + \tan x)'$

$\frac{d}{dx}\sec x = \frac{d}{dx}(\cos x)^{-1}$

$= -1(\cos x)^{-2} \cdot -\sin x$

$= \frac{\sin x}{\cos^2 x}$

$= \sec x \tan x$

$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ ✓ correct application of chain rule

$= \frac{\sec x \, \cancel{(\tan x + \sec x)}}{\cancel{\sec x + \tan x}}$ ✓ Correct 'show' (simplify)

$= \sec x$

(ii) From (i), $\frac{d}{dx}(\ln(\sec x + \tan x)) = \sec x$

then $\int \sec x \, dx = \ln(\sec x + \tan x) + C$

Now, $\int_{\pi/6}^{\pi/3} \sec x \, dx = \left[\ln(\sec x + \tan x)\right]_{\pi/6}^{\pi/3}$

$= \ln\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) - \ln\left(\sec\frac{\pi}{6} + \tan\frac{\pi}{6}\right)$ ✓ correct substitution

$= \ln\left(2 + \sqrt{3}\right) - \ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$

$= \ln(2 + \sqrt{3}) - \ln\left(\frac{3}{\sqrt{3}}\right)$

$= \ln(2+\sqrt{3}) - \ln\sqrt{3}$

$= \ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$ ✓ Correct 'show' (simplify)

(c) $\int_{-1}^{0} x\sqrt{1+x} \, dx$

Let $u = 1+x \rightarrow x = u-1$

$du = dx$

$\begin{cases} x = 0, u = 1 \\ x = -1, u = 0 \end{cases}$

$= \int_0^1 (u-1) \cdot u^{1/2} \, du$ ✓ correct substitution

$= \int_0^1 u^{3/2} - u^{1/2} \, du$

$= \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_0^1$ ✓ Correct integration

$= \frac{2}{5}(1) - \frac{2}{3}(1) - 0$

$= \frac{-4}{15}$ ✓ Correct answer

(d) $\underset{\sim}{v} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ $\quad A(-1,3)$ & $B(-2,-6)$

$\overrightarrow{AB} = -\underset{\sim}{i} - 9\underset{\sim}{j}$ ✓ Correct $\overrightarrow{AB}$

$\text{Proj}_{\underset{\sim}{x}}\underset{\sim}{u} = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{x}}{\underset{\sim}{x} \cdot \underset{\sim}{x}} \times \underset{\sim}{x}$

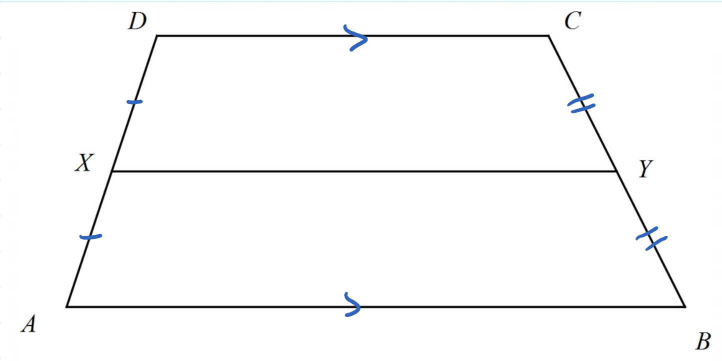
$$\text{Proj}_{\underset{\sim}{v}}\overrightarrow{AB} = \frac{(-\underset{\sim}{i} - 9\underset{\sim}{j}) \cdot (2\underset{\sim}{i} - 3\underset{\sim}{j})}{(2\underset{\sim}{i} - 3\underset{\sim}{j}) \cdot (2\underset{\sim}{i} - 3\underset{\sim}{j})} \times (2\underset{\sim}{i} - 3\underset{\sim}{j})$$

$$= \frac{-1 \times 2 + -9 \times -3}{2^2 + (-3)^2} \times (2\underset{\sim}{i} - 3\underset{\sim}{j})$$ ✓ Correct use of projection formula

$$= \frac{25}{13}(2\underset{\sim}{i} - 3\underset{\sim}{j})$$ ✓ Correct answer

Question 9

(a) (i) $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{DC}$, $\underset{\sim}{c} = \overrightarrow{CB}$, $\underset{\sim}{d} = \overrightarrow{AD}$



$$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB}$$
$$\underset{\sim}{a} = \underset{\sim}{d} + \underset{\sim}{b} + \underset{\sim}{c}$$
$$\underset{\sim}{d} + \underset{\sim}{c} = \underset{\sim}{a} - \underset{\sim}{b}$$

✓ Correct relationship between $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}, \underset{\sim}{d}$

$$\overrightarrow{AB} = \overrightarrow{AX} + \overrightarrow{XY} + \overrightarrow{YB}$$
$$= \frac{1}{2}\overrightarrow{AD} + \overrightarrow{XY} + \frac{1}{2}\overrightarrow{CB}$$
$$\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{d} + \overrightarrow{XY} + \frac{1}{2}\underset{\sim}{c}$$
$$= \frac{1}{2}(\underset{\sim}{c} + \underset{\sim}{d}) + \overrightarrow{XY}$$

✓ Second expression involving $\overrightarrow{XY}$

$$= \frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{b}) + \overrightarrow{XY}$$
$$\overrightarrow{XY} = \underset{\sim}{a} - \frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{b})$$
$$= \underset{\sim}{a} - \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$$
$$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$$

✓ Correct 'show'

(ii) Since $AB \parallel DC$, $\underset{\sim}{b} = \lambda \underset{\sim}{a}$, λ scalar

✓ Correct relationship between $\underset{\sim}{a}$ & $\underset{\sim}{b}$

$$\overrightarrow{XY} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$$
$$= \frac{1}{2}(\underset{\sim}{a} + \lambda \underset{\sim}{a})$$
$$= \frac{\lambda + 1}{2} \times \underset{\sim}{a}$$
$$= \mu \underset{\sim}{a}, \quad \mu \text{ scalar}$$

✓ Correct 'show'

$\therefore XY \parallel AB$

(b) $f(x) = \sin^{-1}(x^2)$

(i) $D: \; -1 \le x^2 \le 1$

$\therefore \; -1 \le x \le 1$ ✓ Correct answer

(ii) $f'(x) = \dfrac{1}{\sqrt{1-(x^2)^2}} \cdot (x^2)'$

$= \dfrac{2x}{\sqrt{1-x^4}}$ ✓ Correct answer

(iii) Max/min value when $f'(x) = 0$

$f'(x) = \dfrac{2x}{\sqrt{1-x^4}} = 0$

$2x = 0$

$x = 0$ ✓ Correct $x=0$

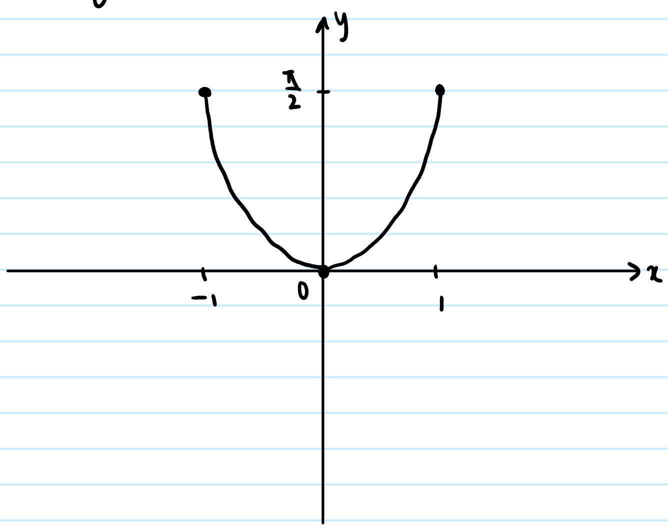
When $x=0$, $y = f(0) = \sin^{-1} 0 = 0$

x	$-1/2$	0	$1/2$
$f'(x)$	-1.03	0	1.03
	↘	→	↗

✓ Sub & test for min

$\therefore$ minimum value is 0.

(iv) $y = \sin^{-1}(x^2)$



✓ Correct min SP

✓ Correct endpoints

Note: Should be vert. tangents at $x = \pm 1$.

Question 10

(a) $\int \sin 7x \cos 4x \, dx = \int \frac{1}{2}\left(\sin(7x+4x) + \sin(7x-4x)\right) dx$

$= \frac{1}{2}\int \sin 11x + \sin 3x \, dx$ ✓ correct 'product to sum' formula

$= \frac{1}{2}\left(-\frac{\cos 11x}{11} + \frac{-\cos 3x}{3}\right) + C$ ✓ correct integration

$= -\frac{1}{2}\left(\frac{\cos 3x}{3} + \frac{\cos 11x}{11}\right) + C$

(b) (i) RHS $= 4-(x-3)^2$

$= 4-(x^2-6x+9)$

$= 4-x^2+6x-9$

$= 6x-x^2-5$

$=$ LHS ✓ Correct 'show'

(ii) $\int \frac{-dx}{\sqrt{6x-x^2-5}} = \int \frac{-1}{\sqrt{4-(x-3)^2}} dx$ ✓ Correct progress

$= -\sin^{-1}\left(\frac{x-3}{2}\right) + C$ ✓ Correct integral

$= \cos^{-1}\left(\frac{x-3}{2}\right) + C$

(c) $\int_{1/2}^{1} \frac{1}{x^2\sqrt{1-x^2}} dx$

Let $x = \cos u$

$dx = -\sin u \, du$

$\begin{cases} x=1, & u=0 \\ x=1/2, & u=\pi/3 \end{cases}$ ✓ Correct substitution

$= \int_{\pi/3}^{0} \frac{1}{\cos^2 u\sqrt{1-\cos^2 u}} \cdot -\sin u \, du$

$= \int_{0}^{\pi/3} \frac{\cancel{\sin u}}{\cos^2 u \sqrt{\cancel{\sin^2 u}}} du$

$= \int_{0}^{\pi/3} \sec^2 u \, du$ ✓ Correct simplification

$= \left[\tan u\right]_0^{\pi/3}$

$= \tan\frac{\pi}{3} - \tan 0$

$= \sqrt{3} - 0$

$= \sqrt{3}$ ✓ Correct answer

(d) RTP: $|\underset{\sim}{a}-\underset{\sim}{b}| = \sqrt{3}$ given $|\underset{\sim}{a}| = |\underset{\sim}{b}| = |\underset{\sim}{a}+\underset{\sim}{b}| = 1$

$\underset{\sim}{u}\cdot\underset{\sim}{u} = |\underset{\sim}{u}|^2$

✓ Recognition of need to use $\underset{\sim}{u}\cdot\underset{\sim}{u} = |\underset{\sim}{u}|^2$

Consider $|\underset{\sim}{a}+\underset{\sim}{b}|^2 = (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})$

$$1^2 = \underset{\sim}{a}\cdot\underset{\sim}{a} + \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b}$$

$$1 = |\underset{\sim}{a}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{b}|^2$$

$$= 1^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + 1^2$$

$$2\underset{\sim}{a}\cdot\underset{\sim}{b} = -1$$

✓ Correct expression for $2\underset{\sim}{a}\cdot\underset{\sim}{b}$

Now, $|\underset{\sim}{a}-\underset{\sim}{b}|^2 = (\underset{\sim}{a}-\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b})$

$$= \underset{\sim}{a}\cdot\underset{\sim}{a} - \underset{\sim}{a}\cdot\underset{\sim}{b} - \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b}$$

$$= |\underset{\sim}{a}|^2 - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{b}|^2$$

$$= 1^2 - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + 1^2$$

$$= 2 - (-1)$$

$$= 3$$

$\therefore |\underset{\sim}{a}-\underset{\sim}{b}| = \sqrt{3}$

✓ Correct 'show'