Caringbah High School

Year 12 2023

Mathematics Extension 1

HSC Course

Assessment Task 2

General Instructions

- Reading time – 5 minutes
- Working time – 75 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided.
- In Questions 6-10, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 45

Section I **5 marks**

Attempt Questions 1-5

Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **40 marks**

Attempt Questions 6-10

Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: _____ **Class:** _____

Marker's Use Only							
Section I	**Section II**					**Total**	
Q 1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**		
/5	/8	/8	/8	/8	/8	/45	%

Section I

5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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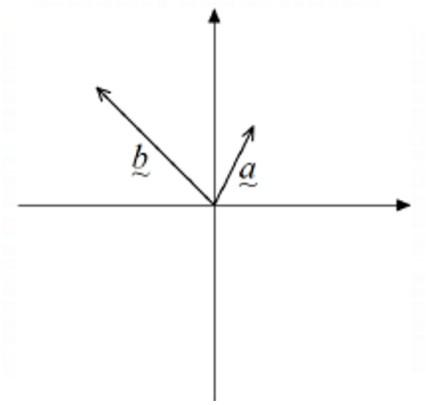
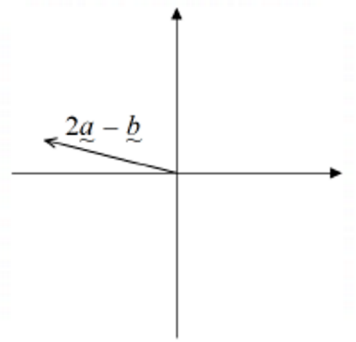
1. Vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown in the diagram below.

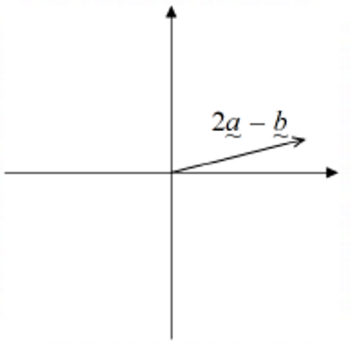


Which diagram best represents the vector $2\underset{\sim}{a} - \underset{\sim}{b}$?

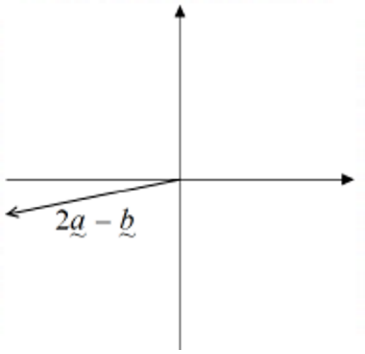
(A)



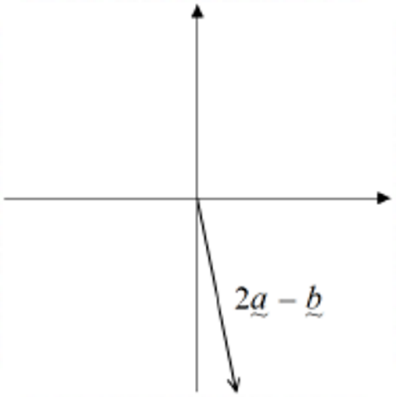
(B)



(C)



(D)



2. Given the points $A(-1,-4)$ and $B(2,5)$, which of the following represents the position vector $\overrightarrow{BA}$?

(A) $-3\underset{\sim}{i}-9\underset{\sim}{j}$

(B) $3\underset{\sim}{i}+9\underset{\sim}{j}$

(C) $-3\underset{\sim}{i}+9\underset{\sim}{j}$

(D) $3\underset{\sim}{i}-9\underset{\sim}{j}$

3. Two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel. Given that $\left|\underset{\sim}{a}\right|=78$ and $\underset{\sim}{b}=10\underset{\sim}{i}+24\underset{\sim}{j}$, which of the following could represent $\underset{\sim}{a}$?

(A) $20\underset{\sim}{i}+48\underset{\sim}{j}$

(B) $30\underset{\sim}{i}+72\underset{\sim}{j}$

(C) $48\underset{\sim}{i}+20\underset{\sim}{j}$

(D) $72\underset{\sim}{i}+30\underset{\sim}{j}$

4. Given $\underset{\sim}{a}=4\underset{\sim}{i}+4\underset{\sim}{j}$ and $\underset{\sim}{b}=2\underset{\sim}{i}-\underset{\sim}{j}$, which of the following represents $proj_{\underset{\sim}{b}}\underset{\sim}{a}$

(A) $\frac{1}{2}\underset{\sim}{i}-\frac{1}{2}\underset{\sim}{j}$

(B) $\frac{1}{16}\underset{\sim}{i}-\frac{1}{32}\underset{\sim}{j}$

(C) $2\underset{\sim}{i}-1\underset{\sim}{j}$

(D) $\frac{8}{5}\underset{\sim}{i}-\frac{4}{5}\underset{\sim}{j}$

5. The points *P, Q* and *R* lie on a straight line. The vector $\overrightarrow{PQ}=\begin{bmatrix}-2\\3\end{bmatrix}$ and the vector $\overrightarrow{PR}=2\overrightarrow{PQ}$. The coordinates of *P* are (-1,-1). What are the coordinates of *R*?

(A) (-10,10)

(B) (-3,3)

(C) (-5,5)

(D) (-3,4)

End of Section I

Section II

40 marks
Attempt Questions 6–10
Allow about 1 hour 7 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (8 marks) Use a SEPARATE writing booklet.

(a) A triangle ABC has vertices $A(2,2)$ $B(10,4)$ and $C(8,12)$.

(i) Write $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as column vectors **1**

(ii) Hence show that $\angle BAC = 45°$ **2**

(iii) Find the area of the triangle. **1**

(b) Given $\underset{\sim}{a} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ find

(i) $\left| \underset{\sim}{a} \right|$ **1**

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(c) Given the vectors $\underset{\sim}{c} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{d} = 9\underset{\sim}{i} + \underset{\sim}{j}$, **2**
Find $proj_{\underset{\sim}{c}} d$.

End of Question 6

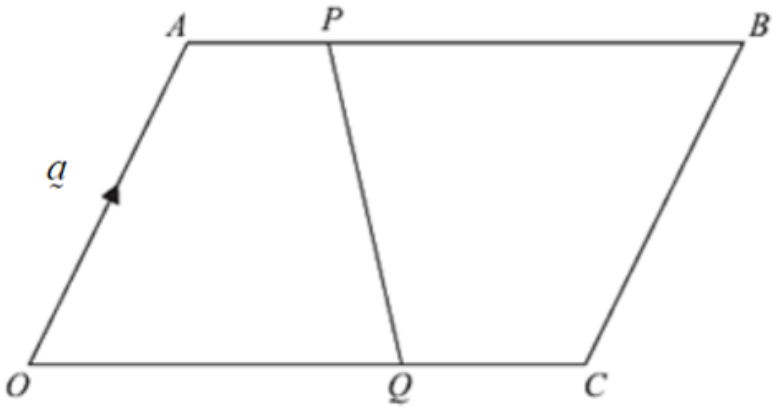
Question 7 (8 marks) Use a SEPARATE writing booklet.

(a) In the diagram below, $OABC$ is a parallelogram. $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. **3**

P is the point on $\overrightarrow{AB}$ such that $\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$.

Q is the point on $\overrightarrow{OC}$ such that $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{OC}$.

Find $\overrightarrow{PQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$, giving your answer in simplest form.

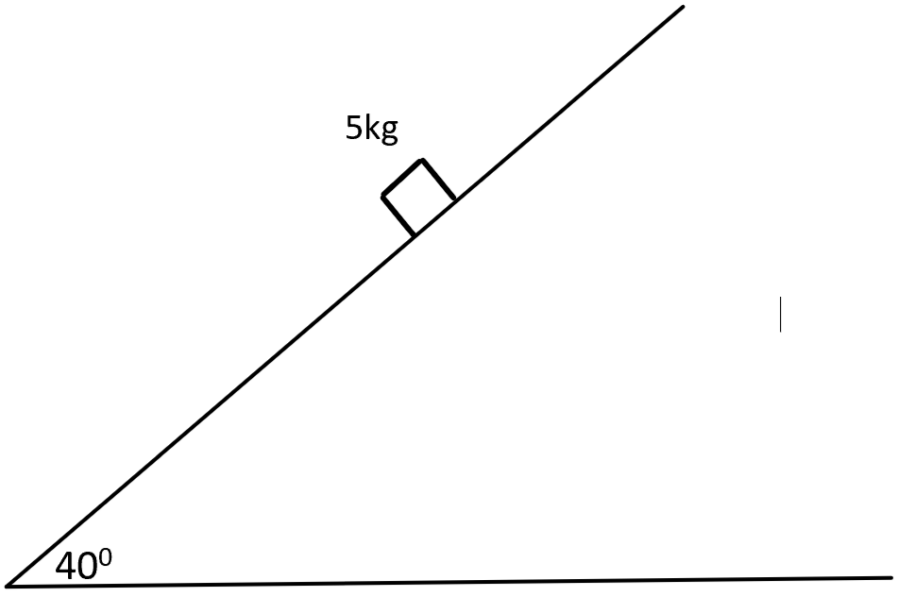


(b) The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $\left|\underset{\sim}{u}\right| = 6$ and $\left|\underset{\sim}{v}\right| = 8$. **3**

If $\underset{\sim}{u} \cdot \left(\underset{\sim}{u} + \underset{\sim}{v}\right) = 30$ determine the size of the angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ correct to the nearest minute.

Question 7 continues on page 6

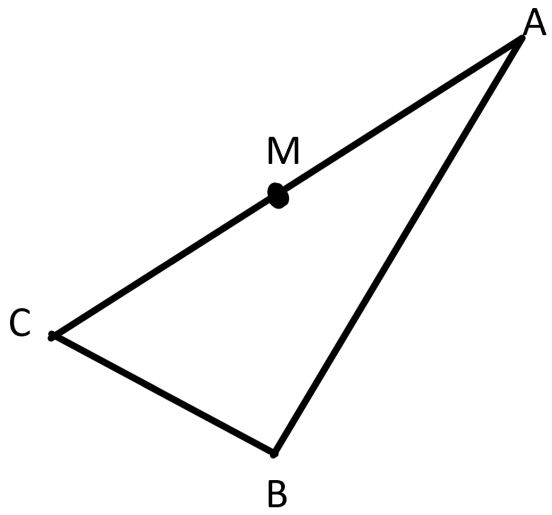
(c) A 5kg weight is stationary on an incline plane that is 40 degrees in slope. What is the force that is acting up the plane that is stopping the weight from sliding down the plane? Gravity is 9.8m/s^2. Answer correct to one decimal place. **2**



End of Question 7

Question 8 (8 marks) Use a SEPARATE writing booklet.

(a) ΔABC is a right-angled triangle with M being the midpoint of the hypotenuse AC, as shown below. Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



(i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Prove that M is equidistant from the three vertices of ΔABC. **2**

(b) The vectors $\underset{\sim}{p} = \begin{bmatrix} a \\ 3 \end{bmatrix}$ and $\underset{\sim}{q} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ are parallel.

(i) Find the value of a. **1**

(ii) The vector $\underset{\sim}{r} = \begin{bmatrix} x \\ y \end{bmatrix}$ is perpendicular to vector $\underset{\sim}{q}$ and $\left| \underset{\sim}{q} - \underset{\sim}{r} \right| = \sqrt{65}$ **3**

Find the possible values of x and y.

End of Question 8

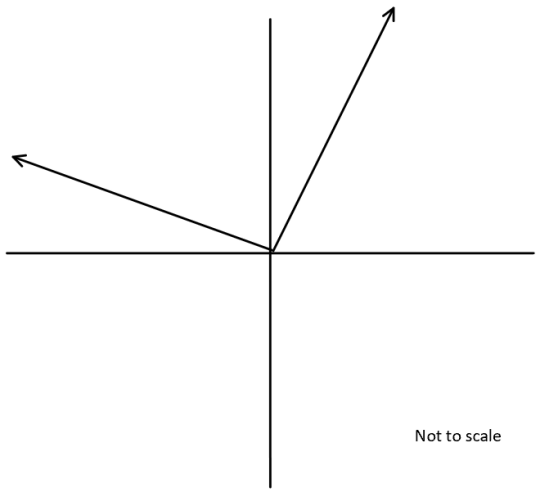
Question 9 (8 marks) Use a SEPARATE writing booklet.

(a) The position of a jet ski relative to a stationary ship is plotted on the ship's radar. One unit on the radar represents a distance of 1 metre along the sea level, and the origin represents the ship's position.
The jet ski travels at a constant speed in a straight line, and is observed at point A on a position vector of $-10\underset{\sim}{i} + 15\underset{\sim}{j}$. One second later, the jet ski is observed at point B on a position vector of $-5\underset{\sim}{i} + 20\underset{\sim}{j}$.
A crew member on the ship observes the motion of the jet ski using binoculars for five seconds starting from point A, without taking his eyes off the jet ski.

(i) Show that the position vector of the jet ski after the five seconds is $15\underset{\sim}{i} + 40\underset{\sim}{j}$. **2**

(ii) Hence find the angle through which the crew member's binoculars move through during this five seconds, correct to the nearest minute. **2**

(b) Two forces $F_1 = 400$N at N65W and $F_2 = 500$N at N30E are acting on an object. Two vectors have been drawn on the diagram.



(i) Copy this diagram into your answer booklet indicating the forces correctly and then draw the horizontal and vertical components of these forces on the diagram **1**

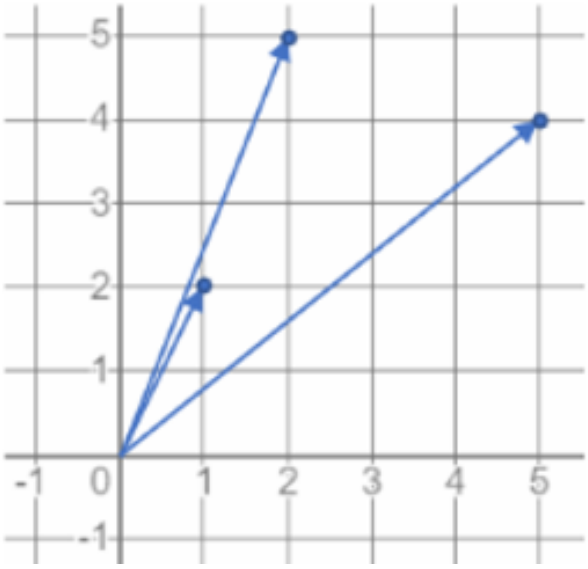
(ii) Calculate these horizontal and vertical components and then calculate magnitude of the resultant force to the nearest Newton. **2**

(iii) Calculate the direction of this resultant force to the nearest minute as a direction bearing or a true bearing. **1**
Note : do not use approximated horizontal or vertical components.

End of Question 9

Question 10 (8 marks) Use a SEPARATE writing booklet.

(a) Three position vectors $\overrightarrow{OA}, \overrightarrow{OB}$ *and* $\overrightarrow{OC}$ are $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}$ and $\begin{bmatrix}5\\4\end{bmatrix}$ respectively.



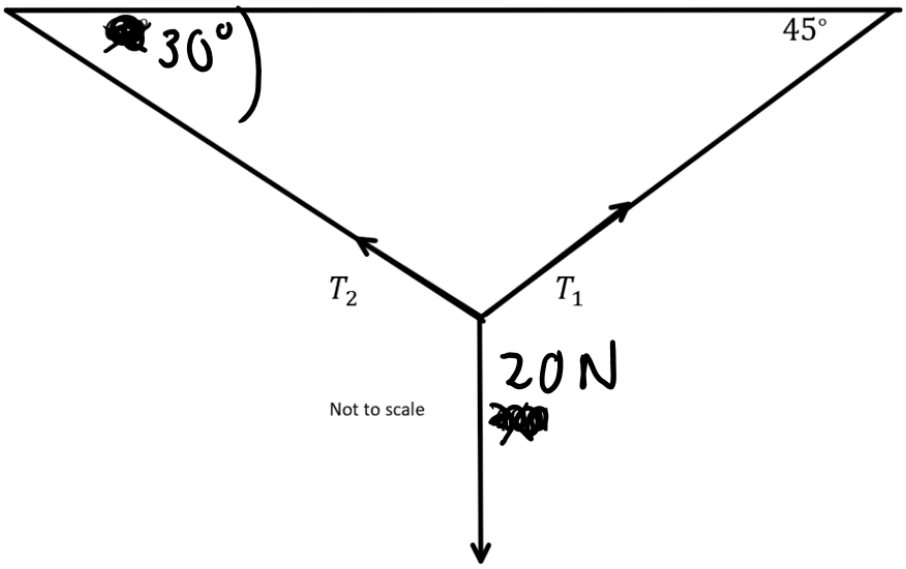
(i) Express vector $\overrightarrow{AB}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$. **1**

(ii) Given that *ABCD* is a parallelogram, use vector operations to show that the position vector $\overrightarrow{OD}$ must be $4\underset{\sim}{i} + \underset{\sim}{j}$. **1**

(iii) By using the properties of vectors, demonstrate that *ABCD* is a rectangle. **2**

Question 10 continues on page 10

(b) A particle experiences a downward force by gravity of 20*N* is suspended by two strings attached at two points in the same horizontal plane. The two strings make angles of 30° and 45°respectivley to the horizontal.



(i) Resolve the tension on the strings T_1 and T_2 into their horizontal and vertical components. **1**

(ii) Use these horizontal and vertical components of the tension to find the exact values of T_1 and T_2. **3**

End of Paper

Multiple Choice Solutions

Friday, 24 March 2023 12:51 PM

Section I

5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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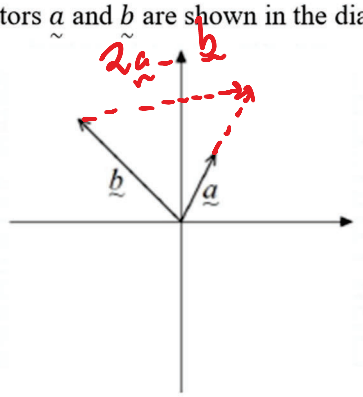
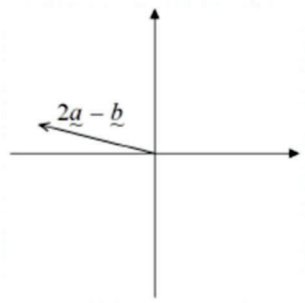
1. Vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown in the diagram below.

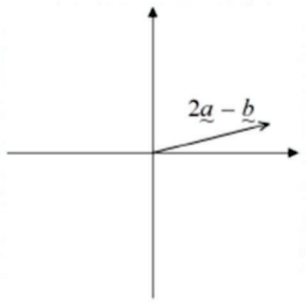


Which diagram best represents the vector $2\underset{\sim}{a} - \underset{\sim}{b}$?

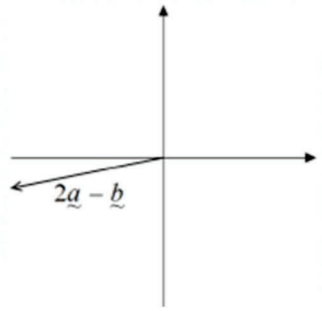
(A)



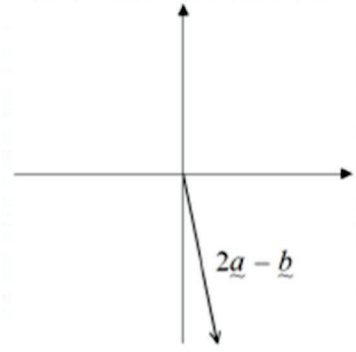
(B)



(C)



(D)



2. Given the points $A(-1,-4)$ and $B(2,5)$, which of the following represents the position vector $\overrightarrow{BA}$?

$\vec{BA} = \underset{\sim}{a} - \underset{\sim}{b} = (-3,-9)$

(A) $-3\underset{\sim}{i} - 9\underset{\sim}{j}$

(B) $3\underset{\sim}{i} + 9\underset{\sim}{j}$

(C) $-3\underset{\sim}{i} + 9\underset{\sim}{j}$

(D) $3\underset{\sim}{i} - 9\underset{\sim}{j}$

3. Two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel. Given that $\left|\underset{\sim}{a}\right| = 78$ and $\underset{\sim}{b} = 10\underset{\sim}{i} + 24\underset{\sim}{j}$, which of the following could represent $\underset{\sim}{a}$?

$\frac{24}{10} = \frac{12}{5}$ $\quad \frac{48}{20} = \frac{12}{5}$ $\quad \frac{72}{30} = \frac{12}{5}$

$\frac{20}{48}$ X $\quad \frac{30}{72}$ X

$\sqrt{30^2+72^2} = 78$

(A) $20\underset{\sim}{i} + 48\underset{\sim}{j}$

(B) $30\underset{\sim}{i} + 72\underset{\sim}{j}$

(C) $48\underset{\sim}{i} + 20\underset{\sim}{j}$

(D) $72\underset{\sim}{i} + 30\underset{\sim}{j}$

4. Given $\underset{\sim}{a} = 4\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j}$, which of the following represents $proj_{\underset{\sim}{b}}\underset{\sim}{a}$

$\frac{a \cdot b}{|b|^2}\underset{\sim}{b} = \frac{8-4}{5} = \frac{4}{5}$

$\frac{4}{5}(2\underset{\sim}{i} - \underset{\sim}{j})$ $\quad \frac{8}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

(A) $\frac{1}{2}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{j}$

(B) $\frac{1}{16}\underset{\sim}{i} - \frac{1}{32}\underset{\sim}{j}$

(C) $2\underset{\sim}{i} - 1\underset{\sim}{j}$

(D) $\frac{8}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

5. The points P, Q and R lie on a straight line. The vector $\overrightarrow{PQ} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and the vector $\overrightarrow{PR} = 2\overrightarrow{PQ}$. The coordinates of P are (-1,-1). What are the coordinates of R?

(A) (-10,18)

(B) (-3,4)

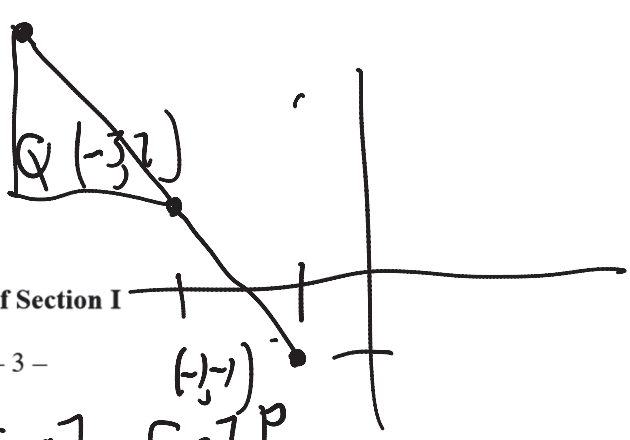
-5, 5

(A) (-10,18)

(B) (-3,4)

(C) (-5,5)

(D) (-3,4)



End of Section I

$\overrightarrow{PQ}$

$Q - P = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$R - P =$

$2\overrightarrow{PQ} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix} \quad \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

Q6 solutions

Friday, 24 March 2023 12:52 PM

Section II

40 marks
Attempt Questions 6–10
Allow about 1 hour 7 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (8 marks) Use a SEPARATE writing booklet.

(a) A triangle ABC has vertices $A(2,2)$ $B(10,4)$ and $C(8,12)$.

(i) Write $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as column vectors **1**

(ii) Hence show that $\angle BAC = 45°$ **2**

(iii) Find the area of the triangle. **1**

(b) Given $\underset{\sim}{a} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ find

(i) $\left|\underset{\sim}{a}\right|$ **1**

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(c) Given the vectors $\underset{\sim}{c} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{d} = 9\underset{\sim}{i} + \underset{\sim}{j}$, **2**
Find $proj_{\underset{\sim}{c}}\underset{\sim}{d}$.

End of Question 6

a) i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$ $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$ ✓

ii) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 48 + 20 = 68$ ✓

(ii) $\underset{\sim}{AB} \cdot \underset{\sim}{AC} = 48 + 20 = 68$ ✓

$|AB||AC| \cos\theta = \sqrt{68} \times \sqrt{136}$

$= 68\sqrt{2}$

$\therefore \cos\theta = \frac{68}{68\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\therefore \theta = 45^0$ ✓

(iii) $\frac{1}{2} \times a \times b \sin C = \frac{1}{2} \sqrt{68} \times \sqrt{136} \times \sin 45$

$= 34\sqrt{2}/\sqrt{2} = 34$ ✓

(b) Given $\underset{\sim}{a} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ find

(i) $|\underset{\sim}{a}|$ **1**

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(c) Given the vectors $\underset{\sim}{c} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{d} = 9\underset{\sim}{i} + \underset{\sim}{j}$, **2**
Find $proj_{\underset{\sim}{c}} \underset{\sim}{d}$.

b) (i) $|\underset{\sim}{a}| = \sqrt{29}$ ✓

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = -10 + 8 = -2$ ✓

c) $proj_{\underset{\sim}{c}} \underset{\sim}{d} = \frac{\underset{\sim}{c} \cdot \underset{\sim}{d}}{|c|^2} \underset{\sim}{c} = \frac{18+3}{13} \left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$ ✓

$= \frac{21}{13} \left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

$$= \frac{42\hat{i}}{13} + \frac{63\hat{j}}{13}$$

Q7 solutions

Friday, 24 March 2023 12:53 PM

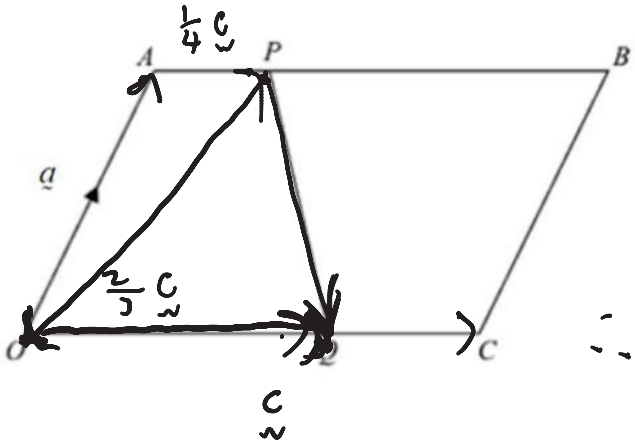
Question 7 (8 marks) Use a SEPARATE writing booklet.

(a) In the diagram below, $OABC$ is a parallelogram. $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. **3**

P is the point on AB such that $AP = \frac{1}{4}AB$.

Q is the point on OC such that $OQ = \frac{2}{3}OC$.

Find $\overrightarrow{PQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$, giving your answer in simplest form.



$\frac{1}{4}\underset{\sim}{c}$ $\underset{\sim}{a}$ $\frac{2}{3}\underset{\sim}{c}$ $\underset{\sim}{c}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ ✓

$\overrightarrow{OP} = \underset{\sim}{a} + \frac{1}{4}\underset{\sim}{c}$

$\therefore \overrightarrow{PQ} = \frac{2}{3}\underset{\sim}{c} - \left(\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{c}\right)$ ✓

$= \frac{5}{12}\underset{\sim}{c} - \underset{\sim}{a}$ ✓

(b) The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $\left|\underset{\sim}{u}\right| = 6$ and $\left|\underset{\sim}{v}\right| = 8$. **3**

If $\underset{\sim}{u} \cdot \left(\underset{\sim}{u} + \underset{\sim}{v}\right) = 30$ determine the size of the angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$. to the nearest minute

$\underset{\sim}{u} \cdot \underset{\sim}{u} = |\underset{\sim}{u}|^2 = 36$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 6 \times 8 \cos\theta$

$= 48\cos\theta$ ✓

$36 + 48\cos\theta = 30$ ✓

$48\cos\theta = 6$

Question 7 continues on page 6

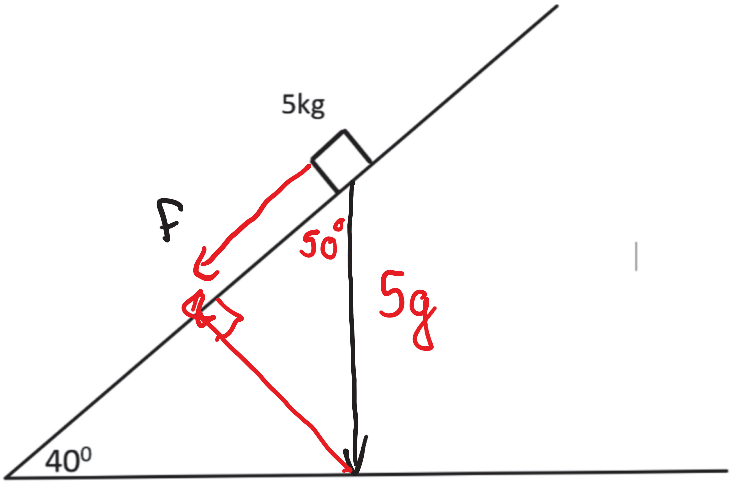
$48\cos\theta = 6$

$\cos\theta = \frac{1}{8}$ 1

$\theta = 82^0 49'$

(c) A 5kg weight is stationary on an incline plane that is 40 degrees in slope. What is the force that is acting up the plane that is stopping the weight from sliding down the plane? Gravity is 9.8m/s^2. **2**

(Answer correct to one decimal place)



End of Question 7

$F = 5g\cos 50$ 1

$= 49\cos 50\ N$

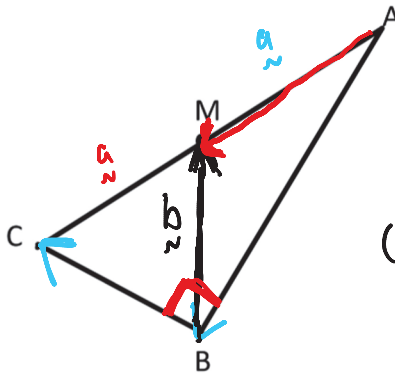
$= 31.5\ N$ 1

Q8 Solutions

Friday, 24 March 2023 12:54 PM

Question 8 (8 marks) Use a SEPARATE writing booklet.

(a) ΔABC is a right-angled triangle with M being the midpoint of the hypotenuse AC, as shown below. Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



(i) $\overrightarrow{AB} = \underset{\sim}{a} - \underset{\sim}{b}$ ✓1

$\overrightarrow{BC} = \underset{\sim}{a} + \underset{\sim}{b}$ ✓1

(ii) $\overrightarrow{AB} \cdot \overrightarrow{BC} = (\underset{\sim}{a} - \underset{\sim}{b})(\underset{\sim}{a} + \underset{\sim}{b}) = 0$ and $\overrightarrow{AB} \perp \overrightarrow{BC}$ (right angle triangle given)

$= |a|^2 - \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{a}\cdot\underset{\sim}{b} - |b|^2$ ✓1

$\therefore |a| = |b|$

$\therefore$ M is equidistant from A, B, C ✓1

(i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Prove that M is equidistant from the three vertices of ΔABC. **2**

(b) The vectors $\underset{\sim}{p} = \begin{bmatrix} a \\ 3 \end{bmatrix}$ and $\underset{\sim}{q} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ are parallel.

(i) Find the value of a. **1**

(ii) The vector $\underset{\sim}{r} = \begin{bmatrix} x \\ y \end{bmatrix}$ is perpendicular to vector $\underset{\sim}{q}$ and $\left|\underset{\sim}{q} - \underset{\sim}{r}\right| = \sqrt{65}$ **3**

Find the possible values of x and y.

End of Question 8

b)

i) $\underset{\sim}{p} \begin{bmatrix} a \\ 3 \end{bmatrix}$ $\underset{\sim}{q} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\frac{3}{a} = \frac{4}{2} \therefore a = \frac{3}{2}$ ✓1

ii) $\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 0$ $(x-2)^2 + (y-4)^2 = 65$

$(-2x-2)^2 + (4-y)^2 = 65$

$\begin{bmatrix} y \\ 7 \end{bmatrix}$

$2x + 4y = 0$

$x = -2y$ ✓

$(6, -3)$ $(-6, 3)$ ✓

$(-2y-2)^2 + (y-4)^2 = 65$

$4y^2 + 8y + 4 + y^2 - 8y + 16 = 65$

$5y^2 = 45$ ✓

$y^2 = 9$

$y = \pm 3$ $x = \mp 6$

Q9 Solutions

Friday, 24 March 2023 12:55 PM

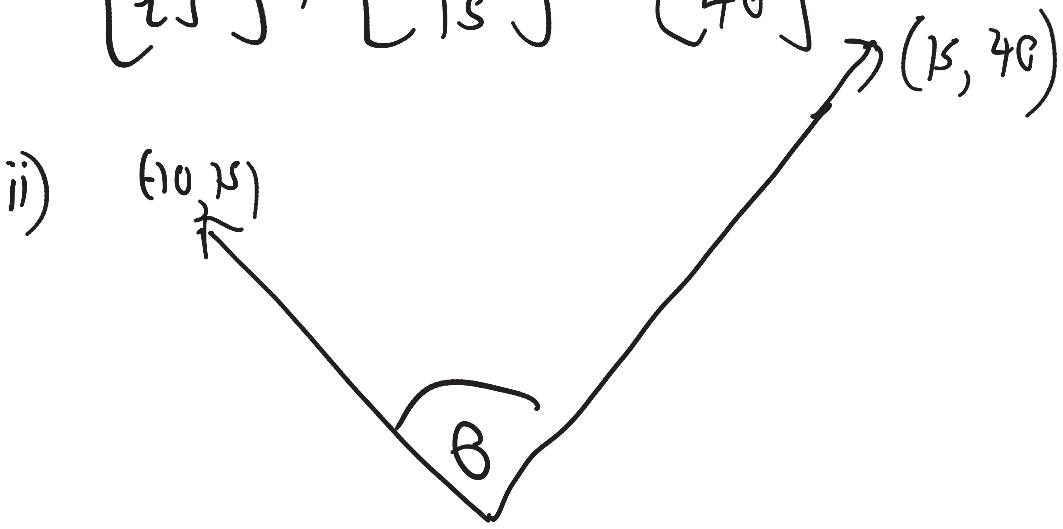
(a)	The position of a jet ski relative to a stationary ship is plotted on the ship's radar. One unit on the radar represents a distance of 1 metre along the sea level, and the origin represents the ship's position. The jet ski travels at a constant speed in a straight line, and is observed at point A on a position vector of $-10\underset{\sim}{i} + 15\underset{\sim}{j}$. One second later, the jet ski is observed at point B on a position vector of $-5\underset{\sim}{i} + 20\underset{\sim}{j}$. A crew member on the ship observes the motion of the jet ski using binoculars for five seconds starting from point A, without taking his eyes off the jet ski.		
	(i)	Show that the position vector of the jet ski after the five seconds is $15\underset{\sim}{i} + 40\underset{\sim}{j}$.	**2**
	(ii)	Hence find the angle through which the crew member's binoculars move through during this five seconds, correct to the nearest minute.	**2**

(i) $\begin{bmatrix} -10 \\ 15 \end{bmatrix}$ goes to $\begin{bmatrix} -5 \\ 20 \end{bmatrix}$ ✓

So speed is $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ in 5 secs it goes $\begin{bmatrix} 25 \\ 25 \end{bmatrix}$

$\begin{bmatrix} 25 \\ 25 \end{bmatrix} + \begin{bmatrix} -10 \\ 15 \end{bmatrix} = \begin{bmatrix} 15 \\ 40 \end{bmatrix}$ ✓

ii)



$$\underset{\sim}{a}\cdot\underset{\sim}{b} = |a||b|\cos\theta = -150 + 600$$
$$= 450$$

$$|a| = \sqrt{325} \qquad |b| = \sqrt{1825}$$
$$25\times13 \qquad 25\times73$$

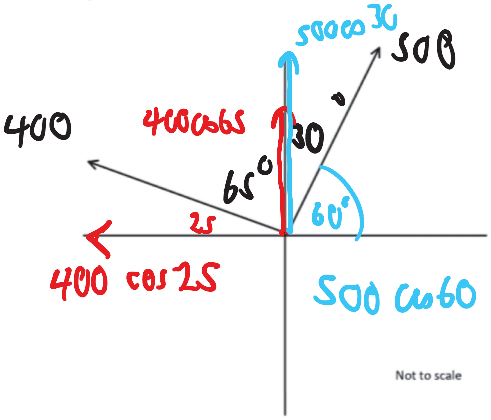
$$\therefore \quad \cos\theta = \frac{450}{\sqrt{325}\sqrt{1825}} \quad \checkmark$$

$$= \frac{45}{5\sqrt{949}}$$

$$= \frac{9}{\sqrt{949}}$$

$$\theta = 73^\circ 1' \quad \checkmark$$

(b) Two forces $F_1 = 400\text{N}$ at N65W and $F_2 = 500\text{N}$ at N30E are acting on an object. Two vectors have been drawn on the diagram.



(i)	Copy this diagram into your answer booklet indicating the forces correctly and then draw the horizontal and vertical components of these forces on the diagram	**1**
(ii)	Calculate these horizontal and vertical components and then calculate magnitude of the resultant force. to the nearest Newton.	**2**
(iii)	Calculate the direction of this resultant force as a direction or compass bearing.	**1**

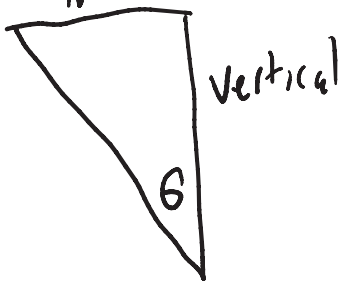
Horizontal

ii) $-400\cos 25 + 500\cos 60 = -112.523$

Vertical: $400\cos 65 + 500\cos 30 = 602.06$

Resultant $= \sqrt{(-112.523)^2 + (602.06)^2} = 612\text{ N}$

iii) Direction



$\tan\theta = \dfrac{\text{horizontal}}{\text{vertical}}$

$= \dfrac{500\cos 60 - 400\cos 25}{400\cos 65 + 500\cos 30}$

$\theta = \tan^{-1}(\quad)$

$= 10^\circ 35'$

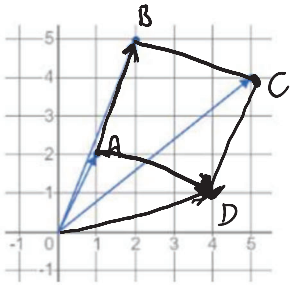
N $10^\circ 35'$ W or $349^\circ 25'$ T.

Q10 Solutions

Friday, 24 March 2023 12:55 PM

Question 10 (8 marks) Use a SEPARATE writing booklet.

(a) Three position vectors $\overrightarrow{OA}, \overrightarrow{OB}$ *and* $\overrightarrow{OC}$ are $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}$ and $\begin{bmatrix}5\\4\end{bmatrix}$ respectively.



(i) Express vector $\overrightarrow{AB}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$. **1**

(ii) Given that $ABCD$ is a parallelogram, use vector operations to show that the position vector $\overrightarrow{OD}$ must be $4\underset{\sim}{i} + \underset{\sim}{j}$. **1**

(iii) By using the properties of vectors, demonstrate that $ABCD$ is a rectangle. **2**

Question 10 continues on page 10

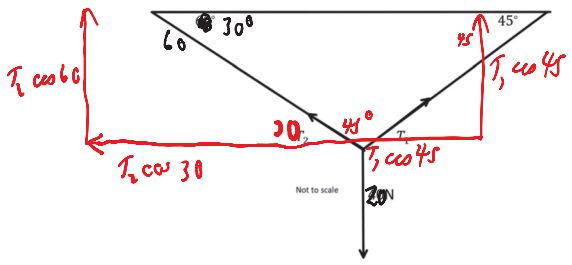
$\underset{\sim}{a} = \begin{bmatrix}1\\2\end{bmatrix}$ $\underset{\sim}{b} = \begin{bmatrix}2\\5\end{bmatrix}$ $\underset{\sim}{c} = \begin{pmatrix}5\\4\end{pmatrix}$

i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{bmatrix}2\\5\end{bmatrix} - \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix} = \underset{\sim}{i} + 3\underset{\sim}{j}$ ✓

ii) $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{AD} = \begin{bmatrix}5\\4\end{bmatrix} - \begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix} = 4\underset{\sim}{i} + \underset{\sim}{j}$ ✓

iii) $\overrightarrow{AB} \cdot \overrightarrow{AD} = \begin{bmatrix}1\\3\end{bmatrix} \cdot \begin{bmatrix}3\\-1\end{bmatrix} = 3 - 3 = 0 \therefore AB \perp AD$ so $ABCD$ is a rectangle ✓ ✓

(b) A particle experiences a downward force by gravity of $20N$ is suspended by two strings attached at two points in the same horizontal plane. The two strings make angles of 30° and 45°respectivley to the horizontal.



✓

(i) Resolve the tension on the strings T_1 and T_2 into their horizontal and vertical components. **1**

See Diagram

(ii) Use these horizontal and vertical components of the tension to find the exact values of T_1 and T_2. **3**

(i) Resolve the tension on the strings T_1 and T_2 into their horizontal and vertical components. **1**

See Diagram

(ii) Use these horizontal and vertical components of the tension to find the exact values of T_1 and T_2. **3**

End of Paper

i) Horizontal:

$T_2\cos 30 = T_1\cos 45$

$\frac{\sqrt{3}}{2}T_2 = \frac{T_1}{\sqrt{2}}$

$T_1 = \frac{\sqrt{6}}{2}T_2$ ✓

Vertical

$T_2\cos 60 + T_1\cos 45 = 20$

$\frac{T_2}{2} + \frac{\sqrt{6}}{2}\frac{T_2}{\sqrt{2}} = 20$

$T_2\left(\frac{1+\sqrt{3}}{2}\right) = 20$

$\therefore T_2 = \frac{40}{1+\sqrt{3}}$

$= \frac{40\sqrt{3}-40}{2}$

$= (20\sqrt{3}-20)\,N$ ✓

$T_1 = \frac{\sqrt{6}}{2}\left(20\sqrt{3}-20\right)$

$= 10\sqrt{18}-10\sqrt{6}$

$T_1 = (30\sqrt{2}-10\sqrt{6})\,N$ ✓