Caringbah High School

Year 12 2024

Mathematics Extension 1

HSC Course

Assessment Task 2

General Instructions

- Reading time – 5 minutes
- Working time – 75 minutes
- Write using a black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 6-10, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 45

Section I **5 marks**

Attempt Questions 1-5

Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **40 marks**

Attempt Questions 6-10

Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: _____ **Class:** _____

Marker's Use Only							
Section I	**Section II**					**Total**	
Q 1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**		
/5	/8	/9	/8	/7	/8	/45	%

Section I

5 marks
Attempt Questions 1-5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

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1 Which of the following vectors is parallel to $\underset{\sim}{u} = \underset{\sim}{i} - 2\underset{\sim}{j}$?

(A) $\underset{\sim}{i} + 2\underset{\sim}{j}$

(B) $-\underset{\sim}{i} - 2\underset{\sim}{j}$

(C) $-3\underset{\sim}{i} + 6\underset{\sim}{j}$

(D) $5\underset{\sim}{i} + 10\underset{\sim}{j}$

2 Find $|\underset{\sim}{v}|$ for the vector $\underset{\sim}{v} = -\underset{\sim}{i} - 6\underset{\sim}{j}$.

(A) 7

(B) $\sqrt{7}$

(C) 37

(D) $\sqrt{37}$

3 Which of the following vectors has a magnitude of 5?

(A) $\frac{\sqrt{2}}{10}\left(-5\underset{\sim}{i} + 5\underset{\sim}{j}\right)$

(B) $5\left(-5\underset{\sim}{i} + 5\underset{\sim}{j}\right)$

(C) $\frac{\sqrt{2}}{2}\left(-5\underset{\sim}{i} + 5\underset{\sim}{j}\right)$

(D) $\frac{5\sqrt{2}}{2}\left(-5\underset{\sim}{i} + 5\underset{\sim}{j}\right)$

4 The vectors $\underset{\sim}{a} = x\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} + y\underset{\sim}{j}$ are perpendicular.
What are the possible values of x and y?

(A) $x = 1$ and $y = 3$

(B) $x = 1$ and $y = -3$

(C) $x = -2$ and $y = -6$

(D) $x = 2$ and $y = 6$

5 A ship leaves a port *O* and sails on a bearing of N30°E for 100 km to a point *P*.
If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the east and north directions respectively, the position vector $\overrightarrow{OP}$ is equal to:

(A) $50\sqrt{3}\underset{\sim}{i} + 50\underset{\sim}{j}$

(B) $70\underset{\sim}{i} + 70\underset{\sim}{j}$

(C) $30\underset{\sim}{i} + 70\underset{\sim}{j}$

(D) $50\underset{\sim}{i} + 50\sqrt{3}\underset{\sim}{j}$

End of Section I

Section II

40 marks
Attempt Questions 6–10
Allow about 65 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (8 marks) Use a SEPARATE writing booklet.

(a) If $\underset{\sim}{u} = 3\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j}$, find:

(i) $\underset{\sim}{u} - \underset{\sim}{v}$ **1**

(ii) $(\underset{\sim}{u} - \underset{\sim}{v}) \cdot \underset{\sim}{v}$ **1**

(iii) Hence, what can be said about the vectors $\underset{\sim}{v}$ and $\underset{\sim}{u} - \underset{\sim}{v}$? **1**

(b) The position vectors of points A and B are $\overrightarrow{OA} = 8\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\overrightarrow{OB} = -5\underset{\sim}{i} - 6\underset{\sim}{j}$.

(i) Find $\overrightarrow{AB}$ **1**

(ii) Hence find $\left|\overrightarrow{AB} - \overrightarrow{OA}\right|$ **2**

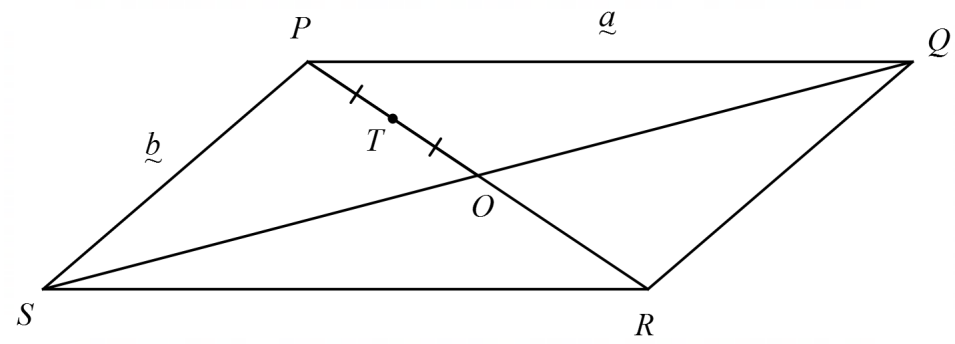
(c) Find the unit vector $\hat{\underset{\sim}{v}}$ in the direction of $\underset{\sim}{v} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$. **2**

End of Question 6

Question 7 (9 marks) Use a SEPARATE writing booklet.

(a) By substituting $u = 1 + x^2$, find $\int 6x\sqrt{1+x^2}\,dx$. **2**

(b) $PQRS$ is a parallelogram with $\overrightarrow{PQ} = \underset{\sim}{a}$ and $\overrightarrow{PS} = \underset{\sim}{b}$. The diagonals PR and SQ intersect at the point O. The point T is the midpoint of OP.



Find $\overrightarrow{ST}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(c) By substituting $u = x + e$, find $\int \dfrac{x}{(x+e)^2}\,dx$ **2**

(d) The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $|\underset{\sim}{u}| = 3$ and $|\underset{\sim}{v}| = 7$. **3**

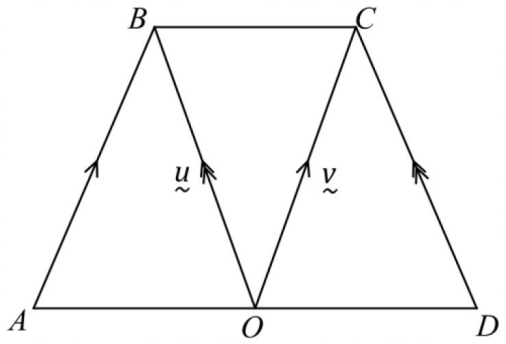
If $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v}) = 25$, determine the size of the acute angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, correct to the nearest degree.

End of Question 7

Question 8 (8 marks) Use a SEPARATE writing booklet.

(a) In the following diagram $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$ and $\overrightarrow{DC}$ is parallel to $\overrightarrow{OB}$.

$\overrightarrow{OB} = \underset{\sim}{u}$, $\overrightarrow{OC} = \underset{\sim}{v}$ and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{OB}\right| = \left|\overrightarrow{OC}\right| = \left|\overrightarrow{DC}\right|$.



Express $\overrightarrow{DB}$ in terms of $\underset{\sim}{u}$ and $\underset{\sim}{v}$. **2**

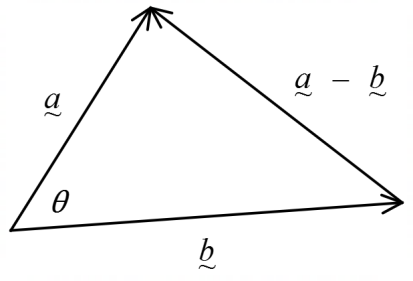
(b) For $\underset{\sim}{a} = -3\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 5\underset{\sim}{j}$, find:

(i) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(ii) the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ **2**

(c) In the following triangle, use vectors to prove the Cosine Rule, **3**

$$\left|\underset{\sim}{a} - \underset{\sim}{b}\right|^2 = \left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2 - 2\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$$



End of Question 8

Question 9 (7 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\int_1^{\sqrt{5}} \frac{x}{\sqrt{1+3x^2}}\,dx$ using the substitution $u = 1+3x^2$ **3**

(b) Consider the points $P(a,2a)$, $Q(-a,5a)$, $R(3a,4a)$ and $S(9a,12a)$, where a is a positive real number.

(i) Express $\overrightarrow{PQ}$ in component form. **1**

(ii) Given the length of the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{RS}$ is 6, find the value of a. **3**

End of Question 9

Question 10 (8 marks) Use a SEPARATE writing booklet.

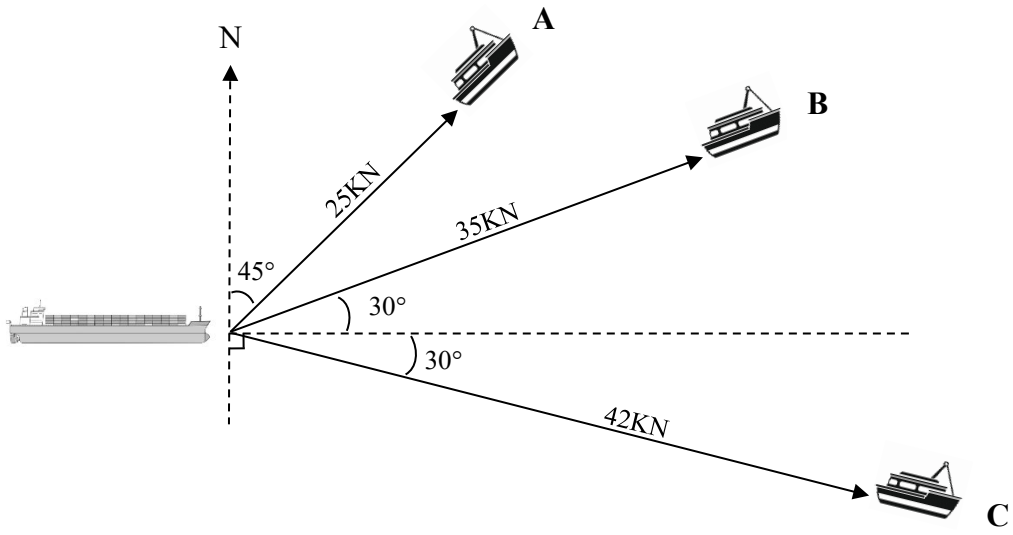
(a) Evaluate $\int_3^8 \frac{x}{\sqrt{1+x}}\,dx$ using the substitution $x = u - 1$ **3**

(b) Three tugboats are pulling a much larger container ship out of port as represented in the diagram below.

Tugboat A is pulling at a force of 25KN,

Tugboat B is pulling at a force of 35KN and

Tugboat C is pulling at a force of 42KN.



(i) Show that the resultant force can be represented by **3**

$$\underset{\sim}{F} = \frac{1}{2}\left[\left(25\sqrt{2} + 77\sqrt{3}\right)\underset{\sim}{i} + \left(25\sqrt{2} - 7\right)\underset{\sim}{j}\right]$$

(ii) Calculate the magnitude and direction of $\underset{\sim}{F}$, correct to the nearest KN and degree respectively. **2**

End of Exam

Name: _____

Mathematics Extension 1

Section I – Multiple Choice

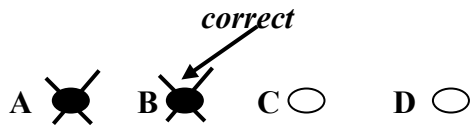
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample **2 + 4 = ?** **(A) 2 (B) 6 (C) 8 (D) 9**

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



Start ➔ Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

2024 Year 12 Extension 1 AT2 Solutions

Section I

Q1) C

Q2) D

$$|\underset{\sim}{v}| = \sqrt{(-1)^2 + 6^2}$$
$$= \sqrt{37}$$

Q3) C

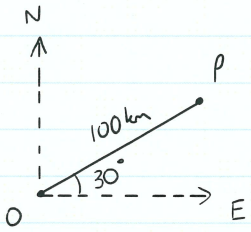
$$\frac{\sqrt{2}}{2} \times \sqrt{(-5)^2 + 5^2}$$
$$= \frac{\sqrt{2}}{2} \times 5\sqrt{2}$$
$$= 5$$

Q4) B

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$$
$$-6x - 2y = 0$$
$$6x = -2y$$

$\therefore$ $x = 1$ and $y = -3$

Q5) A



$$\sqrt{(50\sqrt{3})^2 + 50^2}$$
$$= 100$$

$\overrightarrow{OP} = 50\sqrt{3}\underset{\sim}{i} + 50\underset{\sim}{j}$ since the horizontal component is clearly longer than the vertical component in our case.

Section II

Q6 a) i) $\underset{\sim}{u} - \underset{\sim}{v} = 3\underset{\sim}{i} - \underset{\sim}{j} - (\underset{\sim}{i} - 2\underset{\sim}{j})$

$= 3\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{i} + 2\underset{\sim}{j}$

$= 2\underset{\sim}{i} + \underset{\sim}{j}$

ii) $(\underset{\sim}{u} - \underset{\sim}{v}) \cdot \underset{\sim}{v} = (2\underset{\sim}{i} + \underset{\sim}{j}) \cdot (\underset{\sim}{i} - 2\underset{\sim}{j})$

$= 2 \times 1 + 1 \times -2$

$= 2 - 2$

$= 0$

iii) They are perpendicular.

b) i) $\overrightarrow{OA} = 8\underset{\sim}{i} - 3\underset{\sim}{j}$

$\overrightarrow{OB} = -5\underset{\sim}{i} - 6\underset{\sim}{j}$

$\overrightarrow{AB} = (-5-8)\underset{\sim}{i} + (-6--3)\underset{\sim}{j}$

$= -13\underset{\sim}{i} - 3\underset{\sim}{j}$

ii) $|\overrightarrow{AB} - \overrightarrow{OA}|$

$= |-13\underset{\sim}{i} - 3\underset{\sim}{j} - (8\underset{\sim}{i} - 3\underset{\sim}{j})|$

$= |-13\underset{\sim}{i} - 3\underset{\sim}{j} - 8\underset{\sim}{i} + 3\underset{\sim}{j}|$

$= |-21\underset{\sim}{i}|$ (1)

$= \sqrt{21^2}$

$= 21$ (1)

c) $|\underset{\sim}{v}| = \sqrt{3^2 + (-4)^2}$

$= 5$ (1)

$\hat{\underset{\sim}{v}} = \frac{1}{5} \times (3\underset{\sim}{i} - 4\underset{\sim}{j})$ (1)

$\hat{\underset{\sim}{v}} = \frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

Q7) a) $u = 1 + x^2$ $\quad x^2 = u - 1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$3\int u^{\frac{1}{2}}\,du$ ① Substituting

$= 3 \times \frac{2u^{\frac{3}{2}}}{3} + C$

$= 2u^{\frac{3}{2}} + C$

$= 2(1+x^2)^{\frac{3}{2}} + C$ ①

b) $\overrightarrow{ST} = \overrightarrow{SP} + \overrightarrow{PT}$

$= -\underset{\sim}{b} + \overrightarrow{PT}$

$\overrightarrow{PT} = \frac{1}{4}(\overrightarrow{PS} + \overrightarrow{SR})$

$= \frac{1}{4}(\underset{\sim}{b} + \underset{\sim}{a})$ ①

$\therefore \quad \overrightarrow{ST} = -\underset{\sim}{b} + \frac{1}{4}(\underset{\sim}{b} + \underset{\sim}{a})$

$= \frac{1}{4}\underset{\sim}{a} - \frac{3}{4}\underset{\sim}{b}$ ①

c) $u = x + e$ $\quad x = u - e$

$\frac{du}{dx} = 1$

$du = dx$

$\int \frac{u-e}{u^2}\,du$ ①

$\int \frac{1}{u} - eu^{-2}\,du$

$= \ln u + \frac{e}{u} + C$

$= \ln|x+e| + \frac{e}{x+e} + C$ ①

d) $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v}) = 25$

$\underset{\sim}{u} \cdot \underset{\sim}{u} + \underset{\sim}{u} \cdot \underset{\sim}{v} = 25$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 25 - |\underset{\sim}{u}|^2$ since $\underset{\sim}{u} \cdot \underset{\sim}{u} = |\underset{\sim}{u}|^2$ (1)

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 25 - 3^2$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = 16$ (1)

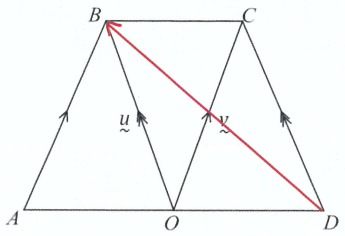
$\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}$

$\therefore \cos\theta = \dfrac{16}{3 \times 7}$

$\theta = \cos^{-1}\left(\dfrac{16}{21}\right)$

$\theta = 40°$ (1)

(Q8) a)



$\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB}$

$= \underset{\sim}{u} - \underset{\sim}{v} + \underset{\sim}{u}$

$= 2\underset{\sim}{u} - \underset{\sim}{v}$ (1)

$\overrightarrow{DO} = \overrightarrow{DC} + \overrightarrow{CO}$

$= \underset{\sim}{u} - \underset{\sim}{v}$ (1)

b) i) $\underset{\sim}{a} = -3\underset{\sim}{i} - 2\underset{\sim}{j}$

$\underset{\sim}{b} = 4\underset{\sim}{i} + 5\underset{\sim}{j}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = -3 \times 4 + -2 \times 5$

$= -12 - 10$

$= -22$

ii) $\text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$

$= \dfrac{-22}{4^2 + 5^2} (4\underset{\sim}{i} + 5\underset{\sim}{j})$ (1)

$= \dfrac{-22}{41} (4\underset{\sim}{i} + 5\underset{\sim}{j})$ (1)

c) LHS $= |\underset{\sim}{a} - \underset{\sim}{b}|^2$

$= (\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b})$ (1)

$= \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b}$ (1)

$= |\underset{\sim}{a}|^2 - 2\underset{\sim}{a} \cdot \underset{\sim}{b} + |\underset{\sim}{b}|^2$

$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - 2|\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$ (1)

$= \text{RHS}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

Q9) a) $u = 1 + 3x^2$

$u = 1 + 3 \times 1^2 = 4$ $\quad u = 1 + 3 \times (\sqrt{5})^2 = 16$

$\frac{du}{dx} = 6x$

$x\,dx = \frac{du}{6}$

$\frac{1}{6}\int_4^{16} u^{-\frac{1}{2}}\,du$ ①

$= \frac{1}{6}\left[2u^{\frac{1}{2}}\right]_4^{16}$ ①

$= \frac{1}{3}\left(\sqrt{16} - \sqrt{4}\right)$

$= \frac{1}{3} \times (4 - 2)$

$= \frac{2}{3}$ ①

b) i) $\overrightarrow{PQ} = (-a-a)\underset{\sim}{i} + (5a-2a)\underset{\sim}{j}$

$= -2a\underset{\sim}{i} + 3a\underset{\sim}{j}$

ii) $\overrightarrow{PQ} = -2a\underset{\sim}{i} + 3a\underset{\sim}{j}$

$\overrightarrow{RS} = 6a\underset{\sim}{i} + 8a\underset{\sim}{j}$

$\overrightarrow{PQ} \cdot \overrightarrow{RS} = -2a \times 6a + 3a \times 8a$

$= -12a^2 + 24a^2$

$= 12a^2$ (1)

$|\overrightarrow{RS}| = \sqrt{(6a)^2 + (8a)^2}$

$= \sqrt{100a^2}$

$= 10a$ (1)

Method 1

$\text{Proj}_{\overrightarrow{RS}} \overrightarrow{PQ} = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{RS}}{|\overrightarrow{RS}|}$

$6 = \dfrac{12a^2}{10a}$

$\dfrac{6}{5}a = 6$

$a = 5$ (1)

Method 2

$\text{Proj}_{\overrightarrow{RS}} \overrightarrow{PQ} = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{RS}}{\overrightarrow{RS} \cdot \overrightarrow{RS}} \times \overrightarrow{RS}$

$\underset{\sim}{v} = \dfrac{12a^2}{100a^2}(6a\underset{\sim}{i} + 8a\underset{\sim}{j})$

$= \dfrac{3}{25}(6a\underset{\sim}{i} + 8a\underset{\sim}{j})$

$|\underset{\sim}{v}| = 6 = \dfrac{3}{25} \times \sqrt{(6a)^2 + (8a)^2}$

$6 = \dfrac{3}{25} \times 10a$

$10a = 50$

$a = 5$

Q10) a) $x = u - 1$
$u = x + 1$

$u = x + 1$
$\frac{du}{dx} = 1$
$du = dx$

$u = x + 1$
$u = 5 + 1$
$= 6$

$u = x + 1$
$u = 10 + 1$
$= 11$

$$\int_4^9 \frac{u-1}{\sqrt{u}}\, du \qquad (1)$$

$$\int_4^9 \frac{u}{u^{\frac{1}{2}}} - \frac{1}{u^{\frac{1}{2}}}\, du$$

$$\int_4^9 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\, du$$

$$= \left[\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}}\right] \qquad (1)$$

$$= 2\left[\frac{u^{\frac{3}{2}}}{3} - u^{\frac{1}{2}}\right]_4^9$$

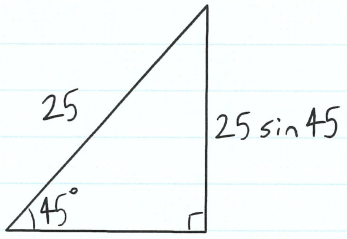
$$= 2\left[\left(\frac{9^{\frac{3}{2}}}{3} - \sqrt{9}\right) - \left(\frac{4^{\frac{3}{2}}}{3} - \sqrt{4}\right)\right]$$

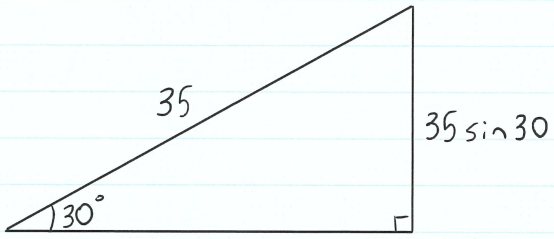
$$= 2\left[\left(\frac{27}{3} - 3\right) - \left(\frac{8}{3} - 2\right)\right]$$

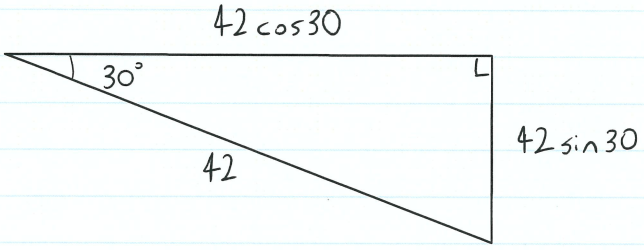
$$= 2\left(6 - \frac{2}{3}\right)$$

$$= \frac{32}{3} \qquad (1)$$

b) i)







① for calculating horizontal and vertical components

$$\underset{\sim}{F} = (25\cos 45 + 35\cos 30 + 42\cos 30)\underset{\sim}{i} + (25\sin 45 + 35\sin 30 - 42\sin 30)\underset{\sim}{j} \quad ①$$

$$= \left(25\times\frac{1}{\sqrt{2}} + 35\times\frac{\sqrt{3}}{2} + 42\times\frac{\sqrt{3}}{2}\right)\underset{\sim}{i} + \left(25\times\frac{1}{\sqrt{2}} + 35\times\frac{1}{2} - 42\times\frac{1}{2}\right)\underset{\sim}{j}$$

$$= \left(\frac{25\sqrt{2}}{2} + \frac{77\sqrt{3}}{2}\right)\underset{\sim}{i} + \left(\frac{25\sqrt{2}}{2} - \frac{7}{2}\right)\underset{\sim}{j}$$

$$= \frac{1}{2}\left[(25\sqrt{2} + 77\sqrt{3})\underset{\sim}{i} + (25\sqrt{2} - 7)\underset{\sim}{j}\right] \quad \text{as required.} \quad ①$$

ii) $|\underset{\sim}{F}| = \frac{1}{2}\times\sqrt{(25\sqrt{2} + 77\sqrt{3})^2 + (25\sqrt{2} - 7)^2}$

$= 85.55$ KN ① Accept answers rounded to the nearest KN.

$\theta = \tan^{-1}\left(\frac{25\sqrt{2} - 7}{25\sqrt{2} + 77\sqrt{3}}\right)$

$= \text{N } 9°32' \text{ E}$ ① Accept answers rounded to the nearest degree.

Also accept $080°28'$ T