
Student Number

Cheltenham Girls High School

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics EXTENSION 1

Assessment Task 3

General Instructions

- Working time – 45 minutes
- Write using black pen only
- Calculators approved by NESA may be used
- Use the multiple-choice answer sheet attached for Question 1-5
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 33

Section I – 5 marks (pages 1 - 2)

- Attempt Questions 1 - 5
- Allow about 5 minutes for this section

Section II – 28 marks (pages 3 - 5)

- Attempt Questions 6 - 8
- Allow about 40 minutes for this section

Multiple Choice Q1-5	Q6	Q7	Q8	Total
/5	/9	/8	/11	/33

Section I

5 marks
Attempt Questions 1–5
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1– 5.

1 What is the value of k such that $\int_0^k \frac{1}{\sqrt{4-x^2}}\, dx = \frac{\pi}{3}$

A. 1

B. $\sqrt{3}$

C. 2

D. $2\sqrt{3}$

2 What is the direction of the position vector $(-3\,, 5)$?

A. $31°$

B. $59°$

C. $121°$

D. $211°$

3 Which expression is equal to $\int \sin 3\theta \ \cos\theta \ \ d\theta$?

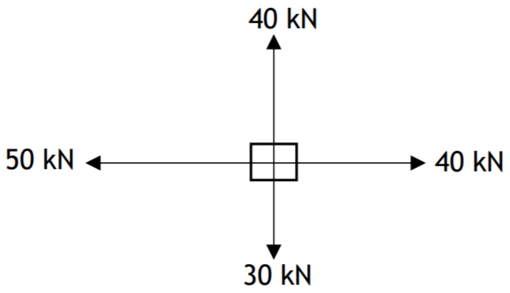
A. $-\frac{\cos 4\theta}{8} - \frac{\cos 2\theta}{4} + C$

B. $-\frac{\cos 4\theta}{4} + \frac{\cos 2\theta}{2} + C$

C. $-\frac{\sin 4\theta}{8} - \frac{\sin 2\theta}{4} + C$

D. $-\frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2} + C$

4 Four tugs boats apply forces to an oilrig as shown.



Which one of these vectors represent the resultant force acting on the oilrig?

A. $\left(10\,\underline{i} + 10\,\underline{j}\right)\ kN$

B. $\left(10\,\underline{i} - 10\,\underline{j}\right)\ kN$

C. $\left(-10\,\underline{i} - 10\,\underline{j}\right)\ kN$

D. $\left(-10\,\underline{i} + 10\,\underline{j}\right)\ kN$

5 Given $\underline{v} = -2\,\underline{i} - 5\,\underline{j}$ and $\underline{w} = 3\,\underline{i} + \underline{j}$, find $proj_{\underline{w}}\underline{v}$

A. $-\frac{11}{9}(\,3\,\underline{i} + \underline{j})$

B. $-\frac{11}{10}(\,3\,\underline{i} + \underline{j})$

C. $\frac{1}{10}(\,3\,\underline{i} + \underline{j})$

D. $-\frac{39}{10}(\,3\,\underline{i} + \underline{j})$

Section II

28 marks
Attempt Questions 6 – 8
Allow about 40 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 6 – 8, your responses should include relevant mathematical reasoning and/ or calculations.

Question 6 (9 marks) Use a new Writing Booklet.

(a) Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = \sin 2x$, the x –axis and the line $x = \frac{\pi}{8}$ is rotated about the x –axis. **3**

(b) Use the substitution $u = e^x$ to find $\int \frac{3e^x}{\sqrt{4-e^{2x}}}\, dx$ **3**

(c) Given that $\underline{a} = 4\underline{i} + 7\underline{j}$ and $\underline{b} = m\underline{i} - 2\underline{j}$, find the possible value (or values) of m if:

(i) $\underline{a}$ and $\underline{b}$ are parallel **2**

(ii) $|\underline{a}| = |\underline{b}|$ **1**

Question 7 (8 marks) Use a new Writing Booklet.

(a) (i) Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **2**

(ii) Hence, solve $\sqrt{2}\sec\theta - \tan\theta = \sqrt{3}$ for $0 < \theta < 2\pi$ **3**

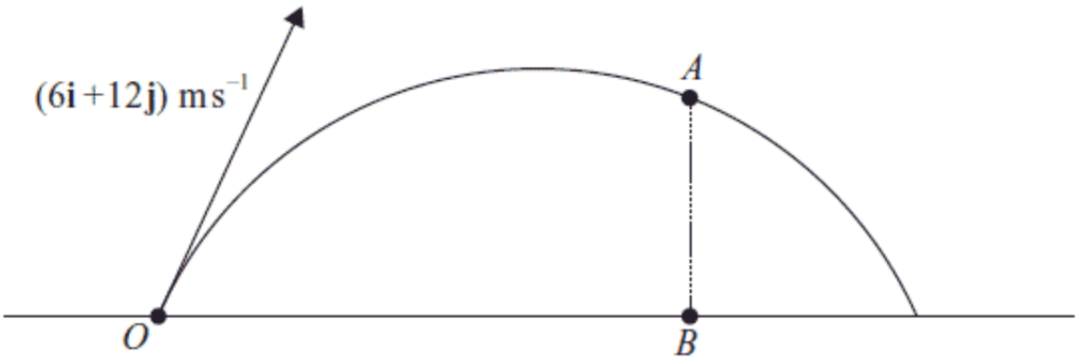
(b) Using the substitution $x = \cos\theta$ for $0 \leq \theta \leq \pi$, **3**

show that $\int \frac{\sqrt{1-x^2}}{x^2}\,dx = \cos^{-1}x - \frac{\sqrt{1-x^2}}{x} + C$

Question 8 (11 marks) Use a new Writing Booklet.

(a) Solve $\sin 3x - \sin x = 0$ for $0 < x < 2\pi$ **3**

(b) The point $\boldsymbol{O}$ is a fixed point on a horizontal plane. A ball is projected from $\boldsymbol{O}$ with a velocity of $(6\underset{\sim}{i} + 12\underset{\sim}{j})\ \mathrm{ms}^{-1}$, and passes through the point A at time t seconds after projection. Point B is on the horizontal plane vertically below A, as shown in the diagram below. Given that $OB = 2AB$



Assume $g = 10\ ms^{-2}$
$\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the x and y directions respectively

(i) Find the value of t **5**

(ii) Find the speed, $V\ \mathrm{ms}^{-1}$ of the ball at the instant it passes through A **3**

End of test

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2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

Assessment Task 3

Section I - Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here ➔

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

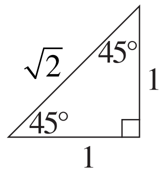
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

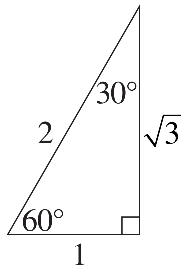
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

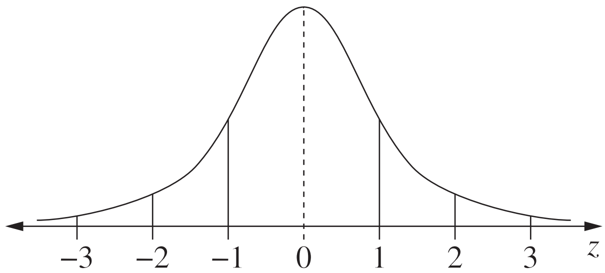
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

$$ \binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

$$ (x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n} $$

Vectors

$$ \left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2} + y^{2}} $$

$$ \underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1 x_2 + y_1 y_2 , $$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$ \underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b} $$

Complex Numbers

$$ \begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned} $$

$$ \begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned} $$

Mechanics

$$ \frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right) $$

$$ x = a\cos(nt + \alpha) + c $$

$$ x = a\sin(nt + \alpha) + c $$

$$ \ddot{x} = -n^{2}(x - c) $$

2020 Mathematics Extension 1 Task 3 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	C
3	A
4	D
5	B

Section II

Question 6 (a)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• Correctly integrates $\sin^2 2x$ using the double angle formula or equivalent merit	**2**
• Correct expression for the volume OR • An attempt to integrate an expression involving $\sin^2 2x$ using the double angle formula	**1**

Sample answer:

$$V = \pi \int_0^{\frac{\pi}{8}} \sin^2 2x \ \ dx$$

$$V = \pi \int_0^{\frac{\pi}{8}} \frac{1-\cos 4x}{2} \ \ dx$$

$$V = \frac{\pi}{2} \left[x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{8}}$$

$$V = \frac{\pi}{2} \left[\frac{\pi}{8} - \frac{\sin\frac{\pi}{2}}{4} \right]_0^{\frac{\pi}{8}}$$

$$V = \pi \left[\frac{\pi}{16} - \frac{1}{8} \right] \text{ cubic units}$$

Question 6 (b)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• Correctly uses the substitution $u = e^x$	**2**
• An attempt to integrate the function or equivalent merit	**1**

Sample answer:

$$\int \frac{3e^x}{\sqrt{4-e^{2x}}}\ dx$$

$$u = e^x$$

$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x}$$

$$\therefore \int \frac{3e^x}{\sqrt{4-e^{2x}}}\ dx = \int \frac{3e^x}{\sqrt{4-u^2}}\ \frac{du}{e^x}$$

$$= \int \frac{3}{\sqrt{4-u^2}}\, du$$

$$= 3\ \sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= 3\ \sin^{-1}\left(\frac{e^x}{2}\right) + C$$

Question 6 (c)(i)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**2**
• Correctly uses the definition of parallel vectors or equivalent merit	**1**

Sample answer:

$\underset{\sim}{a} = 4\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\underset{\sim}{b} = m\underset{\sim}{i} - 2\underset{\sim}{j}$.

Since $\underset{\sim}{a} \parallel \underset{\sim}{b}$

Then $\underset{\sim}{a} = \lambda\underset{\sim}{b}$

i.e. $\left(4\underset{\sim}{i} + 7\underset{\sim}{j}\right) = \lambda(m\underset{\sim}{i} - 2\underset{\sim}{j})$

$4\underset{\sim}{i} + 7\underset{\sim}{j} = \lambda m\underset{\sim}{i} - 2\lambda\underset{\sim}{j}$

$-2\lambda = 7$

$\therefore \lambda = -\frac{7}{2}$

$\lambda m = 4$

$m = \frac{4}{\lambda}$

$\therefore m = -\frac{8}{7}$

Question 6 (c)(ii)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**1**

Sample answer:

$\underset{\sim}{a} = 4\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\underset{\sim}{b} = m\underset{\sim}{i} - 2\underset{\sim}{j}$.

$|\underset{\sim}{a}| = |\underset{\sim}{b}|$

$|\underset{\sim}{a}|^2 = |\underset{\sim}{b}|^2$

$4^2 + 7^2 = m^2 + (-2)^2$

$m^2 = 49 + 16 - 4$

$m^2 = 61$

$m = \pm\sqrt{61}$

Question 7 (a)(i)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**2**
• Correctly finds either a value for R, or a value for α	**1**

Sample answer:

$\sin\theta + \sqrt{3}\ \cos\theta = R\sin(\theta + \alpha)$

$\sin\theta + \sqrt{3}\ \cos\theta = R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$\sin\theta + \sqrt{3}\ \cos\theta = R\sin\theta\cos\alpha + \mathrm{R}\cos\theta\sin\alpha$

$R\cos\alpha = 1\ \longrightarrow (1)$

$R\sin\alpha = \sqrt{3}\ \longrightarrow (2)$

$(1)^2 + (2)^2 : R^2 = 1 + 3 = 4$

$\therefore R = \sqrt{4} = 2$

$\dfrac{(2)}{(1)}$: $\tan\alpha = \sqrt{3}$

$\alpha = \dfrac{\pi}{3}$

$\therefore\ \sin\theta + \sqrt{3}\ \cos\theta = 2\sin\left(\theta + \frac{\pi}{3}\right)$

Question 7 (a)(ii)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• Solves the equation using the result from part (i) providing one solution	**2**
• Attempts to solve the equation using the result from part (i), or equivalent merit	**1**

Sample answer:

$$\sqrt{2}\ \sec\theta - \tan\theta = \sqrt{3}, \quad 0 < \theta < 2\pi$$

$$\frac{\sqrt{2}}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

$$\sqrt{2} - \sin\theta = \sqrt{3}\ \cos\theta$$

$$\sin\theta + \sqrt{3}\ \cos\theta = \sqrt{2}$$

$$2\sin\left(\theta + \frac{\pi}{3}\right) = \sqrt{2}$$

$$\sin\left(\theta + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

$$0 < \theta < 2\pi$$

$$\frac{\pi}{3} < \left(\theta + \frac{\pi}{3}\right) < \frac{7\pi}{3}$$

$$\left(\theta + \frac{\pi}{3}\right) = \frac{3\pi}{4}, \quad \frac{9\pi}{4}$$

$$\therefore \theta = \frac{5\pi}{12}, \quad \frac{23\pi}{12}$$

Question 7 (b)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• Writes $-\int \tan^2\theta\, d\theta = \int 1-\sec^2\theta\, d\theta$ or equivalent merit	**2**
• Writes $\int \frac{\sqrt{1-x^2}}{x^2}\,dx = \int \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta} - \sin\theta\ d\theta$ or equivalent merit	**1**

Sample answer:

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$x = \cos\theta$$

$$\frac{dx}{d\theta} = -\sin\theta$$

$$dx = -\sin\theta\ d\theta$$

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx = \int \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta} - \sin\theta\ d\theta$$

$$= -\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

$$= -\int \tan^2\theta\, d\theta$$

$$= \int 1-\sec^2\theta\, d\theta$$

$$= \theta - \tan\theta + C$$

$$= \cos^{-1}x - \frac{\sqrt{1-x^2}}{x} + C$$

Question 8 (a)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• solves either one of the equations $\cos 2x = 0$ or $\sin x = 0$ correctly	**2**
• Made some progress	**1**

Sample answer:

$\sin 3x - \sin x = 0$

$2\cos 2x \ \sin x = 0$

or $\sin x = 0$

$\cos 2x = 0$, $0 < x < 2\pi$

$0 < 2x < 4\pi$

$2x = \frac{\pi}{2}, \ \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \ \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sin x = 0$

$x = 0, \pi, 2\pi$

Question 8 (b)(i)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**5**
• Correct vector equation for $\underset{\sim}{s}$	**4**
• Correct vector equation for $\underset{\sim}{v}$	**3**
• Correct value for one the constant vectors	**2**
• Can recognise the initial velocity or equivalent merit	**1**

Sample answer:

$\ddot{y} = -10$ and $\ddot{x} = 0$

$\underline{a} = -10\underline{j}$

$\underline{v} = \int -10\underline{j}\, dt$

$\underline{v} = -10t\underline{j} + \underline{C}$

Initially $\underline{v} = (6\underline{i} + 12\underline{j})$

$\therefore \underline{C} = (6\underline{i} + 12\underline{j})$

$\underline{v} = -10t\underline{j} + (6\underline{i} + 12\underline{j})$

$\underline{s} = \int (\, 6\underline{i} + (12 - 10t)\underline{j})\, dt$

$\underline{s} = 6t\underline{i} + (12t - 5t^2)\underline{j}$

At $t = t\ seconds$

$6t = 2(12t - 5t^2)$

$3t = 12t - 5t^2$

$5t^2 - 9t = 0$

$t = o\ or\ t = \frac{9}{5}$

The value of t is $\frac{9}{5}$

Question 8 (b)(ii)

MARKING GUIDELINES

Criteria	Marks
• Correct solution	**3**
• Correct substitution for t or finding $\lvert\underset{\sim}{v}\rvert$	**2**
• Some progress	**1**

Sample answer:

$$\underset{\sim}{v} = -10t\underset{\sim}{j} + (6\underset{\sim}{i} + 12\underset{\sim}{j})$$

$$t = \frac{9}{5}$$

$$\underset{\sim}{v} = -10 \times \frac{9}{5}\underset{\sim}{j} + (6\underset{\sim}{i} + 12\underset{\sim}{j})$$

$$\underset{\sim}{v} = -18\underset{\sim}{j} + (6\underset{\sim}{i} + 12\underset{\sim}{j})$$

$$\underset{\sim}{v} = 6\underset{\sim}{i} - 6\underset{\sim}{j})$$

$$\lvert\underset{\sim}{v}\rvert = \sqrt{6^2 + 6^2}$$

$$\lvert\underset{\sim}{v}\rvert = \sqrt{72}$$

$$\lvert\underset{\sim}{v}\rvert = 6\sqrt{2}$$

$\therefore$ the speed, $V\ \mathrm{ms}^{-1}$ of the ball at the instant it passes through A is $6\sqrt{2}\ ms^{-1}$

Q6 a) Quite well done, just a few cannot integrate $\sin^2 2x$.

b) Quite well done, but some students have forgotten to replace the *u* at the end of integration.

c) Generally well answered.

Marker's Comments for Question 7

7(a) (i)	Generally well done
7(a) (ii)	Some students did not use the result in part (i) to answer the question $\sqrt{2}\ \sec\theta - \tan\theta = \sqrt{3}, \quad 0 < \theta < 2\pi$ $\frac{\sqrt{2}}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sqrt{3}$ $\sqrt{2} - \sin\theta = \sqrt{3}\ \cos\theta$ $\sin\theta + \sqrt{3}\ \cos\theta = \sqrt{2}$ Some students provided one of the solutions Some students were able to successfully answer the question
7(b)	Some students did not derive $x = \cos\theta$ correctly Some students made careless errors showing $\int \frac{\sqrt{1-x^2}}{x^2}\ dx = \int \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta}\ -\sin\theta\ d\theta$ Some students made errors in showing $-\int \tan^2\theta\ d\theta = \int 1 - \sec^2\theta\ d\theta$ Some had difficulty finding $\int 1 - \sec^2\theta\ d\theta$ Some students were able to successfully answer the question

Year 12 EXT1 CT3 Marker's REPORT

Question 8 (11 marks)

a. Students who use the sum to product formula correctly have it easy and could obtain the full solution quickly without mistakes. A proportion of candidates chose to factorise and apply the double angle formula. More than half could reach the final answer correctly but several students lost marks on the way as they struggle to recall applying the correct formulae and factorise correctly.

b. A large proportion of candidates did not appreciate the usefulness of vectors, resulting in them resorting to resolve the velocity component again. In order to do so they also needed to find the magnitude of the velocity. Integration must start from 1st principle to obtain the equations of motion from acceleration to velocity and then to displacement. Several students forgot the constant of integration and lost 1 mark. The 2nd part (ii) is worth 2 marks. Surprisingly some candidates apply velocity = 0 at the maximum height where the question was clearly not talking about this. The 2:1 ratio (condition) was not applied. This led to the wrong time t, being found. For part (ii) it was generally well done.