HK JG GS AT MB MK

Name:	____________________
Class:	11MTX________
Teacher:	____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2014

YEAR 11

AP1 EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 1 HOUR
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions. Marks are indicated to the right of each question.
- Section I (Multiple Choice) 10 MARKS – Use the answer sheet stapled to the back of the question paper. Write your name and class in the space provided. You are advised to spend 15 minutes on this section.
- Section II 39 MARKS – Start each question on a new page, clearly marked Question 1, Question 2 etc at the top of the page. Each page must show your name and class. You are advised to spend 1 hour and 45 minutes on this section.
- Your solutions to Section I and II should be stapled together in one bundle. Make sure that the questions are stapled in order, Section I then Section II Question 1 to 4. Any question not attempted must have a blank page submitted with your name, class and question number clearly indicated.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

SECTION 1

1 What is the exact value of $\tan(\theta - 180^o)$, if $4\cos\theta = -3$ and $\tan\theta > 0$?

(A) $-\dfrac{\sqrt{7}}{3}$

(B) $\dfrac{\sqrt{7}}{3}$

(C) $-\dfrac{3}{\sqrt{7}}$

(D) $\dfrac{3}{\sqrt{7}}$

2 What is the acute angle between the lines $y - \sqrt{3x} - 6 = 0$ and $\sqrt{3}y - x + 2 = 0$?

(A) 30°

(B) 45°`

(C) 60°

(D) 90°

3 What is the solution to the inequality $\dfrac{x^2 + 4x + 5}{x + 1} \geq -1$?

(A) $-3 \leq x \leq -2$ or $x \geq -1$

(B) $-3 \leq x \leq -2$ or $x > -1$

(C) $x \leq -3$ and $x \geq -2$

(D) $-2 \leq x < -1$ or $x \leq -3$

4 What are the solutions to the equation $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$ for $0 \leq \theta \leq 180^\circ$?

(A) $\theta = 30^\circ,\ 135^\circ,\ 150^\circ$

(B) $\theta = 45^\circ,\ 135^\circ,\ 150^\circ$

(C) $\theta = 30^\circ,\ 45^\circ,\ 150^\circ$

(D) $\theta = 60^\circ,\ 45^\circ,\ 120^\circ$

5 If $t=\tan\theta$ which of the following expressions is equivalent to $1-\sin 2\theta$?

(A) $\dfrac{1-t^2}{1+t^2}$

(B) $\dfrac{(t-1)^2}{1+t^2}$

(C) $\dfrac{1+t^2}{1-t^2}$

(D) $\dfrac{(t+1)^2}{1+t^2}$

SECTION 2

Q1 (13 Marks) *Start a new page* — Marks

(a) Solve the inequality $\dfrac{1}{|x-3|}\geq\dfrac{1}{4}$, where $x\neq 3$ **2**

(b) Write $\left(1+\sqrt{5}\right)^3$ in the form $a+b\sqrt{5}$ where a and b are integers **2**

(c) Simplify $\dfrac{9^n-1}{3^n-1}$ **1**

(d) If $\sqrt{10}=3+\dfrac{1}{a}$, show that $a=6+\dfrac{1}{a}$ **2**

(e) Solve the following simultaneous equations **3**

$$\begin{aligned}x+3y-2z&=8\\3x+2y-3z&=15\\4x+2y+3z&=-1\end{aligned}$$

(f) Solve $|x+1|+|x-5|>6$ **3**

Q2 (13 Marks) ***Start a new page*** **Marks**

(a) Expand $(2x+3y)^4$. Show all working. **2**

(b) Find the constant term (that is, the term independent of x) in the expansion of $\left(2x-\frac{1}{x^2}\right)^9$ **2**

(c) Use factorial notation to prove that ${}^{k+1}C_2 = {}^kC_2 + {}^kC_1$ **2**

(d) (i) Write the expansion of $\cos(\alpha+\theta)$ **1**

(ii) Hence or otherwise prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **2**

(iii) Hence solve $8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$ for $0^0 \le \theta \le 360^0$ **2**

(e) Show that $\tan 2x + \dfrac{\tan x}{\sec^2 x} = \dfrac{\tan x(3+\tan^2 x)}{1-\tan^4 x}$ **2**

Q3 (13 Marks) ***Start a new page*** **Marks**

(a) (i) Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x+\alpha)$ where $0^o \le \alpha < 360^o$ **2**

(ii) Hence or otherwise solve the equation $\sqrt{3}\sin x - \cos x = 1$ for $0^o \le x \le 360^o$ **2**

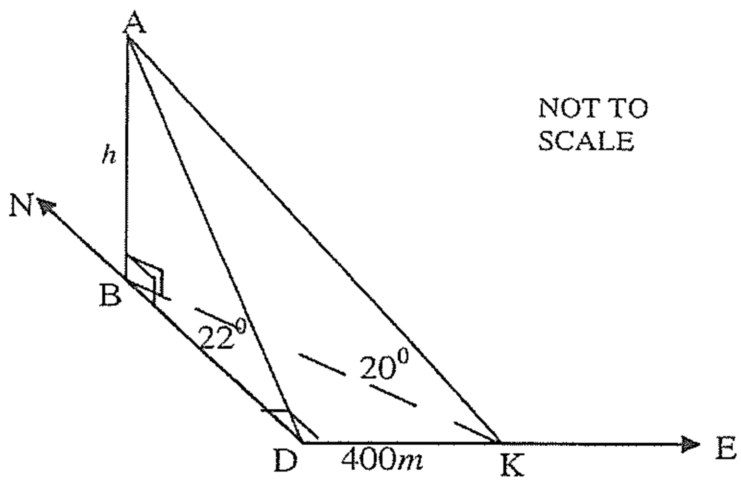
(b) (i) Sketch the graph of $y = \cot 2x$ from $0^0 \le x \le 180^0$ **2**

(ii) Hence find the number of solutions of the equation.
$\cot 2x = 1$ from $0^0 \le x \le 180^0$ (Do not solve the equation) **1**

(c) Find the general solution in degrees of the given equation **2**

$$sin2x = sinx$$

(d) Donna is standing at D and observes the angle of elevation of the tip of a flagpole A to be 22°. Her friend Kate, who is standing at K, 400 metres due east of Donna, finds the angle of elevation of the tip of the flagpole to be 20°. The flagpole is due north of Donna and B is the base of the flagpole. The points B, D and K are on level ground.



(i) Show that the height(h) of the flagpole above the ground is given by **3**

$$h = 400(\cot^2 20^\circ - \cot^2 22^\circ)^{\frac{-1}{2}}$$

(ii) Find the value of h, correct to 3 significant figures. **1**

End of Paper

2014 4U AP1 Ext 1 Solns

1. $\tan(\theta - 180^\circ)$

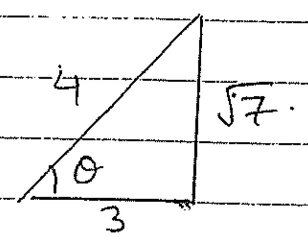
$= \tan -(180 - \theta)$

$= -\tan(180 - \theta)$

$= -(-\tan\theta)$

$= \tan\theta$

$\tan\theta = \frac{\sqrt{7}}{3}$



B

2. $y = \sqrt{3}x + 6$

$m_1 = \sqrt{3}$

$\sqrt{3}y = x - 2$

$m_2 = \frac{1}{\sqrt{3}}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \cancel{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{3}}}}\right|$

$= \left|\frac{3 - 1}{\sqrt{3}(2)}\right| = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

A

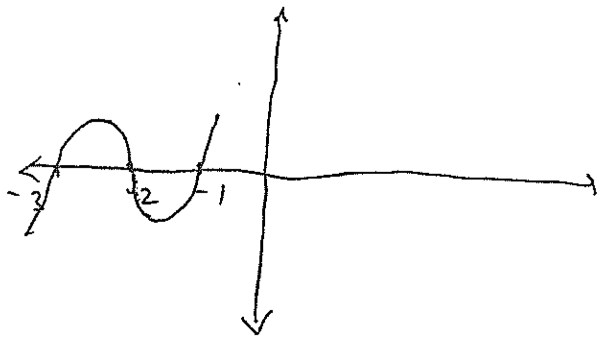
3. $\frac{x^2 + 4x + 5}{x + 1} \geq -1$

$\frac{(x^2 + 4x + 5)(x+1)^2}{\cancel{(x+1)}} \geq -(x+1)^2$

$(x+1)(x^2 + 4x + 5 + x + 1) \geq 0$

$(x+1)(x^2 + 5x + 6) \geq 0$

$(x+1)(x+2)(x+3) \geq 0$



$-3 \leq x \leq -2$ or $x > -1$

B

4. $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$

$3\tan^2\theta(\tan\theta + 1) - 1(\tan\theta + 1) = 0$

$(3\tan^2\theta - 1)(\tan\theta + 1) = 0$

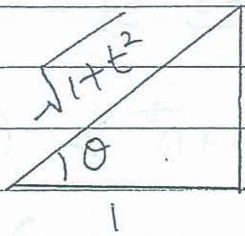
$3\tan^2\theta - 1 = 0$ or $\tan\theta = -1$

$\tan\theta = \frac{\pm 1}{\sqrt{3}}$ $\quad$ $\tan\theta = -1$

$30^\circ, 150^\circ$ $\quad$ 135°

A

(5) $t = \tan\theta$



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2 \times \frac{t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}}$$

$$\sin 2\theta = \frac{2t}{1+t^2}$$

$$1 - \sin 2\theta = 1 - \frac{2t}{1+t^2}$$
$$= \frac{1+t^2-2t}{1+t^2}$$
$$= \frac{(1-t)^2}{1+t^2} \text{ or } \frac{(t-1)^2}{t^2+1}$$

(B)

$\sqrt{125}$
$5\sqrt{5}$

$a^3 + 3a^2b + 3ab^2 + b^3$
$1^3 + 3\times 1^2 \cdot \sqrt{5} + 3\times 1 \times (\sqrt{5})^2 + (\sqrt{5})^3$
$1 + 3\sqrt{5} + + 15 + 5\sqrt{5}$
$16 + 8\sqrt{5}$
$a = 16 \quad b = 8$

Section 2

Q1 (a)

$$\frac{1}{|x-3|} \geq \frac{1}{4}$$

Since $|x-3|$ is positive.

$$4 \geq |x-3|$$
$$|x-3| \leq 4$$
$$-4 \leq x-3 \leq 4$$
$$-1 \leq x \leq 7 \quad \text{(1 mark)}$$

However, $x \neq 3$

$-1 \leq x < 3$ or $3 < x \leq 7$.
(1 mark)

(b) $(1+\sqrt{5})^3$

$1 + (\sqrt{5})^3 + 3\sqrt{5}(1+\sqrt{5})$
$1 + 5\sqrt{5} + 3\sqrt{5} + 15$ (1 mark
$16 + 8\sqrt{5}$

$a = 16$
$b = 8$ (1 mark)

$(3^n)^2 - 1^2$

(c) $$\frac{9^n - 1}{3^n - 1} = \frac{3^{2n} - 1}{3^n - 1}$$

$$= \frac{\cancel{(3^n-1)}(3^n+1)}{\cancel{(3^n-1)}} = 3^n + 1$$
(1 mark)

d) $\sqrt{10} = 3 + \frac{1}{a}$

$\frac{1}{a} = \sqrt{10} - 3$

$a = \frac{1}{\sqrt{10}-3}$ (1 mark)

R.H.S $6 + \frac{1}{a}$

$6 + \sqrt{10} - 3$

$= 3 + \sqrt{10}$

L.H.S $a = \frac{1}{\sqrt{10}-3} = \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$

$= \frac{3+\sqrt{10}}{1}$

L.H.S = R.H.S

(1 mark) Hence Proved.

(e). $x + 3y - 2z = 8 \rightarrow (1)$

$3x + 2y - 3z = 15 - (2)$

$4x + 2y + 3z = -1 \rightarrow (3)$

Considering equations (2) & (3)

$$\begin{aligned} &3x + 2y - 3z = 15 \\ + \; &4x + 2y + 3z = -1 \\ \hline &7x + 4y = 14 \rightarrow (4) \end{aligned}$$

(AW1 mark for elimination of 1 variable)

Considering eqns (1) & (2)

$3(x + 3y - 2z = 8)$

$2(3x + 2y - 3z = 15)$

$$\begin{aligned} &3x + 9y - \cancel{6z} = 24 \\ - \; &6x + 4y - \cancel{6z} = 30 \\ \hline &-3x + 5y = -6 \rightarrow (5) \end{aligned}$$

Multiplying (4) by 3 and (5) by 7.

$$\begin{aligned} &2\cancel{1x} + 12y = 42 \\ &-2\cancel{1x} + 35y = -42 \\ \hline &47y = 0 \end{aligned}$$

$y = 0$

(AW 1 mark for elimination of 2nd variable)

Substituting $y = 0$ in equation (4)

$7x = 14$

$x = +2$

$x + 3y - 2z = 8$

$2 + 0 - 2z = 8$

$-2z = 6$

$z = -3$

$(2, 0, -3)$

1 mark for the final answer

(e) $|x+1| + |x-5| > 6$

$|x+1| + |x-5| - 6 > 0$

$y = |x+1| + |x-5| - 6$.

Region 1 :- when $x \geq 5$.

$|x+1| = x+1$

$|x-5| = x-5$

$y = x+1 + x-5-6$

$y = 2x-10$

(AW 1 mark for regions)

Region 2 when $-1 < x < 5$

$|x+1| = x+1$

$|x-5| = 5-x$

$y = \cancel{x}+1+5-\cancel{x}-6$

$y = 0$

(AW 1 mark for equations in each region)

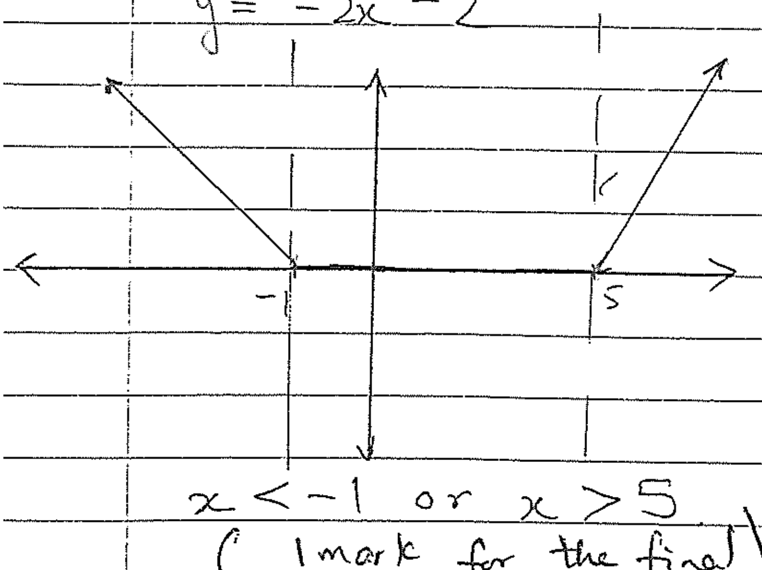
Region 3

when $x \leq -1$

$|x+1| = -(x+1)$ and $|x-5| = 5-x$

$y = -x-1+5-x-6$

$y = -2x-2$



$x < -1$ or $x > 5$

(1 mark for the final answer)

2 (a)

$(2x+3y)^4$

$= (2x)^4 + 4(2x)^3 3y + 6(2x)^2(3y)^2$

$+ 4(2x)(3y)^3 + (3y)^4$

(1 mark)

$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3$

$+ 81y^4$ (1 mark).

(b)

General Term

$^9C_r (2x)^{9-r} \left(\frac{-1}{x^2}\right)^r$

$^9C_r (2)^{9-r} (x)^{9-r} (-1)^r (x)^{-2r}$

$= {}^9C_r (2)^{9-r} (-1)^r (x)^{9-3r}$ (1 mark for the general term)

$9-3r = 0$

$r = 3$

$^9C_3 (2)^{9-3} (-1)^3$

$= -5376$ (1 mark for the final answer)

(c) $\,^{k+1}C_2 = \,^{k}C_2 + \,^{k}C_1$

L.H.S

$$\frac{(k+1)!}{(k-1)!\,2!}$$

$$= \frac{(k+1)k\cancel{(k-1)!}}{\cancel{(k-1)!}\,2!}$$

$$= \frac{k(k+1)}{2}$$

R.H.S

$$\,^{k}C_2 + \,^{k}C_1$$

$$\frac{k(k-1)}{2} + k$$

$$= k\left(\frac{k-1+1}{2}\right)$$

$$= k\left(\frac{k-1+2}{2}\right)$$

$$= \frac{k(k+1)}{2}$$

L.H.S = R.H.S

Hence Proved

(1 mark for factorial notation)

(1 mark for proof)

d

(i). $\cos(\alpha+\theta)$

$= \cos\alpha\cos\theta - \sin\alpha\sin\theta$

(1 mark)

(ii). $\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

Also we know that

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

(1 mark)

$= \cos^3\theta - \cos\theta\sin^2\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\cos\theta\sin^2\theta$

$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

(1 mark)

(iii) $8\cos^3\theta - 6\cos\theta - \sqrt{3} = 0$

$2(4\cos^3\theta - \cos\theta) - \sqrt{3} = 0$

$\cos 3\theta = \frac{\sqrt{3}}{2}$ (1 mark)

$0 \leq \theta \leq 360^\circ$

$0^\circ \leq 3\theta \leq 1080$

$3\theta = 30^\circ, 330^\circ, 390^\circ, 690^\circ, 750^\circ, 1050^\circ$

$\theta = 10^\circ, 110^\circ, 130^\circ, 230^\circ, 250^\circ, 350^\circ$

(1 mark)

e) :- $\tan 2x + \frac{\tan x}{\sec^2 x} = \frac{\tan x(3+\tan^2 x)}{1-\tan^4 x)}$

L.H.S

$\tan 2x + \frac{\tan x}{\sec^2 x}$

$\frac{2\tan x}{1-\tan^2 x} + \frac{\tan x}{1+\tan^2 x}$

(1 mark)

$= \frac{2\tan x(1+\tan^2 x) + \tan x(1-\tan^2 x)}{1-\tan^4 x}$

$= \frac{\tan x(2+2\tan^2 x + 1 - \tan^2 x)}{1-\tan^4 x}$

$= \frac{\tan x(3+\tan^2 x)}{1-\tan^4 x}$

(1 mark)

3

a (i). $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x+\alpha)$.

$\sqrt{3}\sin x - \cos x = R\sin(x+\alpha)$

$\sqrt{3}\sin x - \cos x = R(\sin x\cos\alpha + \cos x\sin\alpha)$

By comparison of coefficients

$R\cos\alpha = \sqrt{3}$

$R\sin\alpha = -1$

$R^2 = 4$

$R = 2$ (1 mark)

$\cos\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = -\frac{1}{2}$

$\therefore \alpha = 330^\circ$ (1 mark)

Hence

$\sqrt{3}\sin x - \cos x = 2\sin(x+330^\circ)$

(ii) $\sqrt{3}\sin x - \cos x = 1$

$2\sin(x + 330^\circ) = 1$ (1 mark)

$0 \le x \le 360$

$330^\circ \le x + 330^\circ \le 690^\circ$

$330^\circ \le u \le 690^\circ$

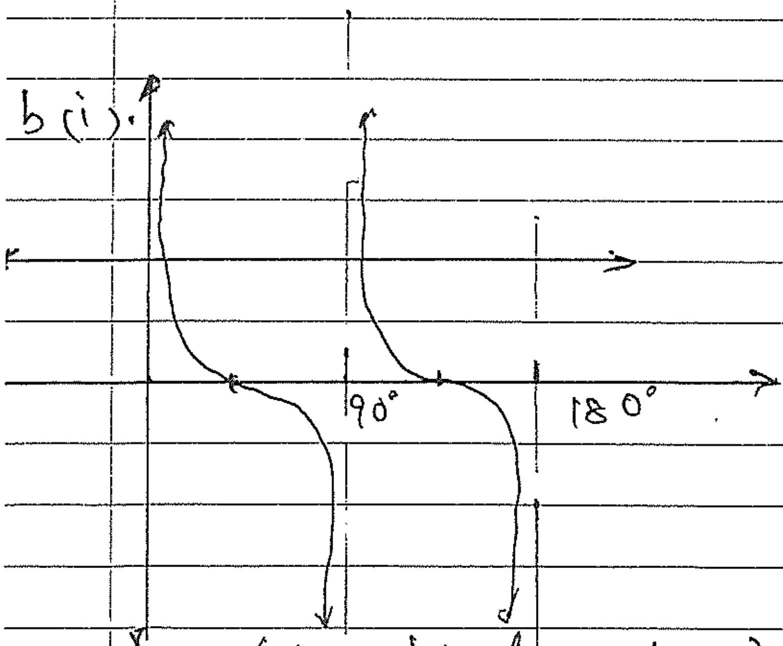
$\sin u = \frac{1}{2}$

$u = 390^\circ, 510^\circ$

$x = 60^\circ, 180^\circ$

(1mark)

b(i).



(1mark for shape)

(1 mark for correct time period and asymptotes)

(ii) 2 solutions

(c) $\sin 2x = \sin x$

$2\sin x\cos x = \sin x$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$, $\cos x = \frac{1}{2}$

$x = 180^\circ n$ (1mark), $x = 360^\circ n \pm 60^\circ$ (1 mark)

where n is an integer

d(i)

$BD = h\cot 22^\circ$

$BK = h\cot 20^\circ$

(1mark)

In ΔDBK

$h^2\cot^2 20^\circ - h^2\cot^2 20^\circ = 400^2$

(1mark)

(AW 1 for pythagoras)

$$h^2 = \frac{400^2}{\cot^2 20^\circ - \cot^2 22^\circ}$$

AW 1 for rearrangement

$$h = 400\left(\cot^2 20^\circ - \cot^2 22^\circ\right)^{-\frac{1}{2}}$$

(ii) 335.36

335 (3 significant figures)