Question 1

a) Solve $\frac{x+2}{x} \geq 3$. **3**

b) i) Graph $y = |2x - 4|$ and $y = x$ on the same number plane. **2**

ii) For what values of c does the equation $|2x - 4| = x + c$ have exactly two solutions ? **2**

c) Solve $|x + 1| + |x - 2| = 3$. **2**

d) i) Expand $(x - 3)^3$. **1**

ii) Write down the general term T_{k+1} in the expansion of $\left(x - \frac{1}{x}\right)^{10}$. **2**

iii) Hence or otherwise, find the coefficient of x^6 in the expansion of $(x - 3)^3\left(x - \frac{1}{x}\right)^{10}$. **3**

End of Question 1

Question 2 (Start a new page)

a) Find the exact value of $\frac{\tan 15°}{1+tan^2 15°}$. **2**

b) Find possible values of a if the lines $ax + 3y - 10 = 0$ and $ax - 2y + 5 = 0$ are inclined to each other at an angle θ, where $\tan\theta = 2$. **3**

c) Using the expansion of $\sin(A + B)$, show that $\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$. **3**

Question 2 *(continued from previous page)*

d) Solve $\cos 4x + \cos 2x = 0$, for $0° \leq x \leq 180°$ **3**

e) i) Sketch the graph of $y = 2 \sin 3x$ for $0° \leq x \leq 180°$. **3**

ii) Hence, by drawing a suitable line on this graph, find the ***number of solutions*** when $\sin 3x = 1$ is solved for $0° \leq x \leq 180°$? **1**

End of Question 2

Question 3 (Start a new page)

a) i) Prove the identity $\dfrac{1}{\tan A+\cot B} + \dfrac{1}{\cot A+\tan B} = \dfrac{\sin(A+B)}{\cos(A-B)}$. **2**

ii) Hence show that $\dfrac{2}{\tan A+\cot A} = \sin 2A$. **1**

b) Using t – formula, solve $2\sec x + \tan x = 2$, correct your answers to the nearest degree. **4**

c) For $0° \leq x \leq 360°$, solve $\cos x - \sqrt{3}\sin x = -\sqrt{3}$ by first expressing it in the form of $R\cos(x - \theta)$. **3**

d) The window of a lighthouse is observed from A, due south of the lighthouse at an angle of elevation of α. From B, which is due east of A, the same window is observed at an angle of elevation of β. **C is the midpoint of AB**.

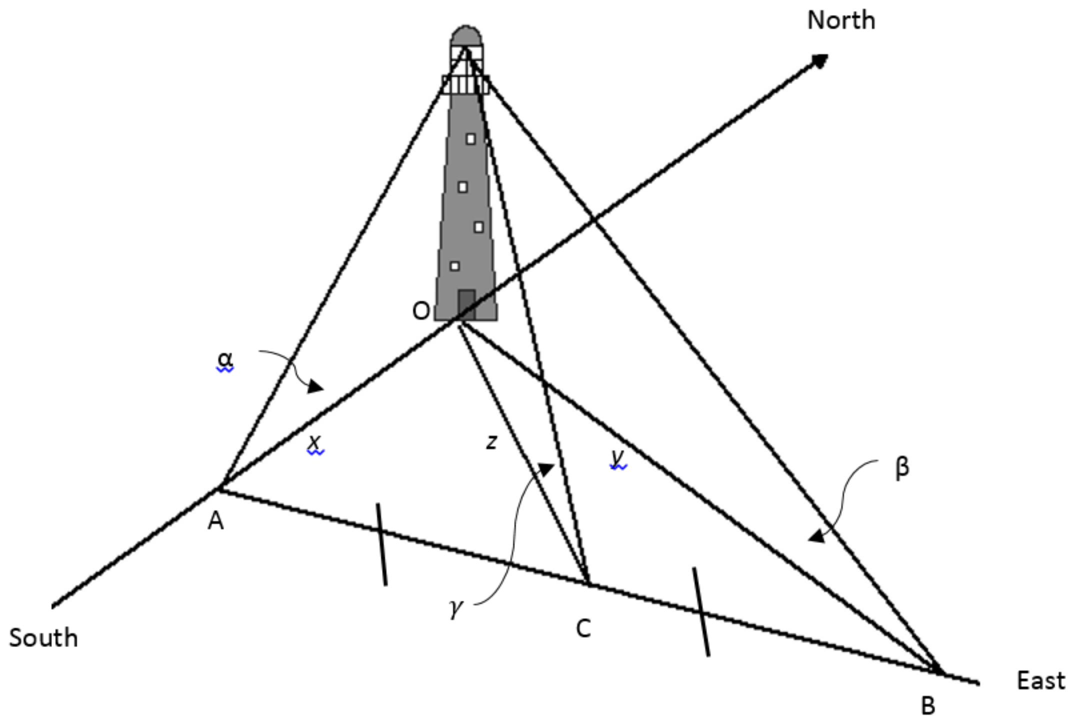
Diagram on next page

Question 3 *(continued from previous page)*

i) Let OA = x, OB = y and OC = z as shown. Show that $x = h\cot\alpha$, $y = h\cot\beta$ and $z = h\cot\gamma$. **1**

ii) Show that AC = $\dfrac{\sqrt{y^2-x^2}}{2}$. **1**

iii) Show that the angle of elevation, γ, of the window from C can be calculated from $\tan\gamma = \left(\dfrac{2}{\sqrt{3\cot^2\alpha + \cot^2\beta}}\right)$. **3**



End of Question 3

End of paper

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Q1

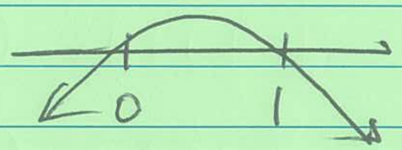
(a) $\dfrac{x+2}{x} \geq 3 \quad , x \neq 0$

$x^2 \times \dfrac{x+2}{x} \geq 3x^2$

$x(x+2) - 3x^2 \geq 0$

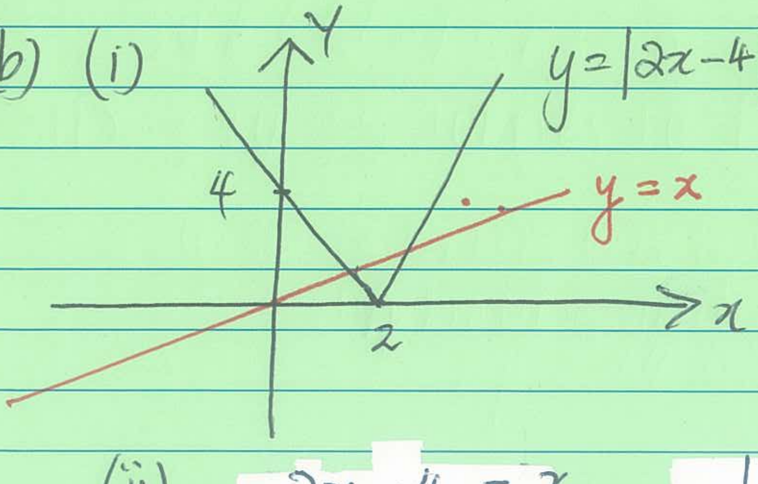
$x(2-2x) \geq 0$

$2x(1-x) \geq 0$



$0 < x \leq 1$

(b) (i)



(ii) $2x-4 = x$

$x = 4$

$-(2x-4) = x$

$4 = 3x$

$\therefore x = \dfrac{4}{3}$

$\therefore$ For $|2x-4| < x$

$\dfrac{4}{3} < x < 4$

Q1

(i)

$$x + y - z = 1 \quad \text{——} \ (1)$$
$$8x + 3y - 6z = 1 \quad \text{——} \ (2)$$
$$-4x - y + 3z = 1 \quad \text{——} \ (3)$$

From (1) $\quad z = x + y - 1 \quad \text{——} \ (4)$

Sub. (4) into (2)

$$8x + 3y - 6(x+y-1) = 1$$
$$8x + 3y - 6x - 6y + 6 = 1$$
$$2x - 3y = -5 \quad \text{——} \ (5)$$

Sub (4) into (3)

$$-4x - y + 3(x+y-1) = 1$$
$$-4x - y + 3x + 3y - 3 = 1$$
$$-x + 2y = 4$$
$$x = 2y - 4 \quad \text{——} \ (6)$$

Sub (6) into (5)

$$2(2y-4) - 3y = -5$$
$$y = -5 + 8$$
$$y = 3$$

Sub. $y = 3$ into (6)

$$x = 2(3) - 4 = 2$$

Sub. x, y into (4), $z = 2 + 3 - 1 = 4$

$\therefore \ x = 2, \ y = 3, \ z = 4$

Q1

(d)

(i) $(x-3)^3 = 1(x)^3 + 3(x)^2(-3) + 3(x)(-3)^2 + (-3)^3$

$= x^3 - 9x^2 + 27x - 27$

(ii) $(2x-1)^6 = \cdots\cdots \, {}^6C_4 (2x)^2(-1)^4$

$+ \binom{6}{C_6}(2x)^0(-1)^6$

$+ \binom{6}{C_5}(2x)^1(-1)^5 + \cdots\cdots$

In the expansion $(x-3)^3(2x-1)^6$, the coeff. of x^2 is

$-9 \times (-1)^6 \binom{6}{C_6} + 27 \times {}^6C_5 (2)(-1)^5 - 27 \binom{6}{C_4}(2)^2(-1)^4$

$= -9 + 27(6)(2)(-1) - 27(15)(4)$

$= -1953$

(e) $(1-(x+1))^6 = (1-x-1)^6 = (-x)^6 = x^6$

Q2

(a) $$\frac{2\tan 15^\circ}{1+\tan^2 15^\circ} = \sin 30^\circ = \frac{1}{2}$$

$$\therefore \frac{\tan 15^\circ}{1+\tan^2 15^\circ} = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

(b) $$m_1 = \frac{a}{-3}, \quad m_2 = \frac{a}{2}$$

$$\tan\theta = 2 = \left| \frac{\frac{a}{2} - \left(-\frac{a}{3}\right)}{1 + \left(-\frac{a}{3}\right)\left(\frac{a}{2}\right)} \right|$$

$$\pm 2 = \frac{5a}{6-a^2}$$

$$2 = \frac{5a}{6-a^2}$$

$$12 - 2a^2 = 5a$$

$$2a^2 + 5a - 12 = 0$$

$$(2a-3)(a+4) = 0$$

$$a = \frac{3}{2}, \ a = -4$$

$$-2 = \frac{5a}{6-a^2}$$

$$2a^2 - 5a - 12 = 0$$

$$(2a+3)(a-4) = 0$$

$$a = -\frac{3}{2}, \ a = 4$$

$$\therefore a = \pm\frac{3}{2}, \pm 4$$

Alternative approach to Q2(c)

$$\cos x - \sqrt{3}\sin x = -\sqrt{3}$$

$$= R\cos(x-\theta)$$

$$= R\cos x\cos\theta + R\sin x\sin\theta$$

Comparing terms,

$$R\cos\theta = 1 \quad \text{— ①}$$

$$R\sin\theta = -\sqrt{3} \quad \text{— ②}$$

$$\frac{②}{①} \Rightarrow \tan\theta = -\sqrt{3} \Rightarrow \theta = -60^\circ$$

$$①^2 + ②^2 \Rightarrow R = 2.$$

$$\therefore \cos x - \sqrt{3}\sin x = -\sqrt{3}$$

$$2\cos(x - -60^\circ) = -\sqrt{3}$$

$$\cos(x+60^\circ) = -\frac{\sqrt{3}}{2}$$

$$x+60^\circ = 150^\circ, 210^\circ$$

$$x = 90^\circ, 150^\circ.$$

Q2

(c) $\cos x - \sqrt{3}\sin x = -\sqrt{3}$

Let $\cos x - \sqrt{3}\sin x = R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha.$

$R\cos\alpha = 1 \cdots ①$

$R\sin\alpha = \sqrt{3} \cdots ②$

$R = \sqrt{1+\sqrt{3}^2} = 2.$

$\frac{②}{①}, \tan\alpha = \sqrt{3}$

$\alpha = 60°$

$\therefore 2\cos(x+60°) = -\sqrt{3}$

$\cos(x+60°) = -\frac{\sqrt{3}}{2}$

$x+60° = 150°, 210°$

$x = 90°, 150°$

(d) $\cos 4x + \cos 2x = 0$

$2\cos^2 2x - 1 + \cos 2x = 0$

$(2\cos 2x - 1)(\cos 2x + 1) = 0$

$\cos 2x = \frac{1}{2}$, $\cos 2x = -1$

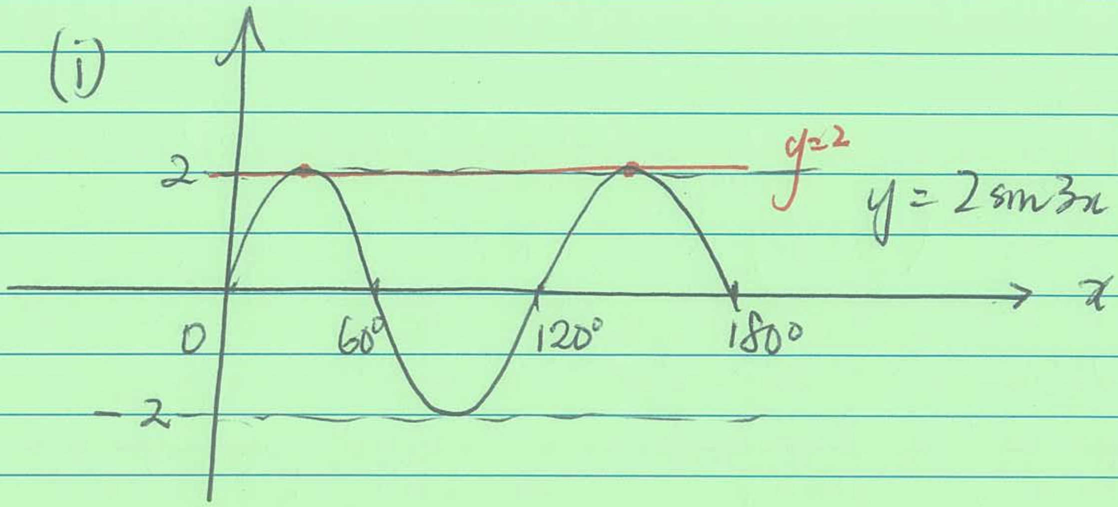
$2x = 60°, 300°$ $\quad$ $2x = 180°$

$x = 30°, 150°$ $\quad$ $x = 90°$

$\therefore x = \underline{30°, 90°, 150°}$

Q2

(e) (i)



(ii) $\sin 3x = 1$

$2\sin 3x = 2.$

Draw $y=2$

$\boxed{\text{2 solutions}}$

Q3

(a) (i) $$LHS = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos B}{\sin B}} \times \frac{\cos A \sin B}{\cos A \sin B} + \frac{1}{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}} \times \frac{\sin A \cos B}{\sin A \cos B}$$

$$= \frac{\cos A \sin B}{\sin B \sin A + \cos B \cos A} + \frac{\sin A \cos B}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\sin A \sin B + \cos A \cos B}$$

$$= \frac{\sin(A+B)}{\cos(A-B)} = RHS$$

(ii) From (i), Let $B = A$

$$\frac{1}{\tan A + \cot A} + \frac{1}{\cot A + \tan A} = \frac{\sin 2A}{\cos 0°}$$

$$\therefore \frac{2}{\tan A + \cot A} = \sin 2A \qquad \text{as } \cos 0° = 1$$

Q3

(b) $2\sec x + \tan x = 2$

$$2\left(\frac{1+t^2}{1-t^2}\right) + \frac{2t}{1-t^2} = 2$$

$$\cancel{1} + t^2 + t = \cancel{1} - t^2$$

$$2t^2 + t = 0$$

$$t(2t+1) = 0$$

$t = 0$ $\qquad$ $t = -\frac{1}{2}$

$\tan\frac{x}{2} = 0$

$\frac{x}{2} = 0°, 180°, 360°$

$x = 0°, 360°$

$\tan\frac{x}{2} = -\frac{1}{2}$

$\frac{x}{2} = 180° - 27°, 360° - 27°$

$x = 306°$

Test $x = 180°$,

$$\frac{2}{\cos 180°} + \tan 180° = \frac{2}{-1} + 0 = -2 \neq 2.$$

$\therefore$ $180°$ is not a solution.

$x = 0°, 306°, 360°$

Q3

(c) (i) $\tan\alpha = \dfrac{h}{x}$

$$x = \frac{h}{\tan\alpha} = h\cot\alpha$$

Similarly $y = h\cot\beta$

$z = h\cot\gamma$

(ii)

$$(AC + CB)^2 + x^2 = y^2$$

$$(2AC)^2 + x^2 = y^2 \qquad \text{as } AC = CB$$

$$4AC^2 + x^2 = y^2$$

$$AC = \frac{\sqrt{y^2 - x^2}}{2}.$$

(iii) $AC^2 + x^2 = z^2$

$$\frac{y^2 - x^2}{4} + x^2 = z^2$$

$$y^2 + 3x^2 = 4z^2$$

$$\cancel{h^2}\cot^2\beta + 3\cancel{h^2}\cot^2\alpha = 4\cancel{h^2}\cot^2\gamma$$

$$4\cot^2\gamma = \cot^2\beta + 3\cot^2\alpha$$

$$\frac{4}{\tan^2\gamma} = \cot^2\beta + 3\cot^2\alpha$$

$$\frac{4}{\cot^2\beta + 3\cot^2\alpha} = \tan^2\gamma$$

$$\therefore \tan\gamma = \frac{2}{\sqrt{\cot^2\beta + 3\cot^2\alpha}}.$$