SECTION 1

1 The solution(s) to the equation $|x-2| = 2x+1$ are?

(A) $x=-3$

(B) $x=-\frac{1}{3}$

(C) $x=\frac{1}{3}$

(D) $x=3$

2 Which of the following is an expression for $\cos(A+B)-\cos(A-B)$?

(A) $-2\cos A\cos B$

(B) $-2\sin A\sin B$

(C) $2\cos A\cos B$

(D) $2\sin A\sin B$

3 Simplify $\frac{\sin(180^\circ+\theta)}{\cos(90^\circ-\theta)}$

(A) -1

(B) 1

(C) $\tan\theta^\circ$

(D) $\tan(90-\theta)^\circ$

4 What is the exact value of $\sin(-105^\circ)$?

(A) $-\frac{1-\sqrt{3}}{2\sqrt{2}}$

(B) $-\frac{1+\sqrt{3}}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

5 The solution to the inequality $\frac{6}{x(x-2)} > 2$ is?

(A) $-1 < x < 0$ or $0 < x < 2$

(B) $x < -1$ or $2 < x < 3$

(C) $x < -1$ or $0 < x < 2$

(D) $-1 < x < 0$ or $2 < x < 3$

SECTION 2

QUESTION 1 (13 MARKS) **Start a new writing booklet** **MARKS**

a) Simplify $\frac{\sqrt{x^2}}{|x|}$ $x \neq 0$ **1**

b) **(i)** Factor $3^{n+1} + 3^n$, and **1**

(ii) hence simplify $\frac{3^{101}+3^{100}}{4}$ **2**

c) Solve the inequality $\frac{|3-x|}{x^2+3} \geq 1$ **3**

d) Solve the inequality $7-3x < \frac{2}{x}$ for x. **3**

e) Sketch carefully $y = |x-3| + |x+2|$ **3**

QUESTION 2 (13 MARKS) **Start a new writing booklet** **MARKS**

a) Express $(3x - \frac{1}{x^3})^8$ in sigma notation. **2**

b) By considering $(2+x-x^2)^4$ as $\left((2+x)-x^2\right)^4$ **3**

Hence, or otherwise find the coefficient of x^4.

c) Show that the sum of the coefficients of the terms in $(2x+y)^5$ is 243. **2**

d) **(i)** If $\sqrt{3}\cos x + \sin x = R\cos(x - \propto)$ where $R > 0$ and $\propto$ is acute, show $R = 2$ and the angle $\propto = 30°$ **2**

(ii) Hence, or otherwise solve **2**

$\sqrt{3}\cos x + \sin x = 1$ for $0° \leq x \leq 360°$.

e) Sketch the graph $y = 3\cos\frac{x}{2}$ for $0° \leq x \leq 360°$. **2**

QUESTION 3 (14 MARKS) **Start a new writing booklet** **MARKS**

a) Express $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}}$ as a single trigonometric ratio. **3**

b) Using the *t*-formulae, solve the equation **2**

$\sin 2x = \tan x$ for $0° \le x \le 180°$.

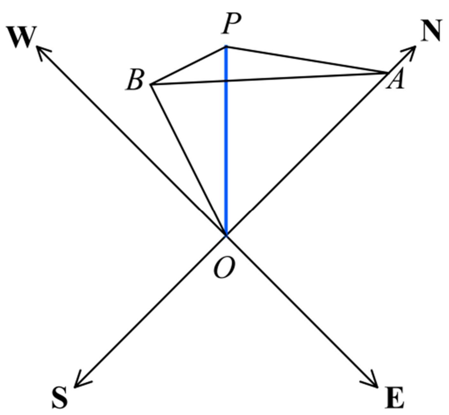
c) If $3\cos\theta + 2 = 0$ and $\tan\theta > 0$, find the exact value of:

(i) $\tan(\theta - 180)°$ **1**

(ii) $\sin(\theta + 180)°$ **1**

d) Find the acute angle between the lines $y = -2x + 3$ and $x - 3y + 2 = 0$. **3**
Answer correct to the nearest degree.

e) A vertical pole *PO* is 10 m high. Point *A* is due north of *O* and *OA* is the shadow of the pole when the angle of elevation to the sun is 67°. *OB* is the shadow cast by the pole when the direction of the sun is on a bearing of 340° and the sun's angle of elevation is 49°.



(i) What is the size of $\angle AOB$? **1**

(ii) Find the distance between *A* and *B*. Answer correct to 1 decimal place. **3**

END OF PAPER

KP

Multiple Choice Answers

Section 1.

1 C 2 B 3 A 4 B 5 D.

$|x-2| = 2x+1$

$x-2 = 2x+1 \qquad x-2 = -2x-1$

$-3 = x \qquad 3x = 1$

$|-3-2| = -6+1$ (F) $\qquad x = \frac{1}{3}$

$|\frac{1}{3}-2| = 2\times\frac{1}{3}+1$ ✓ $\qquad x = \frac{1}{3}$

$1\frac{2}{3} = 1\frac{2}{3}$

$\cos(A+B) - \cos(A-B)$

$\cos A\cos B - \sin A\sin B$

$-(\cos A\cos B + \sin A\sin B)$

$-2\sin A\sin B$ $\qquad$ B

$\sin(180+\theta) = -\sin\theta$

$\cos(90-\theta) \qquad \sin\theta$

$= -1.$

$\sin(105)^\circ = -\sin(75^\circ)$

$= -\sin(45+30)$

$= -(\sin 45\cos 30 + \sin 30\cos 45)$

$= -\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right)$

$= -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ $\qquad$ B

5) $\frac{6}{x(x-2)} > 2 \qquad x\neq 0,\ x\neq 2.$

$x\left[x^2(x-2)^2\right]$

$6x(x-2) > 2x^2(x-2)^2$

$6x(x-2) - 2x^2(x-2)^2 > 0$

$2x\left(3(x-2) - x(x-2)^2\right) > 0$

$2x(x-2)\left(3 - x(x-2)\right) > 0$

$x(x-2)(3 - x^2 + 2x) > 0$

$-x(x-2)(x-3)(x+1) > 0$

$x = 0,\ 2,\ 3,\ -1$

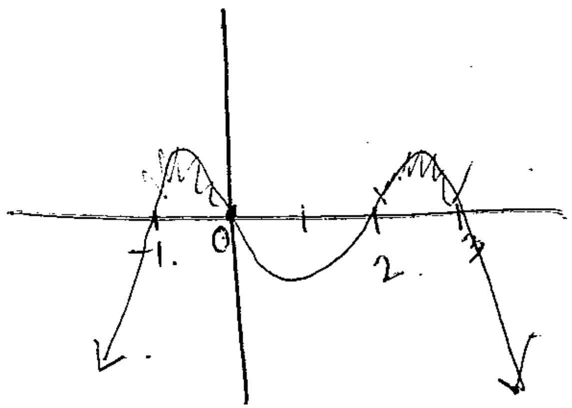
test

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

$x = 1 \qquad -1(-1)(-2).2 > 0$ F.

Solution

$-1 < x < 0 \qquad 2 < x < 3$



Section 2 Q1.

a) $\frac{\sqrt{x^2}}{|x|} = 1$ (1)

b) $3^{n+1} + 3^n = 3^n(3+1)$ (1)

c/m for correct substitn.

$\frac{3^{101} + 3^{100}}{4} = \frac{3^{100}(3+1)}{4}$ (1)

$= 3^{100}$ (1)

c) $\frac{|3-x|}{x^2+3} \geq 1$

for 2 cases (1)

Case 1

$|3-x| = 3-x \quad (x<3)$

$\frac{3-x}{x^2+3} \geq 1$

$3-x \geq x^2+3$

$0 \geq x^2+x$ (1)

$0 \geq x(x+1)$ (1)

Case 2

$|3-x| = -3+x \; (x>3)$

$-3+x \geq x^2+3$

$0 \geq x^2-x+6$

no soln.

$-2 \quad -1 \quad 0 \quad 1$

$x=1 \quad \frac{|3-1|}{1+3} = \frac{2}{4} > 0$

Solution

$-1 \leq x \leq 0$ (1)

d) $7 - 3x < \frac{2}{x}$

c/o $\times x^2$ soln

$7x^2 - 3x^3 < 2x$ (1)

$0 < 3x^3 - 7x^2 + 2x$

$< x(3x^2 - 7x + 2)$

$0 < x(3x-1)(x-2)$

$x = 0 \quad x = \frac{1}{3} \quad x = 2$ (1)

$x = 1 \quad 7-3<2 \quad F$

$0 < x < \frac{1}{3} \qquad x > 2$ (1)

e) $y = |x-3| + |x+2|$

$|x-3| = x-3 \quad x>3$

$= -x+3 \quad x<3$

$|x+2| = x+2 \quad x>-2$

$= -x-2 \quad x<-2$

$-2 \quad 3$

$++ \quad x-3+x+2 = y$

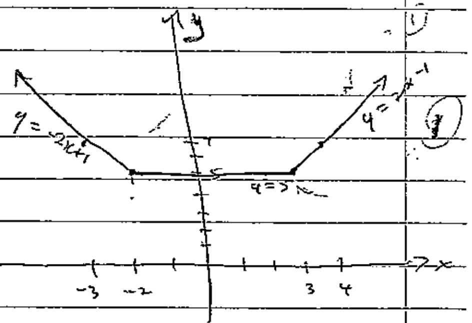
$(x>3) \quad 2x-1 = y$

$-- \quad -x+3-x-2 = y$

$\begin{matrix} x<3 \\ x<-2 \end{matrix} \quad -2x+1 = y$ (1)

$-+ \quad -x+3+x+2 = y$

$(-2<x<3) \quad 5 = y$ (1)



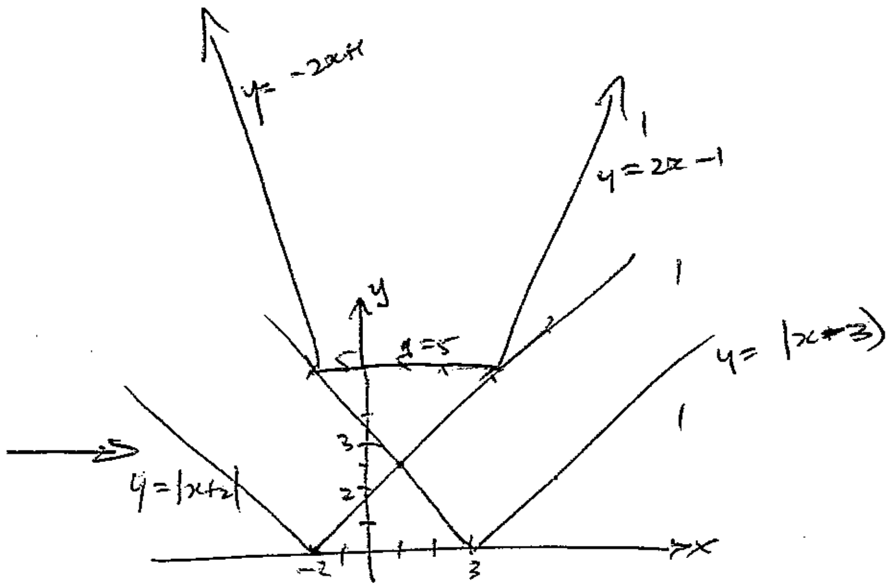
(1)

if graph only needs shape - (lines)

- # for turning.

KP

$\sqrt{x^2} = |x|$.



bold ans. (3)

Ext 1 AP3 2017 Question 2

a) $$\left(3x-\frac{1}{x^3}\right)^8 = \sum_{r=0}^{8}\binom{8}{r}(3x)^{8-r}\left(\frac{-1}{x^3}\right)^r$$ — 1mk. Substitutes appropriate terms into sigma notation, including boundaries of summation and binomial coefficient

$$= \sum_{r=0}^{8}\binom{8}{r}3^{8-r}x^{8-r}(-1)^r x^{-3r}$$

$$= \sum_{r=0}^{8}\binom{8}{r}3^{8-r}(-1)^r x^{8-4r}$$ — 1mk. Simplifies by combining x terms

Alternate solution:

$$\left(3x-\frac{1}{x^3}\right)^8 = \sum_{r=0}^{8}\binom{8}{r}(3x)^r\left(-\frac{1}{x^3}\right)^{8-r}$$

* Students may choose to write summation in reverse order (accepted with full marks if simplified accordingly).

$$= \sum_{r=0}^{8}\binom{8}{r}3^r x^r(-1)^{8-r}(x^{-3})^{8-r}$$

$$= \sum_{r=0}^{8}\binom{8}{r}3^r(-1)^{8-r}x^r x^{-24+3r}$$

$$= \sum_{r=0}^{8}\binom{8}{r}3^r(-1)^{8-r}x^{4r-24}$$

$$= \sum_{r=0}^{8}\binom{8}{r}3^r(-1)^8(-1)^{-r}x^{4r-24}$$

$$= \sum_{r=0}^{8}\binom{8}{r}3^r(-1)^r x^{4r-24}$$

$$= \sum_{r=0}^{8}\binom{8}{r}(-3)^r x^{4r-24}$$

b) $$(2+x-x^2)^4 = \left[(2+x)-x^2\right]^4$$

$$= \sum_{r=0}^{4}\binom{4}{r}(2+x)^r(-x^2)^{4-r}$$

$$= \binom{4}{0}1(x^8) + \binom{4}{1}(2+x)(-x^6) + \binom{4}{2}(4+4x+x^2)(x^4)$$

$$+ \binom{4}{3}(8+12x+6x^2+x^3)(-x^2) + \binom{4}{4}(16+32x+24x^2+8x^3+x^4)(1)$$

— 1mk. correct expansion

$\therefore x^4$ term

$$= \binom{4}{2}\times 4\times(x^4) + \binom{4}{3}\times(6x^2)\times(-x^2) + \binom{4}{4}\times 1(x^4)\times(1)$$ — 1mk. identification of relevant terms

$$= 24x^4 + (-24x^4) + x^4$$

$$= x^4$$ $\therefore$ Coefficient is 1. — 1mk. final answer

c) $$(2x+y)^5 = \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4y + \binom{5}{2}(2x)^3y^2 + \binom{5}{3}(2x)^2y^3$$

$$+ \binom{5}{4}(2x)y^4 + \binom{5}{5}y^5$$

$$= 1(32x^5) + 5(16x^4)y + 10(8x^3)y^2 + 10(4x^2)y^3 + 5(2x)y^4 + 1y^5$$

— 1mk. correct expansion

$\therefore$ Sum of coefficients $= 32+80+80+40+10+1$ — 1mk. evaluates coefficients

$= 243$ as required.

(d)(i) $\sqrt{3}\cos x + \sin x = R\cos(x-\alpha)$ $\quad \{R>0,\ 0^\circ<\alpha<90^\circ\}$

$= R[\cos x\cos\alpha + \sin x\sin\alpha]$

$= R\cos\alpha(\cos x) + R\sin\alpha(\sin x)$

$\therefore$ By comparison of coefficients:

$R\cos\alpha = \sqrt{3}$ — ①

$R\sin\alpha = 1$ — ② —— 1mk.

Identifies coefficients from trigonometric expansion & equates with appropriate values from given expression

①^2 + ②^2 : $R^2 = 4$

$R = 2$ $\quad \{R>0\}$

Sub R into ① & ②:

①: $2\cos\alpha = \sqrt{3}$ $\qquad$ ②: $2\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{2}$ $\qquad$ $\sin\alpha = \frac{1}{2}$

$\therefore \alpha = 30^\circ$ $\quad \{0^\circ<\alpha<90^\circ\}$

1 mk. combines simultaneous equations WITH WORKING (not sufficient for students to directly state $R = \sqrt{a^2+b^2}$ or $\tan\alpha = \frac{b}{a}$ without justification)

(ii) $\sqrt{3}\cos x + \sin x = 1$ $\quad \{0^\circ \le x \le 360^\circ\}$

$2\cos(x-30^\circ) = 1$ $\quad$ from part (i)

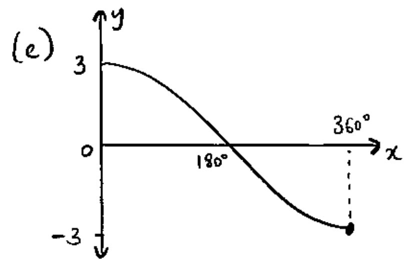
$\cos(x-30^\circ) = \frac{1}{2}$

1 mk. Solves for $x-30^\circ$ or equivalent if (i) not used

$x - 30^\circ = 60^\circ$ or 300° $\quad \{-30^\circ \le x-30^\circ \le 330^\circ\}$

$x = 90^\circ$ or 330° — 1 mk. both solutions

(e)



— 1 mk. Period/frequency, x-intercept

— 1 mk. Range/domain, shape

Question 3

(a)

$$\sqrt{\frac{4+4\cos 2x}{1-\cos 2x}}$$

$$= \sqrt{\frac{4(1+\cos 2x)}{1-\cos 2x}} \quad \text{Factor} ✓$$

$$= \sqrt{\frac{4(1+2\cos^2 x-1)}{1-(1-2\sin^2 x)}} ✓$$

$$= 2\sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$$

$$= 2\cot x ✓$$

OR

$$\sqrt{\frac{4(1+\cos^2 x-\sin^2 x)}{1-(\cos^2 x-\sin^2 x)}}$$

$$= 2\sqrt{\frac{(1-\sin^2 x)+\cos^2 x}{1-\cos^2 x+\sin^2 x}} ✓$$

$$= 2\sqrt{\frac{\cos^2 x+\cos^2 x}{\sin^2 x+\sin^2 x}}$$

$$= 2\sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$$

$$= 2\cot x ✓$$

(b) $\sin 2x = \tan x$

Let $t = \tan x$

$\therefore \sin 2x = \frac{2t}{1+t^2}$

$$\frac{2t}{1+t^2} = t$$

$$2t = t + t^3$$

$$t^3 + t - 2t = 0$$

$$t^3 - t = 0$$

$$t(t^2-1) = 0$$

$$t(t+1)(t-1) = 0$$

$t = 0$, $t = \pm 1$ ✓

$\tan x = 0$

$x = 0$ or 180°

$\tan x = 1$

$x = 45^\circ$

$\tan x = -1$

$x = 135^\circ$

$\therefore x = 0^\circ, 45^\circ, 135^\circ, 180^\circ$ ✓

OR Long method.

Let $t = \tan x/2$.

$$\tan x = \frac{2t}{1-t^2}$$

$$\sin 2x = 2\sin x\cos x$$

$$= 2\left(\frac{2t}{1+t^2}\right)\left(\frac{1-t^2}{1+t^2}\right)$$

$$\therefore \cancel{2}\left(\frac{2t}{1+t^2}\right)\left(\frac{1-t^2}{1+t^2}\right) = \frac{\cancel{2}t}{1-t^2}$$

$$2t(1-t^2)^2 = t(1+t^2)^2$$

$$t\left[2(1-t^2)^2 - (1+t^2)^2\right] = 0$$

$$t\left[\sqrt{2}(1-t^2) + (1+t^2)\right]\left[\sqrt{2}(1-t^2) - (1+t^2)\right] = 0$$

$$t\left[(\sqrt{2}+1) + t^2(1-\sqrt{2})\right]\left[(\sqrt{2}-1) - t^2(\sqrt{2}+1)\right] = 0$$

$t = 0$, $t^2 = \frac{(\sqrt{2}+1)}{\sqrt{2}-1}$, $t^2 = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$= (\sqrt{2}+1)^2$ $\quad = (\sqrt{2}-1)^2$

$t = \pm(\sqrt{2}+1)$ $\quad t = \pm(\sqrt{2}-1)$ ✓

$\tan\frac{x}{2} = 0$ $\quad \tan\frac{x}{2} = \sqrt{2}+1$ $\quad \tan\frac{x}{2} = (\sqrt{2}-1)$

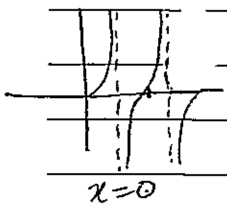
$\frac{x}{2} = 0,$ $\quad \frac{x}{2} = 67.5^\circ$ (using cal) $\quad \frac{x}{2} = 22.5^\circ$

$x = 45^\circ$

$x = 135^\circ$

$\tan x/2 = -(\sqrt{2}+1)$ ↳ Rejected out of domain

$\tan\frac{x}{2} = -(\sqrt{2}-1)$ ↳ Rejected out of domain



Check boundary value $x = 180^\circ$

$0 \leq x \leq 180^\circ$

$0 \leq \frac{x}{2} \leq 90^\circ$

$\left.\begin{array}{l}\sin 2x = 0 \\ \tan 180^\circ = 0\end{array}\right\} \therefore x = 180^\circ$ ✓

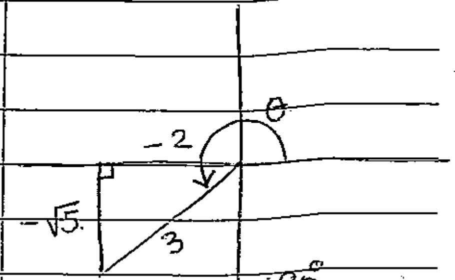
$x = 0, 45^\circ, 135^\circ, 180^\circ$

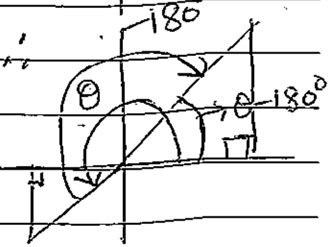
(c) $3\cos\theta + 2 = 0$

$\cos\theta = -\frac{2}{3} < 0$

$\tan\theta > 0$

$\therefore \theta$ is in 3rd Quad.



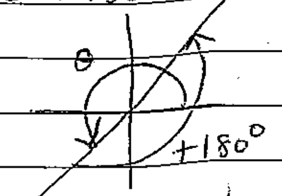


(i) $\theta - 180^\circ$ is in 1st Qua

$\therefore \tan(\theta - 180^\circ) = \frac{\sqrt{5}}{2}$ ✓

(ii) $\sin(\theta + 180^\circ)$

$\theta + 180^\circ$ in 1st Qua



$\sin(\theta + 180^\circ)$

$= \frac{\sqrt{5}}{3}$ ✓

d) $y=-2x+3 \quad x-3y+2=0$

$m_1=-2 \quad m_2=\frac{1}{3}$ ✓

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{-2-\frac{1}{3}}{1-\frac{2}{3}}\right|$ ✓ *including absolute value sign

$= 7$

$\therefore \theta = 82^\circ$ ✓

*should clearly differentiate from prev. step with $\tan\theta =$ and $\theta =$

$OA = \frac{10}{\tan 67^\circ}$ or $10\cot 67^\circ$

or $10\tan 27^\circ$

OA & OB expressions ✓

~~Assump~~

$AB^2 = OA^2 + OB^2 - 2OA \times OB \cos 20^\circ$

$= (10\tan 27^\circ)^2 + (10\tan 41^\circ)^2 - 2(10\tan 27^\circ)(10\tan 41^\circ) \times \cos 20^\circ$ ✓

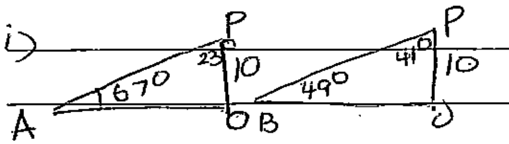
$\approx 4.9m$ ✓

3(e) As per the diagram.

(i) $\angle AOB = 360^\circ - 340^\circ$

$= 20^\circ$

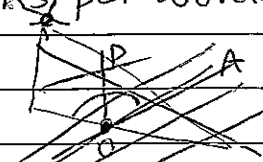
(ii)

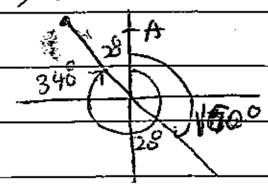


$OB = \frac{10}{\tan 49^\circ}$ or $10\cot 49^\circ$

or $10\tan 41^\circ$

(e) As per wordings





(i) $\angle AOB = 160^\circ$ ✓

OA = OB = ✓ See previous method*

$AB^2 = (10\tan 41^\circ)^2 + (10\tan 23^\circ)^2 - 2 \times (10\tan 41^\circ)(\tan 23^\circ) \cos 160^\circ$ ✓

$= 162.9313279\ldots$

$AB \approx 12.8$ m. ✓