NAME: ___________________________________

CLASS: 11MTX__

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2019

YEAR 11 AP1 EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed – 1 hour plus 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Write your name on the front page of this question/answer booklet.
- Attempt all questions. Marks are indicated to the right of each question.
- Answer in the space below each question or use the additional writing space provided at the back of this booklet, ensuring all answers are clearly labelled with the question number.
- All necessary working should be shown in every question, other than multiple choice.
- Full marks may not be awarded for careless, badly arranged or badly written work.
- NESA approved calculators may be used. You may bring one handwritten A4 reference sheet into the test.

Topic	**Total**
1 – Graphical relationships	/ 12
2 – Inequalities	/ 10
3 – Inverse functions	/ 12
4 – Factor and remainder theorems	/ 10
Total	**/ 44**

Question 1 **F1.1 Graphical relationships** **12 marks**

(a) If and , what is the domain of ? **1**

Circle the correct answer.

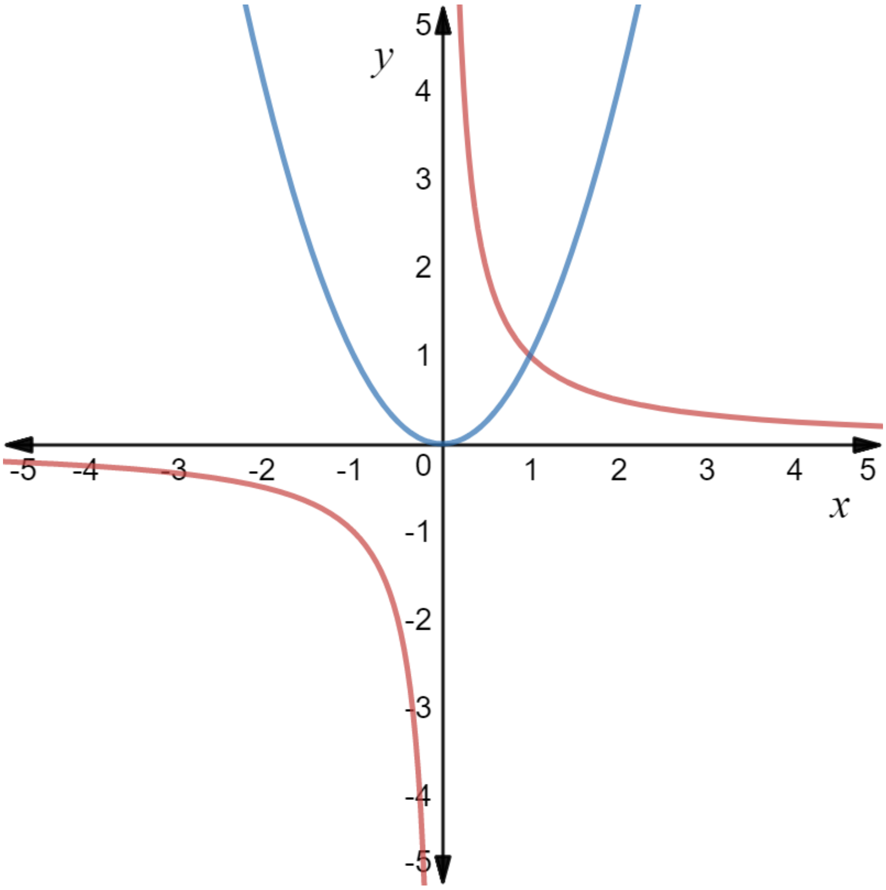
A. all real x

B.

C.

D.

(b) Below are the graphs of and **1**



Circle the correct answer which describes the **range** of .

A. the same as the range of

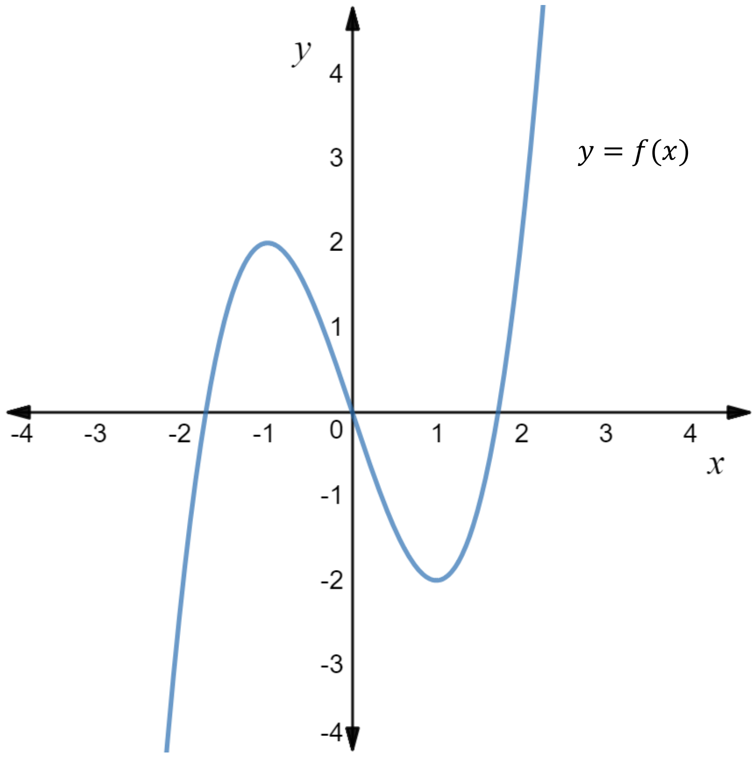
B. the same as the range of .

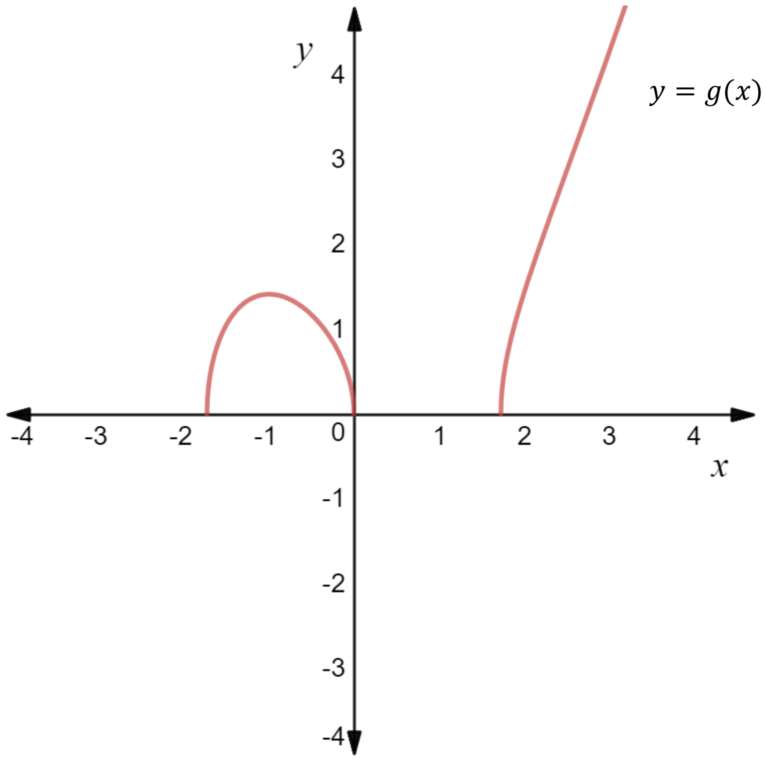
C. the intersection of the two ranges.

D. None of the above.

Question 1 continued

(c) Below is a graph and a new function Which equation describes ?





1

Circle the correct answer:

A.

B.

C.

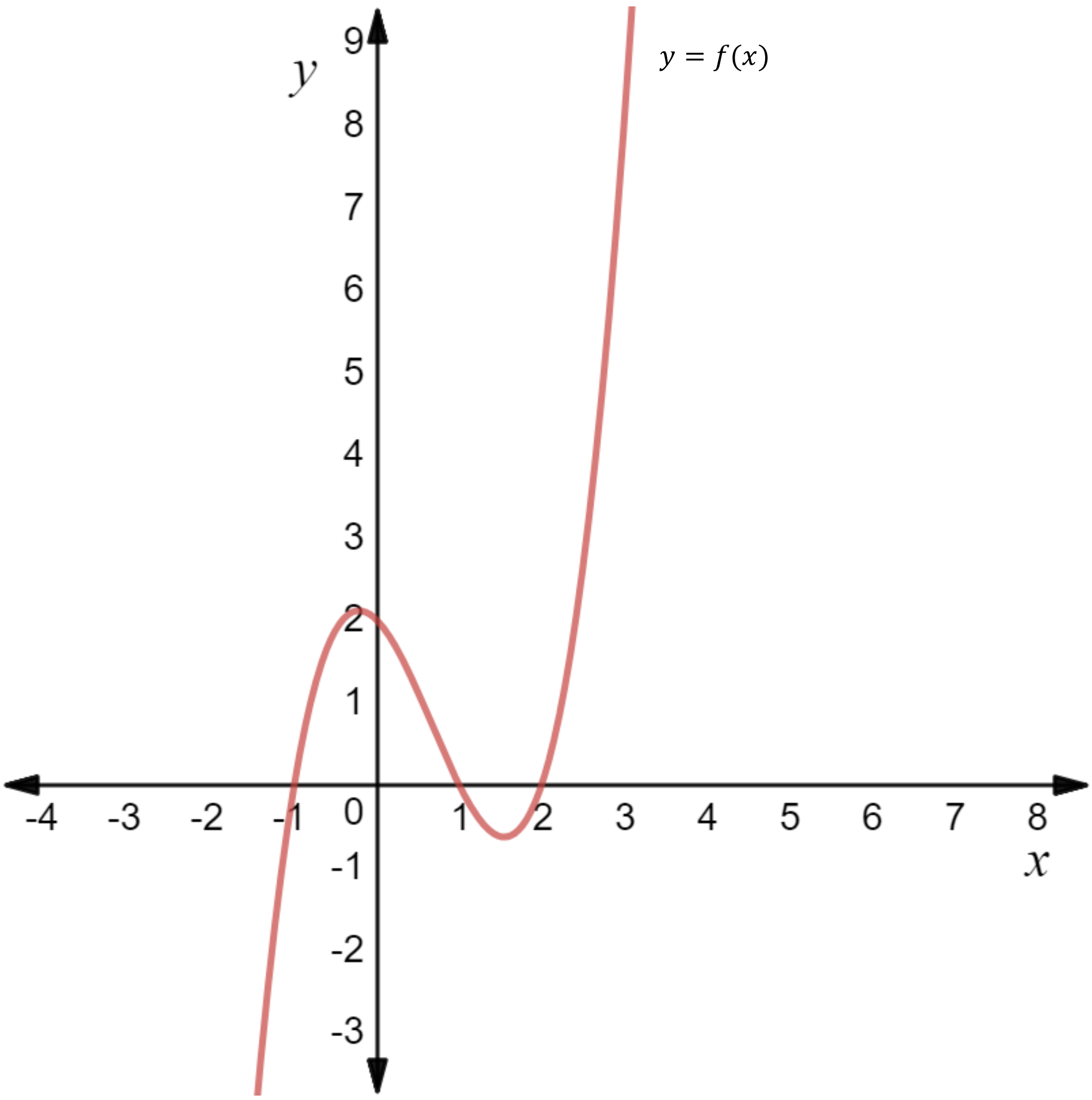
D.

Question 1 continued

(d) Describe in words **OR** illustrate with a suitable diagram, how you would draw the graph of if given the graph of **1**

(e) Below is the graph of a function . **2**

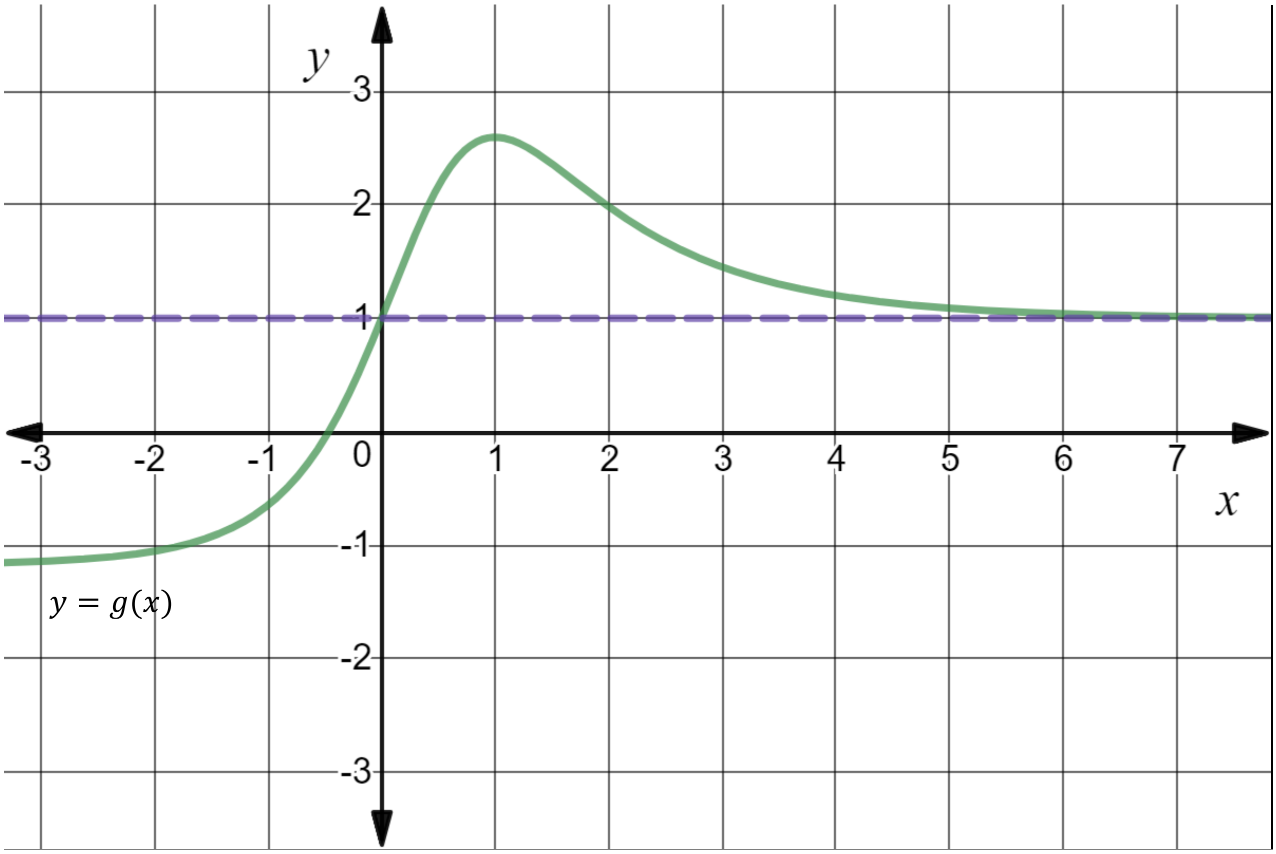
Sketch the graph on the same number plane.



Question 1 continued

(f) Below is the graph of a function . **3**

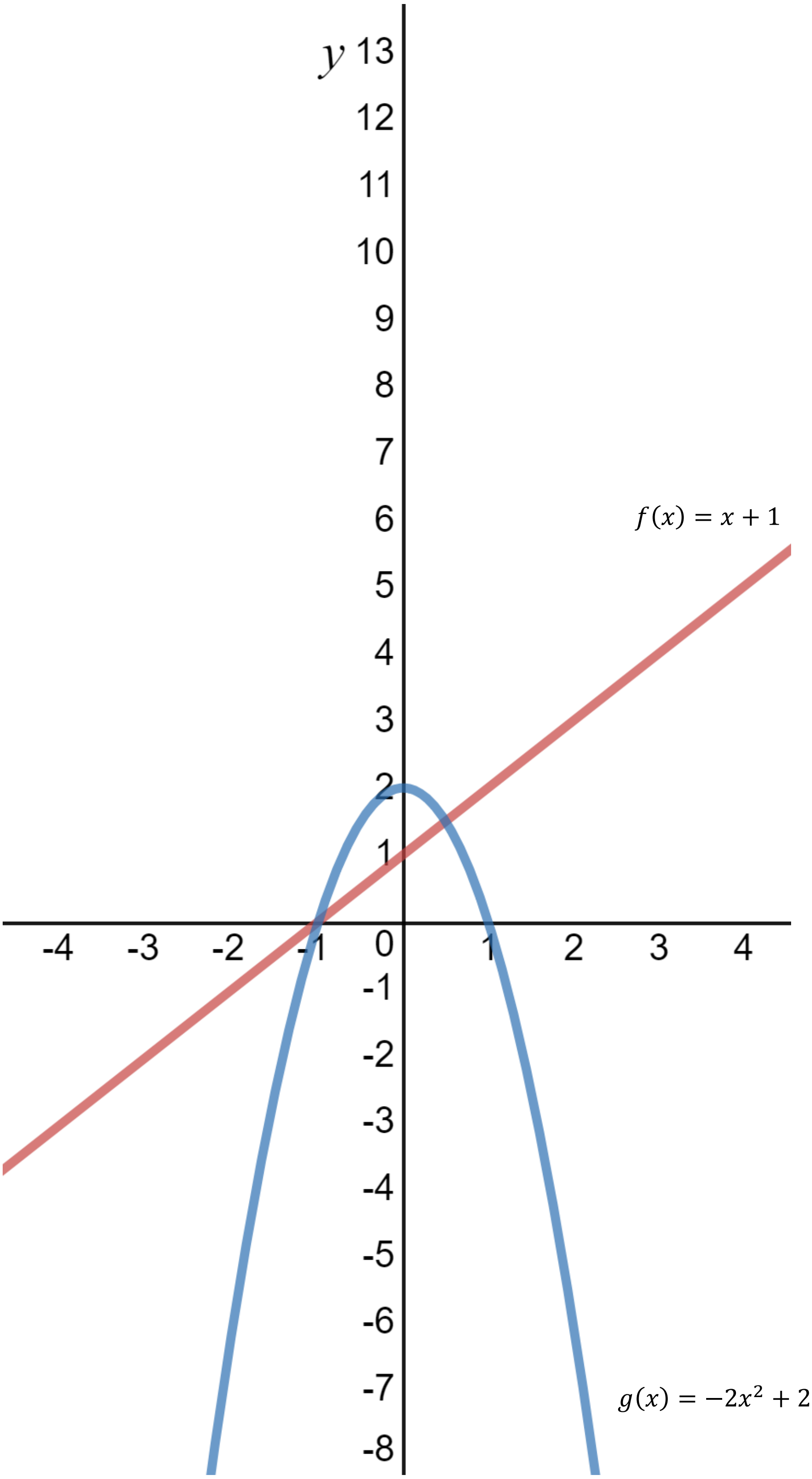
Sketch the graph on the same number plane, clearly marking all intercepts and asymptotes.



Question 1 continued

(g) On the number plane below are the graphs of two functions and . **3**

Sketch the graph of on the same number plane marking all key features.



Question 2 **F1.2 - Inequalities** **10 marks**

(a) Solve the inequation . Circle the correct answer. **1**

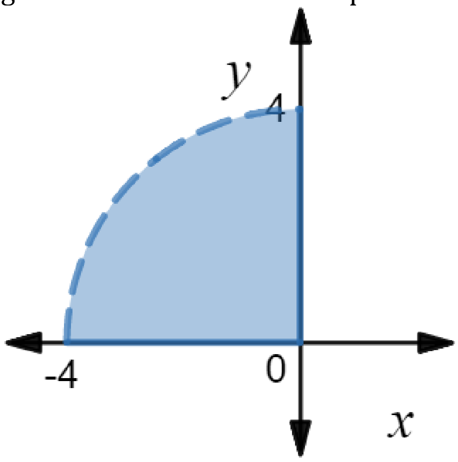
A.

B.

C.

D. No solution

(b) The region shaded on the number plane below is described by which inequalities? **1**



Circle the correct answer

A.

B.

C.

D.

(c) Solve . **2**

Question 2 continued

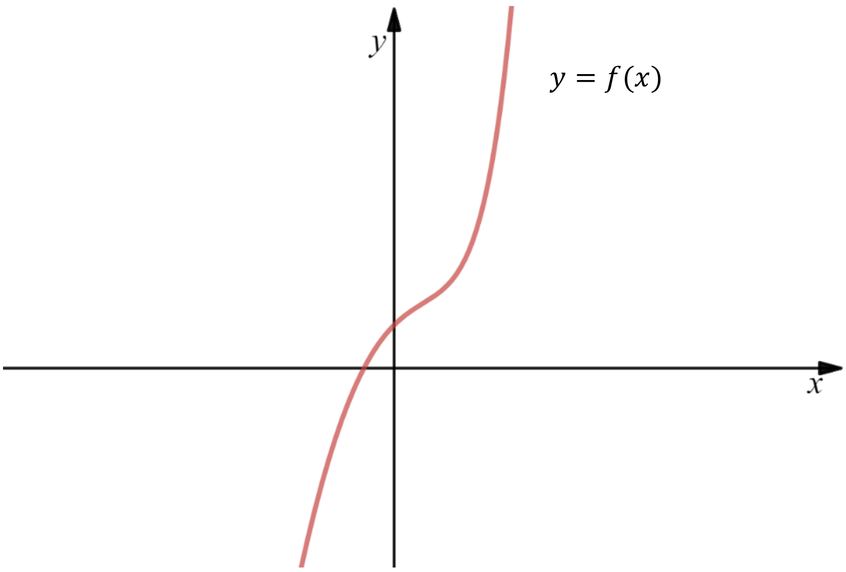
(d) Solve and sketch your solution on the number line below. **3**

(e) Solve **3**

Question 3 **F1.3 - Inverse Functions** **12 marks**

(a) If find the equation of the inverse function . **1**

(b) Below is the graph of a function **1**



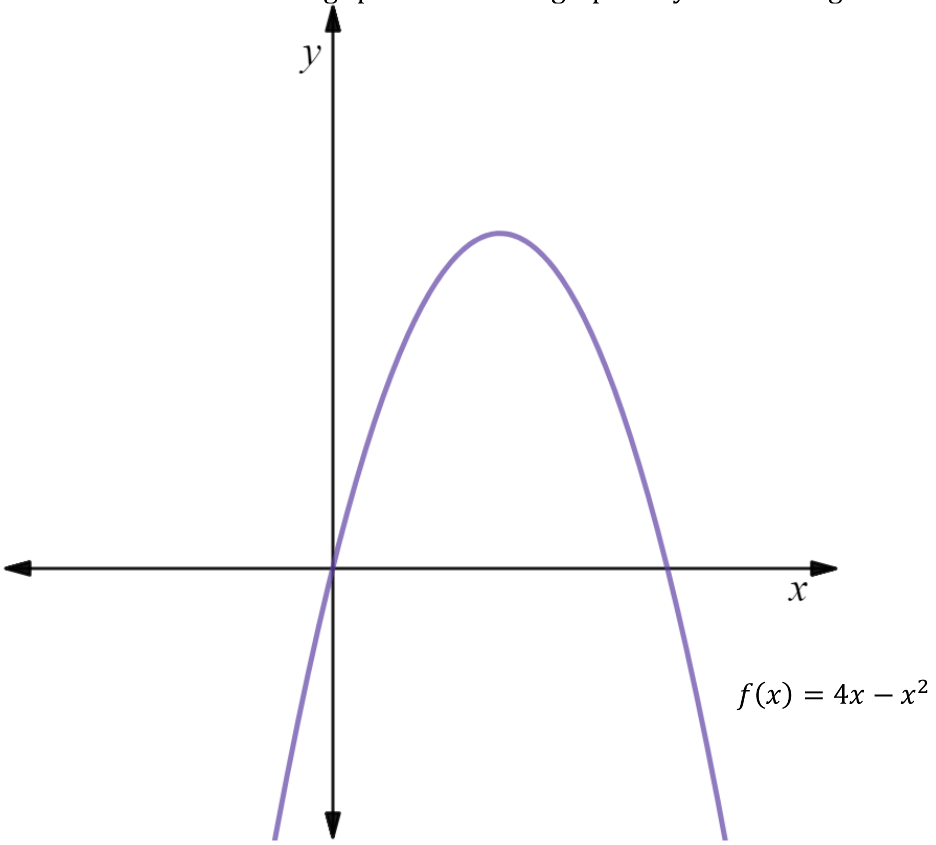
State whether the inverse is a function and **explain** your answer

Question 3 continued

(c) Below is the graph of .

i) Find the value of all intercepts and the vertex and mark these on the diagram. **1**

You should use the writing space below the graph for your working.



ii) Suggest a restriction on the domain of so that the inverse is a function with maximum possible range. **1**

iii) Sketch the inverse function on the graph above (using your restricted domain from part **ii**) ensuring that all critical points are clearly marked. **2**

Question 3 continued

(d) A function is defined with

i) Show that **1**

ii) Hence, show that the inverse and state any restrictions on the range of the inverse function. **2**

Question 3 continued

(d) **iii)** Sketch the graph of clearly marking all critical points. **3**

Question 4 **F2.1 – Remainder and factor theorems** **10 marks**

(a) Which of the following is the correct solution to the equation ? **1**

Circle the correct answer

A.

B.

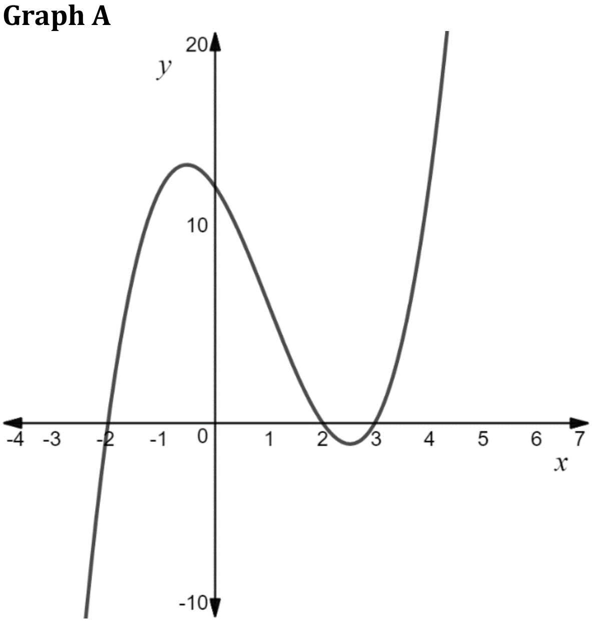
C.

D.

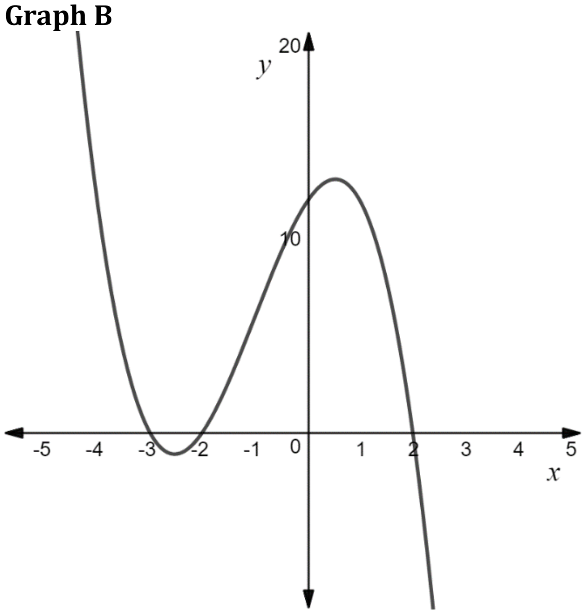
(b) A polynomial has as a factor and .
Which of the following is a possible graph of

Circle the correct graph. **1**

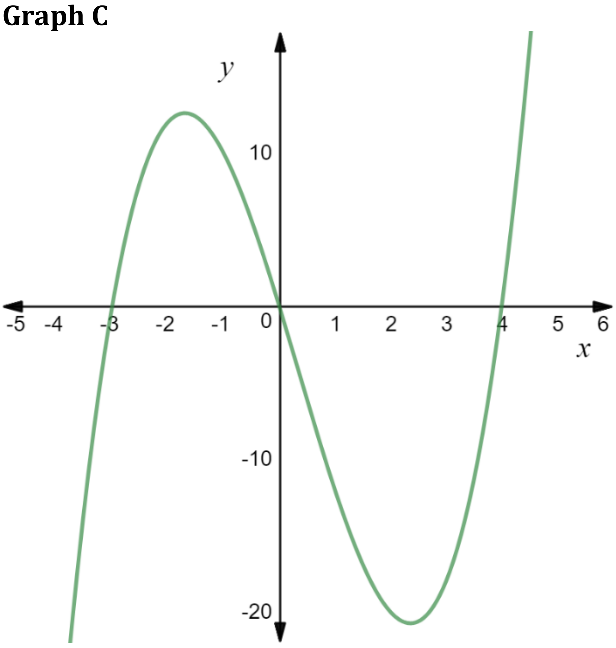
Graph A



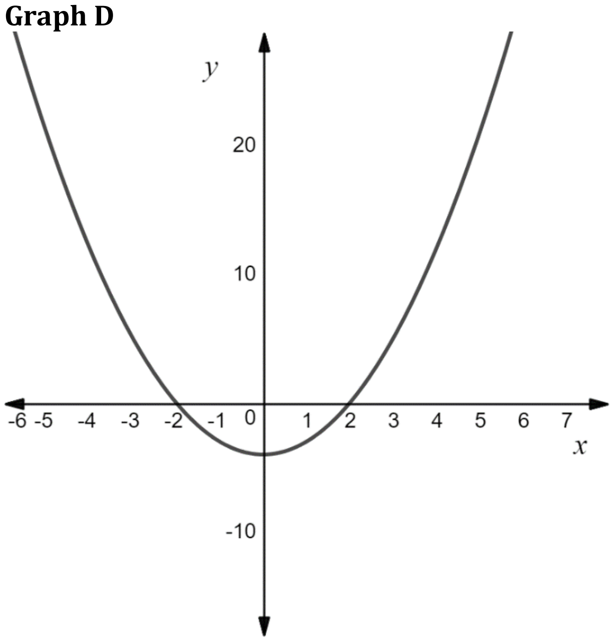
Graph B



Graph C



Graph D



Question 4 continued

(c) Find the remainder when is divided by . **1**

(d) Show that is a factor of the polynomial . **1**

Question 4 continued

(e) Sketch the graph in the blank space below, marking all intercepts. **4**
Use the writing space for your working.

Question 4 continued

(f) The polynomials and have a common factor where .

Show that **2**

END OF TEST

Use the lines below if you need additional space.
Ensure your work is **clearly labelled with the question number**.

NAME: Solutions

CLASS: 11MTX__

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2019

YEAR 11 AP1 EXAMINATION

MATHEMATICS EXTENSION 1

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Topic	Total
1 – Graphical relationships	/ 12
2 – Inequalities	/ 10
3 – Inverse functions	/ 11
4 – Factor and remainder theorems	/ 10
Total	**/ 43**

Question 1 **F1.1 Graphical relationships** **12 marks**

(a) If $f(x) = |1 - 4x|$ and $g(x) = \sqrt{x}$, what is the domain of $f \circ g(x)$?

Circle the correct answer.

A. all real x

B. $x \geq 0$

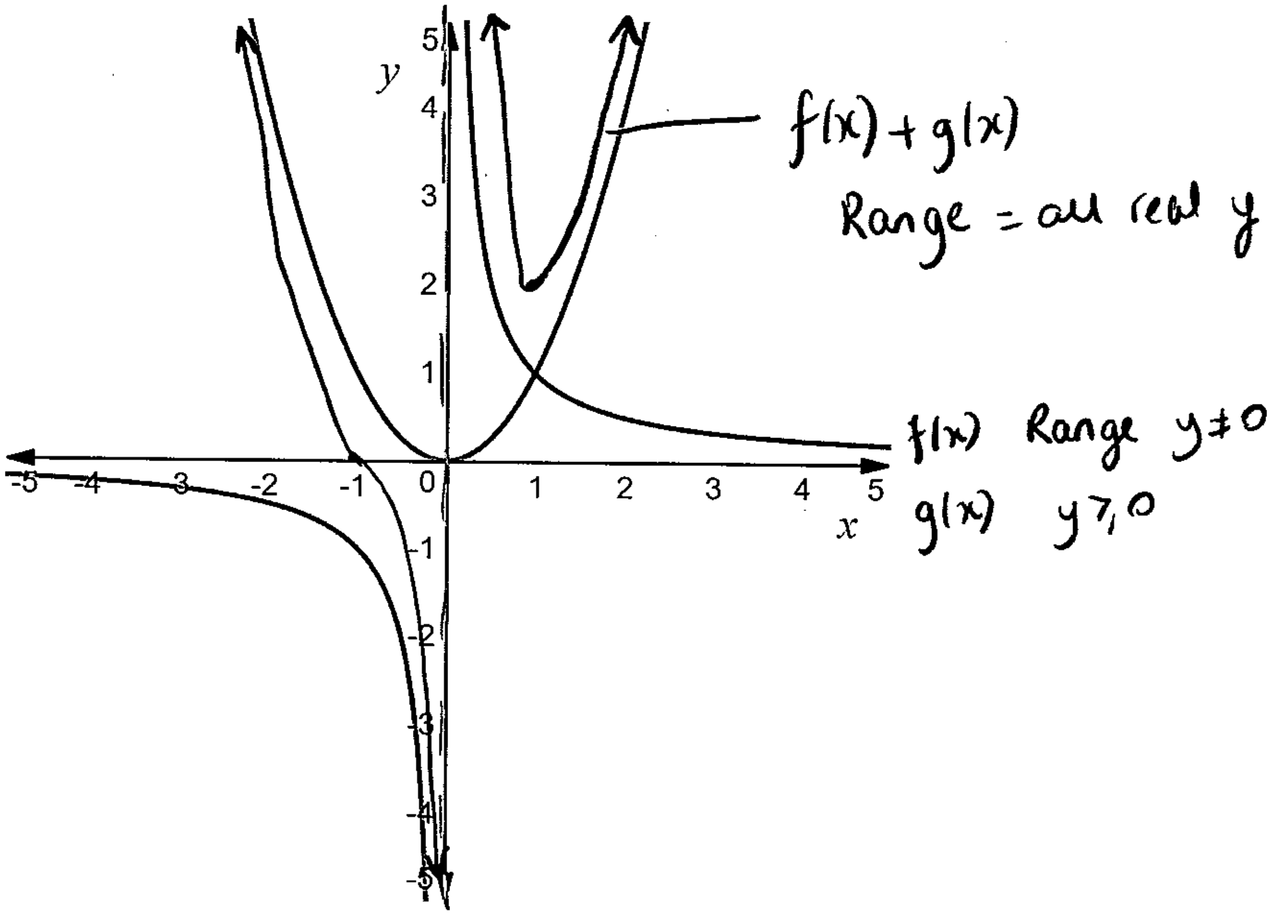
C. $x > 0$

D. $x \geq \frac{1}{4}$

$f \circ g(x) = |1 - 4\sqrt{x}|$

only defined for $x \geq 0$

(b) Below are the graphs of $f(x) = \frac{1}{x}$ and $g(x) = x^2$ **1**



Circle the correct answer which describes the the **range** of $f(x) + g(x)$.

A. the same as the range of $f(x)$. ✗

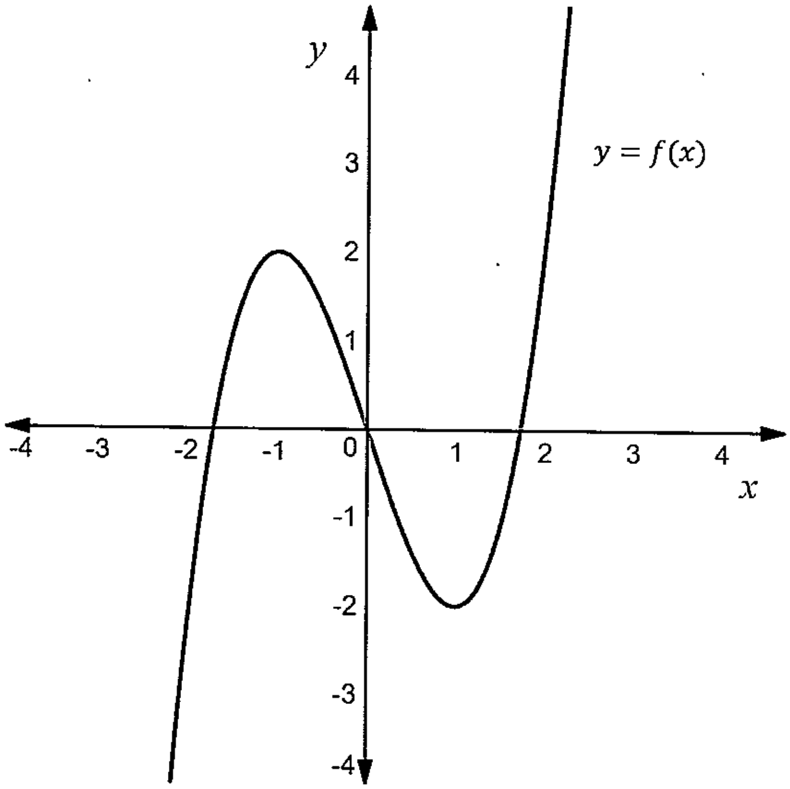
B. the same as the range of $g(x)$. ✗

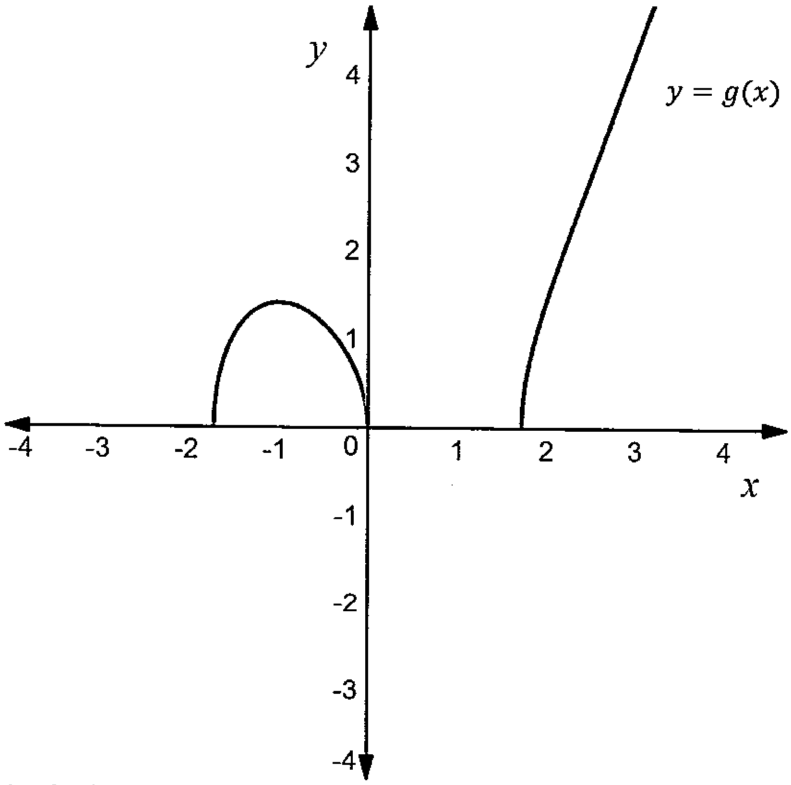
C. the intersection of the two ranges. ✗

D. None of the above.

Question 1 continued

(c) Below is a graph $y = f(x)$ and a new function $y = g(x)$. Which equation describes $g(x)$?





1

Circle the correct answer:

A. $g(x) = \sqrt{f(x)}$

B. $g(x) = |f(x)|$

C. $g(x) = f^{-1}(x)$

D. $g(x) = f(|x|)$

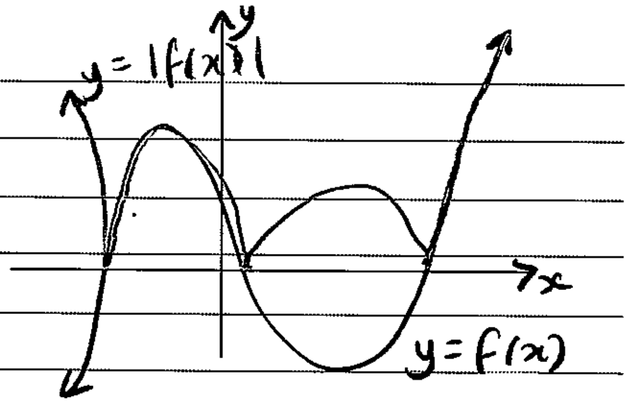
square root function is only defined for $f(x) > 0$

Question 1 continued

(d) Describe in words **OR** illustrate with a suitable diagram, how you would draw the graph of $y = |f(x)|$ if given the graph of $y = f(x)$. **1**

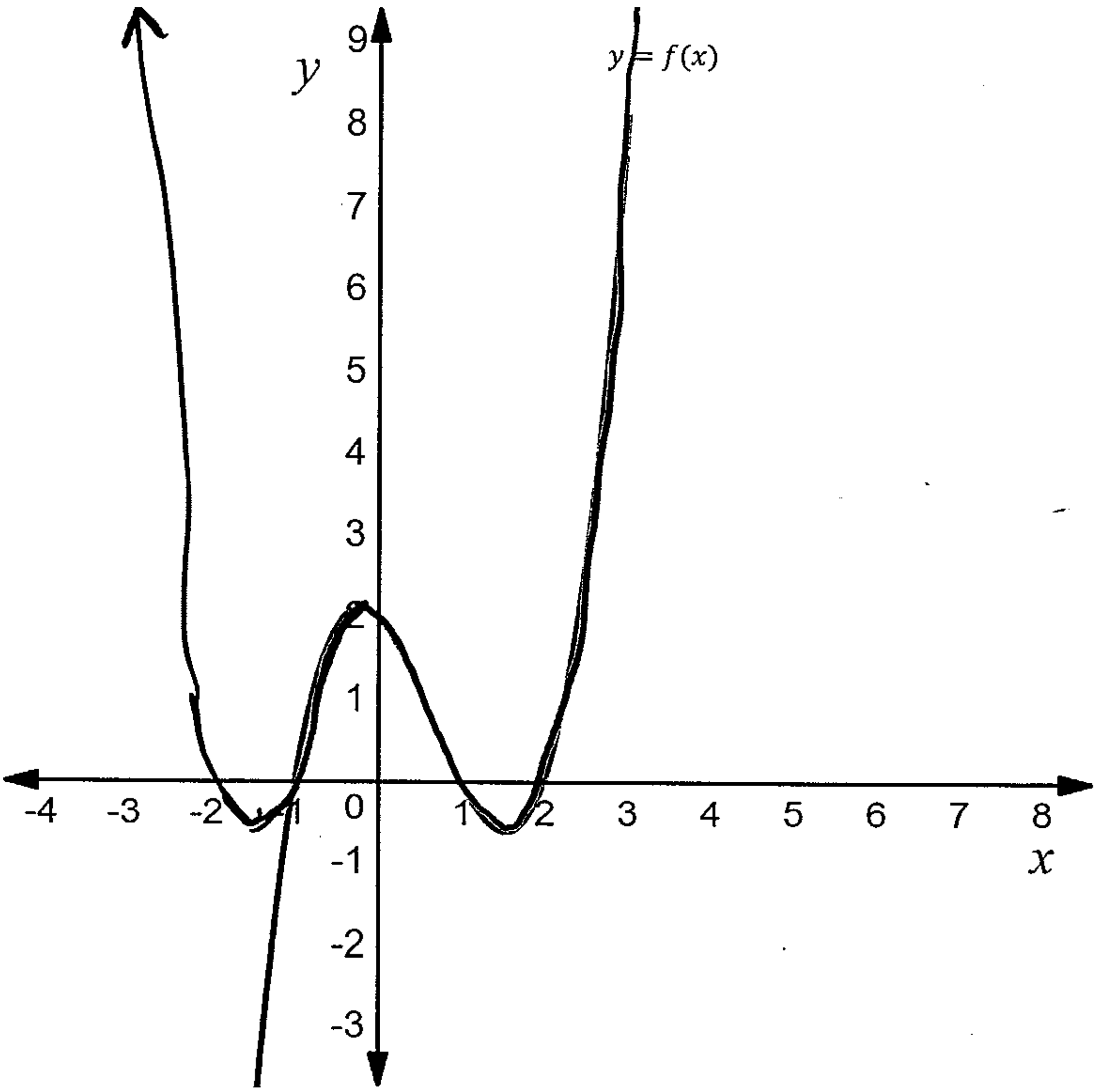
Reflect any parts below the x-axis (Q3, Q4) above the x-axis

OR



(e) Below is the graph of a function $y = f(x)$. **2**

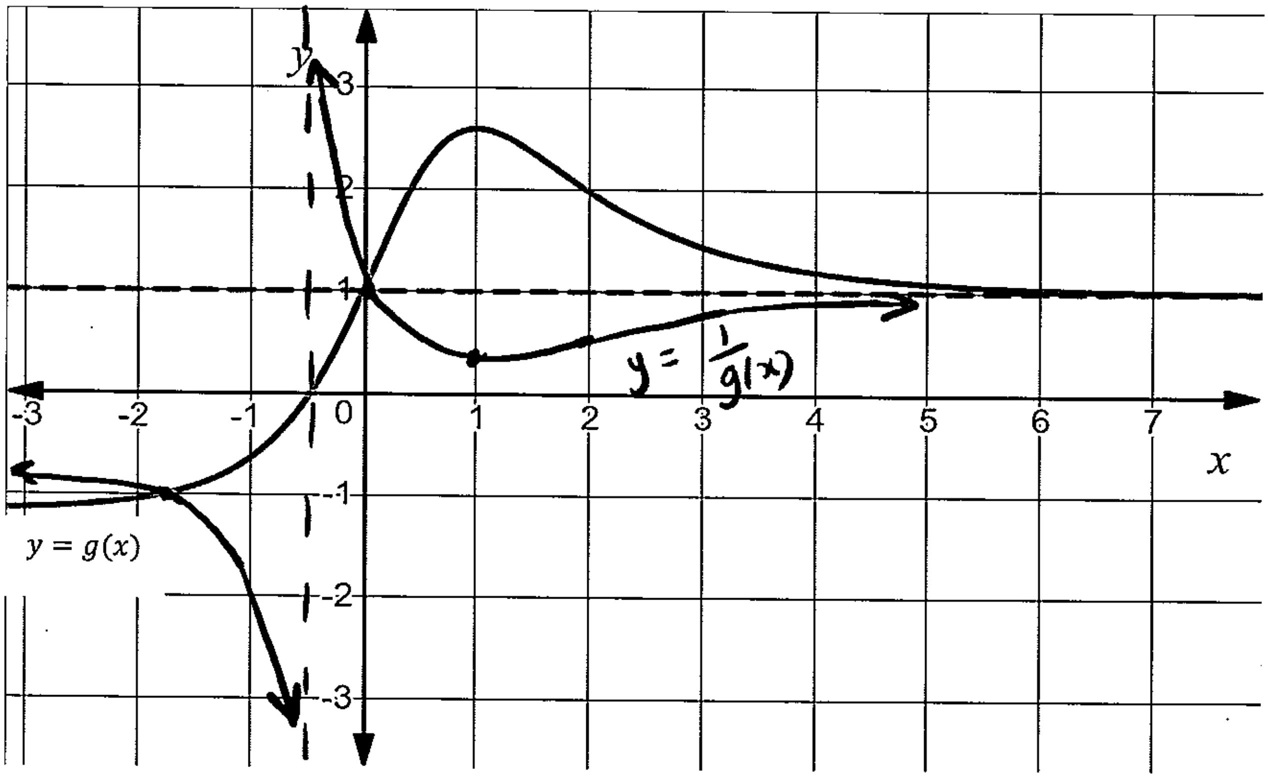
Sketch the graph $y = f(|x|)$ on the same number plane.



Question 1 continued

(f) Below is the graph of a function $y = g(x)$. **3**

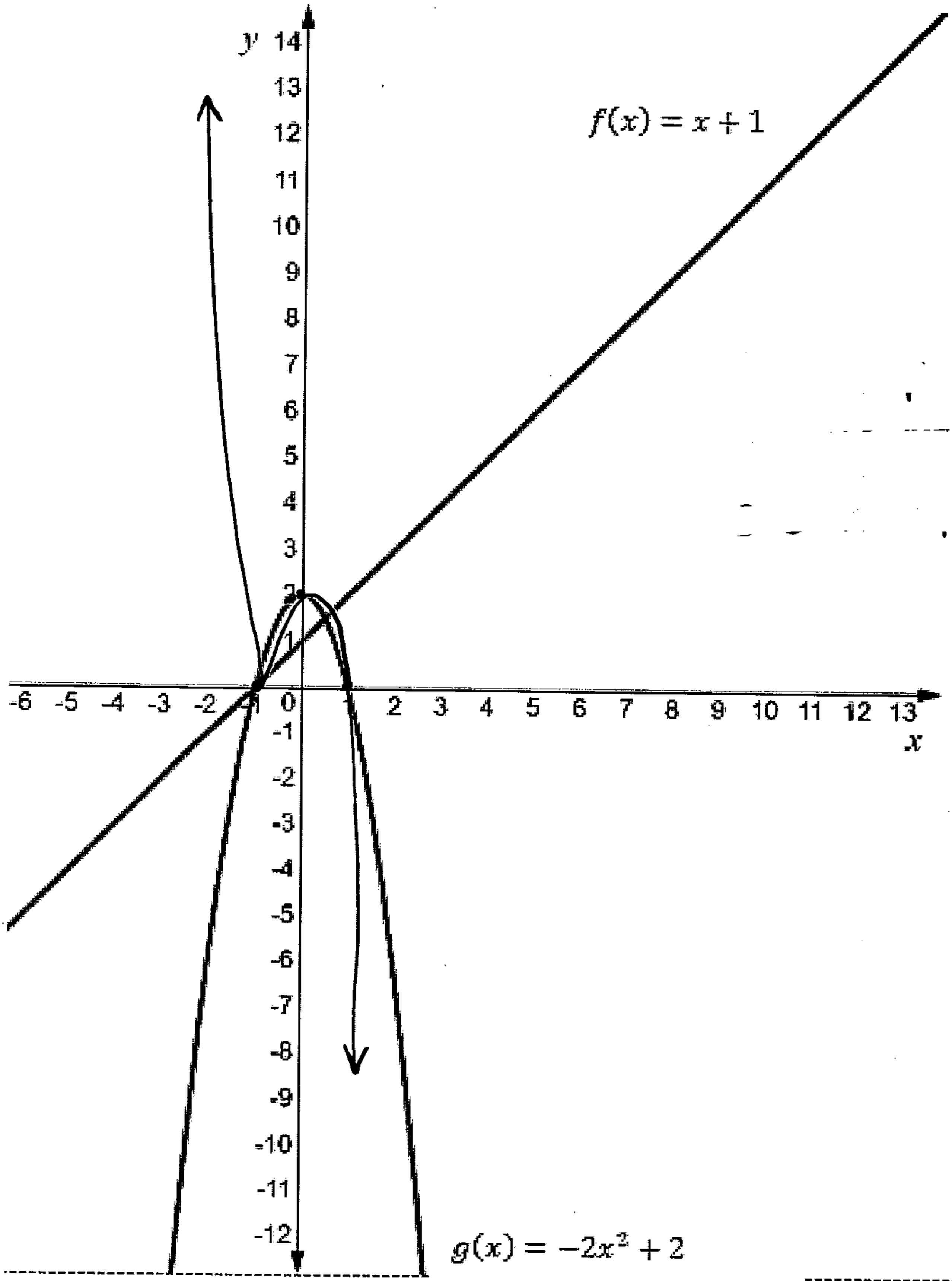
Sketch the graph $y = \dfrac{1}{g(x)}$ on the same number plane, clearly marking all intercepts and asymptotes.



Question 1 continued

(g) t On the number plane below are the graphs of two functions $f(x) = x + 1$ and $g(x) = -2x^2 + 2$. **3**

Sketch the graph of $h(x) = f(x)g(x)$ on the same number plane marking all key features.



Question 2 **F1.2 - Inequalities** **10 marks**

(a) Solve the inequation $\frac{|-x|}{8} \le 2$. Circle the correct answer. **1**

A. $x \ge 16$

B. $x \le -16$

C. $-16 \le x \le 16$

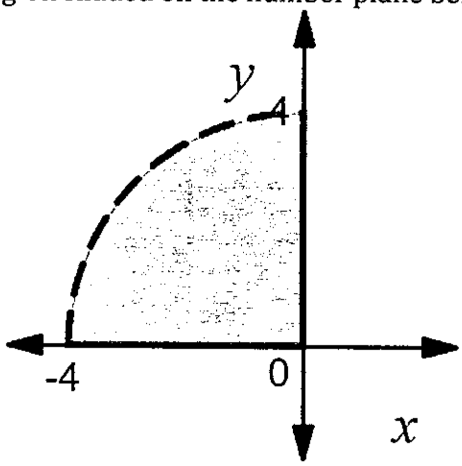
D. No solution

$|-x| \le 16$

$|x| \le 16$

$-16 \le x \le 16$

(b) The region shaded on the number plane below is described by which inequalities? **1**



Circle the correct answer

A. $x^2 + y^2 \ge 16$ and $x \le 0$ or $y \ge 0$

B. $x^2 + y^2 > 4$ and $x \le 0$ and $y \ge 0$

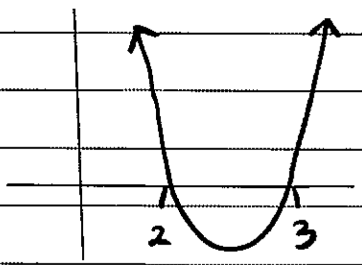
C. $x^2 + y^2 < 16$ and $x \le 0$ and $y \ge 0$

D. $x^2 + y^2 \le 4$ and $x \le 0$ and $y \ge 0$

(c) Solve $x^2 - 5x + 6 < 0$. **2**

$(x-2)(x-3) < 0$

Roots $x = 2, 3$



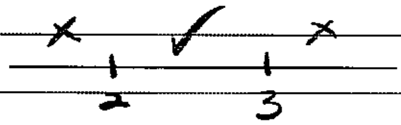
$2 < x < 3$

OR

Let $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$



$\therefore 2 < x < 3$

Question 2 continued

(d) Solve $|2x+1| \geq 8$ and sketch your solution on the number line **3**

$2x+1 \geq 8$, $2x+1 \leq -8$

$2x \geq 7$, $2x \leq -9$

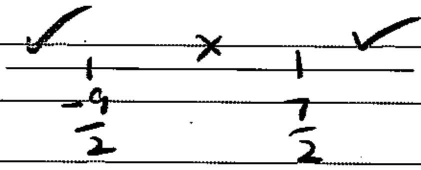
$x \geq \frac{7}{2}$, $x \leq -\frac{9}{2}$

or

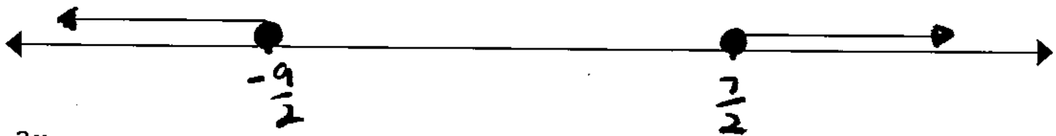
Let $|2x+1| = 8$

$2x+1 = 8$, $2x+1 = -8$

$x = \frac{7}{2}$, $x = -\frac{9}{2}$



$x \leq -\frac{9}{2}$, $x \geq \frac{7}{2}$



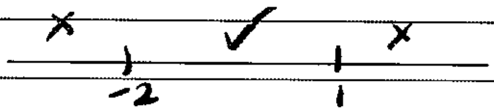
(e) Solve $\frac{3x}{x-1} \leq 2$ **3**

Let $\frac{3x}{x-1} = 2$ $\quad x \neq 1$

$3x = 2(x-1)$

$3x - 2x + 2 = 0$

$x = -2$



$-2 \leq x < 1$

OR

$x \neq 1$

$3x(x-1) \leq 2(x-1)^2$

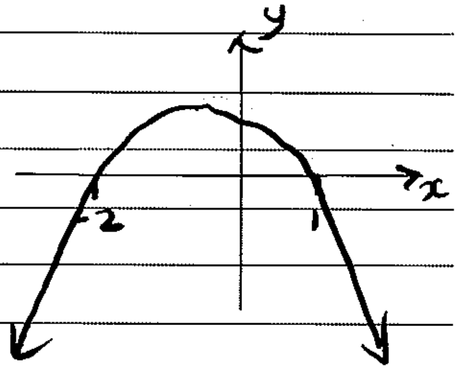
$(x-1)(2(x-1) - 3x) \geq 0$

$(x-1)(-2-x) \geq 0$

$-(x-1)(2+x) \geq 0$

Roots: $x = 1, -2$

concave down



$-2 \leq x < 1$

Question 3 **F1.3 - Inverse Functions** **12 marks**

(a) If $f(x) = \frac{2}{3}x - 5$ find the equation of the inverse function $f^{-1}(x)$. **1**

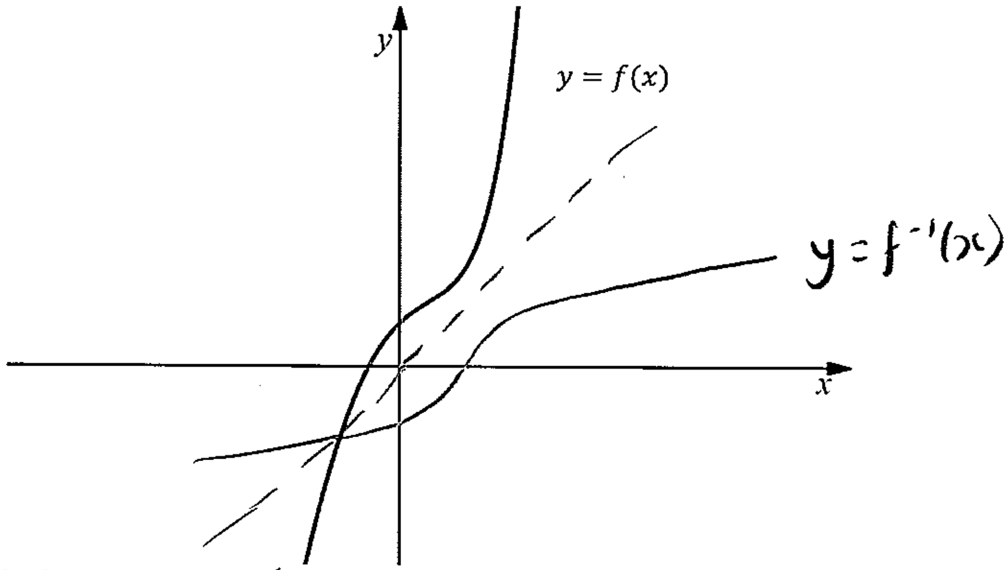
$f(x): \quad y = \frac{2}{3}x - 5$

$f^{-1}(x): \quad x = \frac{2}{3}y - 5$

$x + 5 = \frac{2}{3}y$

$y = \frac{3}{2}(x+5)$ OR $y = \frac{3x}{2} + \frac{15}{2}$

(b) Below is the graph of a function $y = f(x)$. **1**



State whether the inverse $f^{-1}(x)$ is a function and **explain** your answer

$f(x)$ is a function as $f(x)$ passes the horizontal line test.

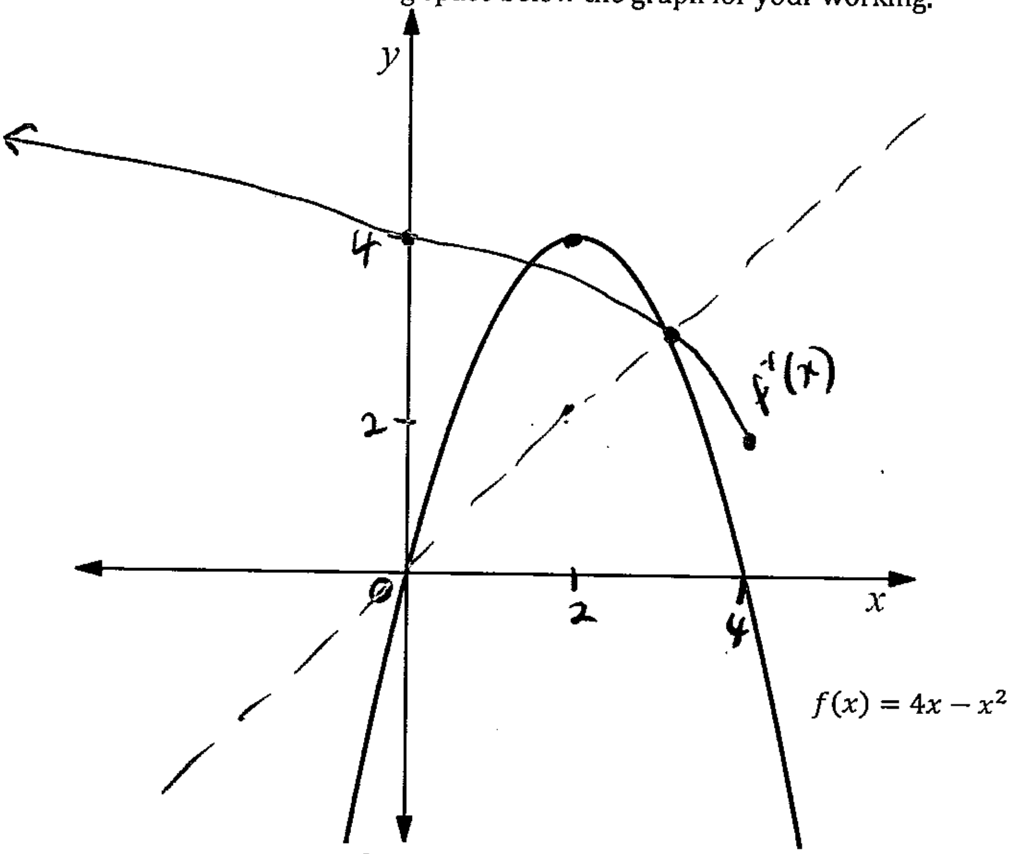
OR $f^{-1}(x)$ is a function as it passes the vertical line test $f^{-1}(x)$ must be drawn on graph as above

Question 3 continued

(c) Below is the graph of $f(x) = 4x - x^2$.

i) Find the value of all intercepts and the vertex and mark these on the diagram. **1**

You should use the writing space below the graph for your working.



x-int: $4x - x^2 = 0$

$x(4-x) = 0$

$x = 0, 4$

vertex $x = \frac{0+4}{2}$

$= 2$

$y = 4(2) - 2^2$

$= 8 - 4$

$= 4$

ii) Suggest a restriction on the domain of $f(x)$ so that the inverse $f^{-1}(x)$ is a function with maximum possible range. **1**

$x \geq 2$ OR $x \leq 2$

iii) Sketch the inverse function $f^{-1}(x)$ on the graph above (using your restricted domain from part **ii)**) ensuring that all critical points are clearly marked. **2**

Question 3 continued

(d) A function is defined $f(x) = \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1}$, $x \neq -1, x \neq \frac{3}{2}$.

i) Show that $\frac{5}{(x+1)(2x-3)} + \frac{1}{x+1} = \frac{2}{2x-3}$ **1**

$$LHS = \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1}$$

$$= \frac{5 + (2x-3)}{(x+1)(2x-3)}$$

$$= \frac{2\cancel{(x+1)}}{\cancel{(x+1)}(2x-3)}$$

$$= \frac{2}{2x-3}$$

SHOW THAT

ii) Hence, show that the inverse $f^{-1}(x) = \frac{1}{x} + \frac{3}{2}$ .and state any restrictions on the range of the ~~function~~. inverse. **2**

$$f(x): \quad y = \frac{2}{2x-3}$$

$$f^{-1}(x): \quad x = \frac{2}{2y-3}$$

$$2y-3 = \frac{2}{x}$$

$$y = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{x} + 3\right)$$

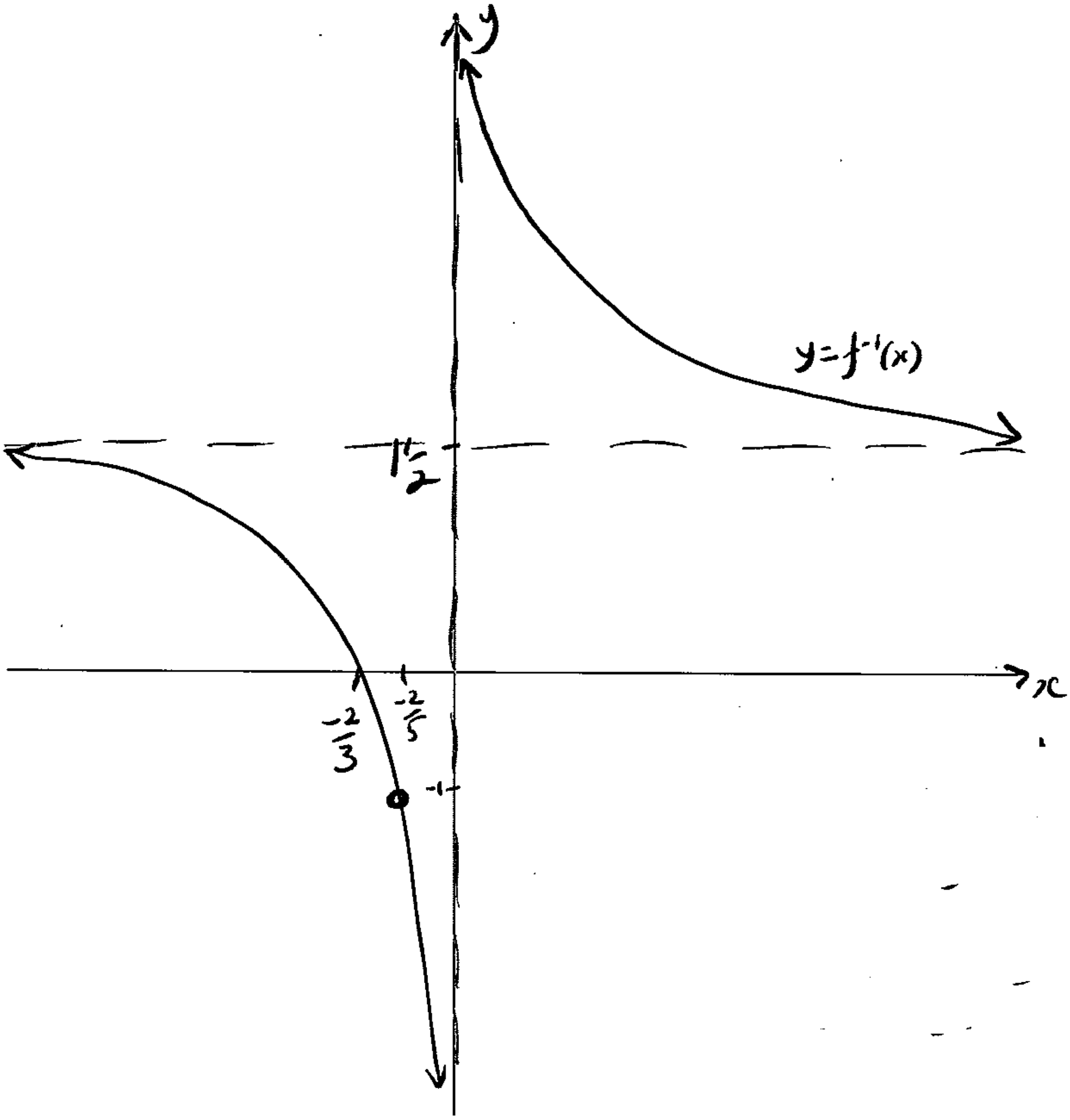
$$y = \frac{1}{x} + \frac{3}{2}$$

SHOW THAT

Restrictions on range $y \neq -1, y \neq \frac{3}{2}$

Question 3 continued

(d) **iii)** Sketch the graph of $f^{-1}(x)$ clearly marking all critical points. 2/3



x-intercept: $y=0$ $\quad \frac{1}{x}+\frac{3}{2}=0 \quad \frac{1}{x}=\frac{-3}{2} \quad x=-\frac{2}{3}$

undefined at $y=-1$ $\quad \frac{1}{x}+\frac{3}{2}=-1 \quad \frac{1}{x}=\frac{-5}{2} \quad x=-\frac{2}{5}$

Question 4 **F2.1 – Remainder and factor theorems** **10 marks**

(a) Which of the following is the correct solution to the equation $x^3 - 3x^2 + 4 = 0$? **1**

Circle the correct answer

$P(-1) = 0 \quad \therefore x+1$ is a factor

A. $x = -1$

B. $x = -1, 2$

C. $x = 1, -2$

D. $x = 1, -2, 2$

$x^3 - 3x^2 + 4 = (x+1)(x^2-4x+4)$

$= (x+1)(x-2)^2$

$\therefore$ roots $-1, 2$

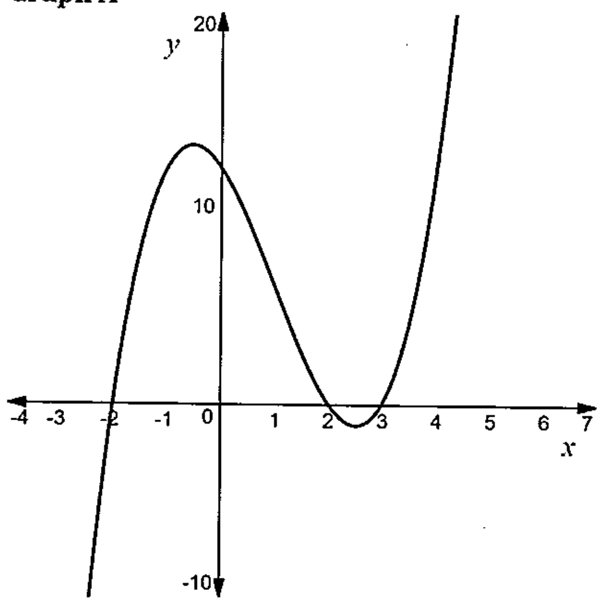
(b) A polynomial $P(x)$ has $x^2 - 4$ as a factor and $P(-3) = 0$. Which of the following is a possible graph of $P(x)$? **1**

$(x-2)(x+2)(x+3)$ are factors

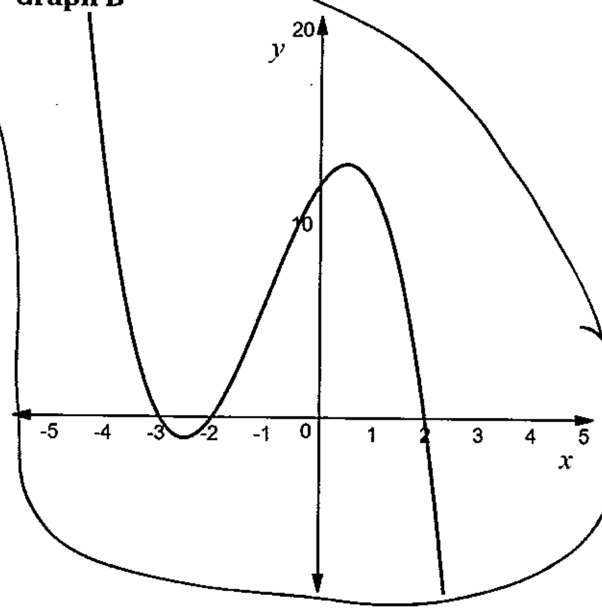
$\therefore$ Roots $2, -2, -3$

Circle the correct graph.

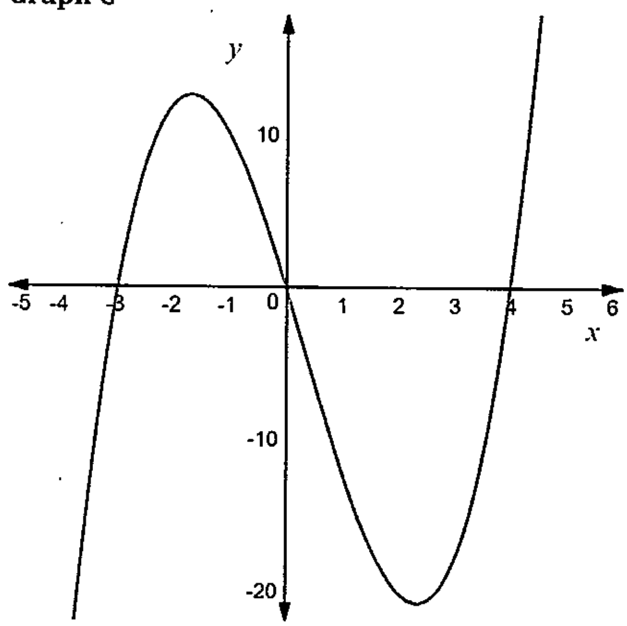
Graph A



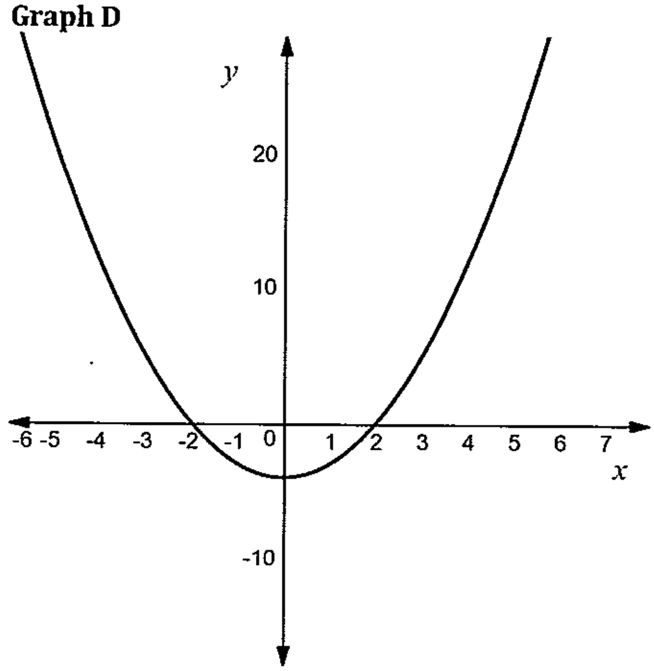
Graph B



Graph C



Graph D



Question 4 continued

(c) Find the remainder when $x^3 - x^2 + x - 4$ is divided by $x - 3$. 1

$P(x) = x^3 - x^2 + x - 4$

$P(3) = 3^3 - 3^2 + 3 - 4$

$= 27 - 9 + 3 - 4$

$= 17$

$\therefore$ Remainder $= 17$

OR

$$\begin{array}{r l}
 & x^2 + 2x + 7 \\
x-3 \,) & x^3 - x^2 + x - 4 \\
- & x^3 - 3x^2 \\
 & 2x^2 + x \\
 & 2x^2 - 6x \\
 & 7x - 4 \\
- & 7x - 21 \\
 & 17
\end{array}$$

$\therefore$ Remainder $= 17$

(d) Show that $x + 1$ is a factor of the polynomial $2x^3 - x^2 - 4x - 1$. 1

$P(x) = 2x^3 - x^2 - 4x - 1$

$P(-1) = 2(-1)^3 - (-1)^2 - 4(-1) - 1$

$= -2 - 1 + 4 - 1$

$= 0$

$P(-1) = 0 \therefore x+1$ is a factor

OR

$$\begin{array}{r l}
 & 2x^2 - 3x - 1 \\
x+1 \,) & 2x^3 - x^2 - 4x - 1 \\
- & 2x^3 + 2x^2 \\
 & -3x^2 - 4x \\
- & -3x^2 - 3x \\
 & -x - 1 \\
- & -x + 1 \\
 & 0
\end{array}$$

$R(x) = 0 \therefore x+1$ is a factor

Question 4 continued

(e) Sketch the graph $y = 2x^3 - 5x^2 - 9x + 18$ in the blank space below, marking all intercepts. **4**
Use the writing space for your working.

$P(x) = 2x^3 - 5x^2 - 9x - 18$

$P(-2) = 2(-2)^3 - 5(-2)^2 - 9(-2) + 18$

$= -16 - 20 + 18 + 18$

$= 0$

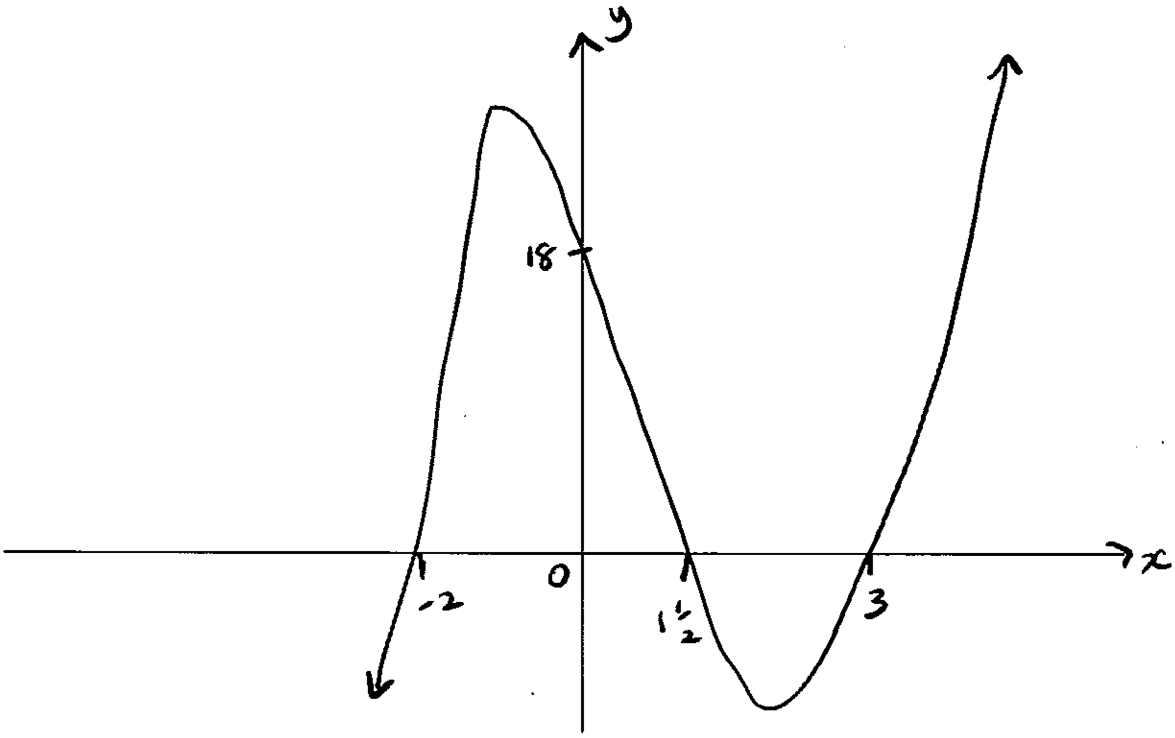
$\therefore$ $x+2$ is a factor

$$\begin{array}{r} 2x^2 - 9x + 9 \\ x+2 \overline{\smash{)}\, 2x^3 - 5x^2 - 9x + 18} \\ - \quad 2x^3 + 4x^2 \\ \hline -9x^2 - 9x \\ - \quad -9x^2 - 18x \\ \hline 9x + 18 \\ 9x + 18 \\ \hline 0 \end{array}$$

$2x^3 - 5x^2 - 9x + 18 = (x+2)(2x^2 - 9x + 9)$

$= (x+2)(2x-3)(x-3)$

$\therefore$ Roots $x = -2, \frac{3}{2}, 3$ y-intercept $= 18$



Question 4 continued

(f) The polnomials $P(x) = x^2 - ax - b$ and $Q(x) = 2x^2 + b$ have a common factor $x + c$ where $a, b, c \neq 0$.

Show that $b = \dfrac{2ac}{3}$. **2**

$P(-c) = 0$ $\quad$ $Q(c) = 0$

$P(-c) = (-c)^2 - a(-c) - b$ $\quad$ $Q(c) = 2(c)^2 + b$

$\therefore c^2 + ac - b = 0$ ① $\quad$ $\therefore 2c^2 + b = 0$ ②

From ② $c^2 = \dfrac{-b}{2}$

sub into ①

SHOW THAT

$\dfrac{-b}{2} + ac - b = 0$

$\dfrac{-3b}{2} + ac = 0$

$\dfrac{3b}{2} = ac$

$b = \dfrac{2ac}{3}$

END OF TEST