GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC ASSESSMENT

TASK 1 2024

(December 2023)

ANSWERS COVER SHEET

Student Number:__________________________________.

QUESTION	MARK	ME12-1	ME12-2	ME12-3	ME12-4	ME12-5	ME12-6	ME12-7
1-5	/5			√				√
6	/12			√				√
7	/11			√				√
8	/13			√				√
9	/8			√				√
10	/16	√						√
11	/8			√				√
TOTALS	/73	/16		/57				/73

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1 applies techniques involving proof or calculus to model and solve problems

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-5 applies appropriate statistical processes to present, analyse and interpret data

ME12-6 chooses and uses appropriate technology to solve problems in a range of contexts

ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

2024 HSC Task 1 *(December 2023)*

Student Number:____

This Booklet contains the answer sheet for Part A and Writing Booklet for Part B.

Part A ANSWER SHEET

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Instructions

- If you need more paper for Questions 6-11, please ask your supervisor.
- Write your name on every booklet you use.
- Write on both sides of each sheet of paper.
- Start each question on a new page.

Total number of booklets used ______________.

GIRRAWEEN HIGH SCHOOL

HSC TASK 1 2024 *(December 2023)*

MATHEMATICS

EXTENSION 1

Time allowed – 90 Minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used. Reference sheets will be provided.
- Each question attempted is to be marked clearly Question 6, Question 7 etc. Each question is to be returned on a separate page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.
- For Questions 6-11: All necessary working should be shown in each question. Marks may be deducted for careless or badly arranged work.

Multiple Choice begins on the following page

For Multiple choice Questions 1-5: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.

Question 1

If $\tan\theta = \sqrt{2}$, the exact value of $\tan\left(\boldsymbol{\theta} - \frac{\pi}{4}\right) =$

(A) $3 + 2\sqrt{2}$

(B) $3 - 2\sqrt{2}$

(C) $\sqrt{2} - 1$

(D) $\sqrt{2} + 1$

Question 2

If $\cos\alpha = \frac{4}{5}$, $0 < \alpha < \frac{\pi}{2}$, then $\sin(\theta + \alpha) =$

(A) $\frac{3\sin\theta + 4\cos\theta}{5}$

(B) $\frac{3\sin\theta - 4\cos\theta}{5}$

(C) $\frac{4\sin\theta + 3\cos\theta}{5}$

(D) $\frac{4\sin\theta - 3\cos\theta}{5}$

Question 3

If $t = tan\left(\frac{\theta}{2}\right), sec^2\boldsymbol{\theta} =$

(A) $\frac{(1-t^2)^2}{4t^2}$

(B) $\frac{4t^2}{(1-t^2)^2}$

(C) $\frac{(1+t^2)^2}{4t^2}$

(D) $\frac{(1+t^2)^2}{(1-t^2)^2}$

Multiple Choice continues on the following page

Multiple Choice *(continued)*

Question 4

$\sin\left(\frac{\pi}{6} + 2x\right) - \sin\left(\frac{\pi}{6} - 2x\right) =$

(A) $\sqrt{3}\sin 2x$

(B) $\frac{1}{2}\sin 2x$

(C) $\sqrt{3}\cos 2x$

(D) $\frac{1}{2}\cos 2x$

Question 5

If $\frac{2\tan\theta}{1-tan^2\theta} = -\sqrt{3}, 0 \leq \boldsymbol{\theta} \leq \frac{\pi}{2}$ then $\tan\theta =$

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $-\sqrt{3}$

(D) $-\frac{1}{\sqrt{3}}$

Examination continues on the following page

Question 6 ***(12 marks) (Start solutions on a new page)*** **Marks**

(a) Use the compound angle and double angle formulae to find the exact value of

(i) $\sin 110^o \cos 10^o + \cos 110^o \sin 10^o$ **2**

(ii) $\cos 40^o \cos 20^o - \sin 40^o \sin 20^o$ **2**

(b) Use the formula for $\sin(\alpha - \beta)$ to show that $\sin 15^o = \frac{\sqrt{6}-\sqrt{2}}{4}$. **3**

(c) If $\tan\theta = -\frac{3}{4}, \frac{\pi}{2} < \boldsymbol{\theta} < \boldsymbol{\pi},$

(i) Show that $\tan 2\theta = -\frac{24}{7}$ **2**

(ii) By using $\tan 3\theta = \tan(2\theta + \theta)$, show that $\tan 3\theta = \frac{117}{144}$ **3**

Question 7 ***(11 marks) (Start solutions on a new page)***

(a) Solve for $\theta, 0 \leq \boldsymbol{\theta} \leq \boldsymbol{2\pi},$

(i) $2\sin 2\theta - \cos\theta = 0$ **3**

(ii) $2\cos 2\theta + 3\sin\theta = 1$ **3**

(b) If $\cos\alpha = \frac{1}{\sqrt{5}}$, $0 < \alpha < \frac{\pi}{2}$ and $\sin\beta = \frac{1}{\sqrt{10}}$, $0 < \beta < \frac{\pi}{2}$

(i) Show using the $\cos(\alpha - \beta)$ formula that $\alpha - \beta = \frac{\pi}{4}$. **3**

(ii) Hence or otherwise, show that $\sin 2\alpha = \cos 2\beta$. **2**

Examination continues on the following page

Question 8 ***(13 marks) (Start solutions on a new page)*** **Marks**

(a) Using the auxiliary angle method and choosing a substitution so that $0 < \alpha < \frac{\pi}{2}$, solve $3\sin\theta - 2\cos\theta = 1$ for $0 \leq \boldsymbol{\theta} \leq \mathbf{2\pi}$. **6**

(b) **(i)** Express $3\cos\theta + 3\sqrt{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and α is in degrees and minutes: **4**

(ii) Hence solve $3\cos\theta + 3\sqrt{3}\sin\theta = 2, 0 \leq \boldsymbol{\theta} \leq \mathbf{360^o}$. **3**

Question 9 ***(8 marks) (Start solutions on a new page)***

(a) Solve using the t formulae for $0 \leq \boldsymbol{\theta} \leq \mathbf{2\pi}$ where $t = \tan\frac{\theta}{2}$,

$5\sin\theta - 3\cos\theta = 3$ **4**

(b) If $t = \tan\frac{\theta}{2}$, prove using the t formulae that $\frac{\cos\theta+\sin\theta-1}{\cos\theta-\sin\theta+1} = tan\frac{\theta}{2}$ **4**

Examination continues on the following page

Question 10 ***(16 marks) (Start solutions on a new page)*** **Marks**

Prove each of the following using the method of mathematical induction:

(a) $\sum_{k=1}^{n}(4k+2) = 2n^2 + 4n$ for all positive integers n. **4**

(b) $5^n + 23$ is divisible by 4 for all positive integers n. **4**

(c) $n^2 + 6n$ is divisible by 8 for all EVEN positive integers n. **4**

(d) $(2^2-1)(3^2-1)\dots.(n^2-1) = \frac{(n!)^2(n+1)}{2n}$ for all integers $n \geq 2$. **4**

Question 11 ***(8 marks) (Start solutions on a new page)***

(a) Use the sums-to-products formulae to show that **2**

$\cos 80^o + \cos 40^o = \cos 20^o$.

(b) Use the sums -to-products formulae to show that **2**

$2\sin 35^o \cos 25^o + 2\cos 20^o \sin 10^o = \frac{\sqrt{3}+1}{2}$.

(c) **(i)** Show that $2\sin x(\cos 2x + \cos 4x + \cos 6x) = \sin 7x - \sin x$. **2**

(ii) Hence show that $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$. **2**

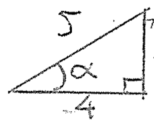
END OF EXAMINATION

Page 1

Mathematics Assessment Solutions Y11 December 2023

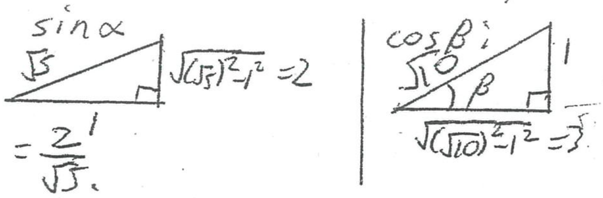
Girraween High School

☐ Year 11	☑ Extension 1	Year: 2024
☑ Year 12	☐ Extension 2	Task Number 1

Suggested solutions	Marker's Feedback
Multiple Choice: (1) If $\tan\theta = \sqrt{2}$, $\tan(\theta - \frac{\pi}{4}) = \dfrac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta\tan\frac{\pi}{4}}$ $= \dfrac{\sqrt{2}-1}{1+\sqrt{2}\times 1} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$ $= \dfrac{3-2\sqrt{2}}{1}$ $= 3-2\sqrt{2}$ B	B
(2) $\sin(\theta+\alpha)$ $= \sin\theta\cos\alpha + \cos\theta\sin\alpha$ $= \frac{3}{5}\cos\alpha + \frac{4}{5}\sin\alpha$ $= \dfrac{3\cos\alpha + 4\sin\alpha}{5}$ $= \dfrac{4\sin\alpha + 3\cos\alpha}{5}$ Note: As $\cos\alpha = \frac{4}{5}$  $\sin\alpha = \frac{3}{5}$.	C
(3) $\sec^2\theta = \dfrac{1}{\cos^2\theta}$ $= \dfrac{1}{\left(\frac{1-t^2}{1+t^2}\right)^2}$ $= \dfrac{(1+t^2)^2}{(1-t^2)^2}$. D	D
(4) By $\frac{1}{2}[\sin(A+B) - \sin(A-B)] = \cos A \sin B$, $\times 2$ $\sin(\frac{\pi}{6}+2x) + \sin(\frac{\pi}{6}-2x) = 2\cos\frac{\pi}{6}\sin 2x$ A $= \sqrt{3}\sin 2x$.	A
(5) As $\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$, A $\tan 2\theta = -\sqrt{3}$. $2\theta = \dfrac{2\pi}{3}$ [as $0 \le 2\theta \le \pi$]. $\theta = \dfrac{\pi}{3} \Rightarrow \tan\theta = \sqrt{3}$.	A

Page 2

Suggested solutions Q.6 = 12	Marker's Feedback
Q.(6)(a)(i) $\sin 110^\circ \cos 10^\circ + \cos 110^\circ \sin 10^\circ$ $= \sin(110^\circ + 10^\circ)$ $= \sin 120^\circ$ $= \frac{\sqrt{3}}{2}$ 2	Few wrote $\sin 120 = \frac{1}{2}$
(ii) $\cos 40^\circ \cos 20^\circ - \sin 40^\circ \sin 20^\circ$ $= \cos(40^\circ + 20^\circ)$ $= \cos 60^\circ$ $= \frac{1}{2}$ 2	Few wrote $\cos 60 = \sqrt{3}$
(b) $\sin 15^\circ$ $= \sin(45^\circ - 30^\circ)$ $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$. 3	Mostly done well.
(c)(i) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ $= \frac{2 \times -\frac{3}{4}}{1-(-\frac{3}{4})^2}$ $= \frac{-\frac{3}{2}}{(\frac{7}{16})}$ $= -\frac{24}{7}$ 2	Those who used calculator got 1 mark. $\tan\theta = -\frac{3}{4}$ $\theta = -0.6435$ $2\theta = -1.287$ $\tan 2\theta = \frac{-24}{7}$
(ii) $\tan 3\theta = \tan(2\theta + \theta)$ 3 $= \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$ $= \frac{-\frac{24}{7} - \frac{3}{4}}{1 - -\frac{24}{7} \times -\frac{3}{4}}$ $\times 28$ / $\times 28$ $= \frac{-96-21}{28-72}$ $= \frac{-117}{-44}$ $= \frac{117}{44}$.	One strange mistake $\tan(2\theta + \theta)$ $= \tan(\frac{-24}{7} - \frac{3}{4})$ $= \frac{\tan(\frac{-24}{7}) + \tan(\frac{-3}{4})}{1 - \tan(\frac{-24}{7})\tan(\frac{-3}{4})}$

Suggested solutions Q7: (11)	Marker's Feedback
Q.(7)(a)(i) $2\sin 2\theta - \cos\theta = 0$ $4\sin\theta\cos\theta - \cos\theta = 0$ $\cos\theta(4\sin\theta - 1) = 0$ $\quad$ 3 Either $\cos\theta = 0$ or $\sin\theta = \frac{1}{4}$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, 0{\cdot}2527, 2{\cdot}8889.$	ai, Expansion well done! Most students are leaving their answers in degree instead of radians.
(ii) $2\cos 2\theta + 3\sin\theta = 1.$ $2(1-2\sin^2\theta) + 3\sin\theta = 1.$ $\rightarrow$ [& simplify] $4\sin^2\theta - 3\sin\theta\ 1 = 0$ $(4\sin\theta+1)(\sin\theta-1) = 0$ $\quad$ 3 $\sin\theta = -\frac{1}{4}$ or $\sin\theta = 1$ $\theta = 3{\cdot}3943, 6{\cdot}0310$ (4DP), $\frac{\pi}{2}$	aii, Expansion well done. Some students struggled to use Quadratic formula correctly. Some students failed to solve the trig equation further hence only wrote $\pi/2$ as their answer
(b)(i) Need $\sin\alpha$ & $\cos\beta$.  $=\frac{2}{\sqrt{5}}$. $\quad$ $\cos\beta = \frac{3}{\sqrt{10}}$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $= \frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}$ $\quad$ 3 $= \frac{3+2}{5\sqrt{2}}$ $= \frac{1}{\sqrt{2}}$. As $\cos(\alpha-\beta) = \frac{1}{\sqrt{2}}$ & $\alpha > \beta$ [as $\cos\alpha < \cos\beta$ & $0 < \alpha$ or $\beta < \frac{\pi}{2}$]. $\alpha - \beta = \frac{\pi}{4}$.	Very well done. Most students got full marks
(ii) If $\alpha - \beta = \frac{\pi}{4}$ $2\alpha - 2\beta = \frac{\pi}{2}$ $\quad +2\beta - \frac{\pi}{2}$ $\quad$ 2 $2\alpha - \frac{\pi}{2} = 2\beta$ $\cos(2\alpha - \frac{\pi}{2}) = \cos 2\beta$. $\cos(\frac{\pi}{2} - 2\alpha) = \cos 2\beta$ [as cos is even $\rightarrow \cos(-\theta) = \cos\theta$]. $\sin 2\alpha = \cos 2\beta$ [as $\cos[\frac{\pi}{2} - \theta] = \sin 2\theta$]. PTO for "otherwise" solution →.	Very well done. Students solved it in a variety of ways and most of them got full marks.

Page 4 Suggested solutions	Marker's Feedback
Q.(7)(b)(ii) Alternative "otherwise" solution: (Or) Noting $\sin\alpha = \frac{2}{\sqrt{5}}$ from (i) and $\cos\alpha = \frac{1}{\sqrt{5}}$, $\sin\beta = \frac{1}{\sqrt{10}}$ [given in question]. LHS $\sin 2\alpha$ $= 2\sin\alpha\cos\alpha$ $= 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$ $= \frac{4}{5}.$ $\cos 2\beta$ $= 1 - 2\sin^2\beta$ Alternative 2 $= 1 - 2 \times \left(\frac{1}{\sqrt{10}}\right)^2$ $= 1 - \frac{1}{5}$ $= \frac{4}{5}.$ LHS = RHS. $\sin 2\alpha = \cos 2\beta$	

Page 5 **Suggested solutions** Q8: (13) **Marker's Feedback**

Q.(8)(a) Let $3\sin\theta - 2\cos\theta \equiv R\sin(\theta - \alpha)$, $R>0$

$\therefore 3\sin\theta - 2\cos\theta \equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$.

Equating parts,

$3\sin\theta = R\sin\theta\cos\alpha$ | $-2\cos\theta = -R\cos\theta\sin\alpha$

$3 = R\cos\alpha$ (1) | $2 = R\sin\alpha$ (2)

Squaring & adding (1) & (2):

$R^2(\cos^2\alpha + \sin^2\alpha) = 3^2 + 2^2$

$R = \sqrt{13}$

6

Sub. $R=\sqrt{13}$ in (1): $3 = \sqrt{13}\cos\alpha$. $\frac{3}{\sqrt{13}} = \cos\alpha$.

Sub. $R=\sqrt{13}$ in (2): $2 = \sqrt{13}\sin\alpha$. $\frac{2}{\sqrt{13}} = \sin\alpha$.

Q_1: $\alpha = 0.58800..$ [$=0.5880$ (4DP)]

Hence solving $3\sin\theta - 2\cos\theta = 1$.

$\sqrt{13}\sin(\theta - 0.5880..) = 1$.

$\sin(\theta - 5.880\_) = \frac{1}{\sqrt{13}}$.

$\theta - 0.5880.. = 0.28103\_$ or $2.86055\_$

$\theta = 0.8690$ or 3.4486 (4DP)

Note: avoid writing radians in terms of π with a decimal coefficient.

* must show full working for R.

* must consider both equations to confirm α is in the first quadrant

* evaluate α do not leave inverse trig form e.g. $\alpha = \cos^{-1}\frac{3}{\sqrt{13}}$

* must be given in radians

(b)(i) Let $3\cos\theta + 3\sqrt{3}\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $(R>0)$.

Equating parts,

$3\cos\theta = R\cos\theta\cos\alpha$ | $3\sqrt{3}\sin\theta = -R\sin\theta\sin\alpha$

$3 = R\cos\alpha$ (1) | $-3\sqrt{3} = R\sin\alpha$ (2)

Squaring & adding (1) & (2)

$R^2(\cos^2\alpha + \sin^2\alpha) = 3^2 + (-3\sqrt{3})^2 = 36$

4

$R = 6$

Sub. $R=6$ in (1): $3 = 6\cos\alpha$. $\frac{1}{2} = \cos\alpha$.

Sub. $R=6$ in (2): $-3\sqrt{3} = 6\sin\alpha$. $-\frac{\sqrt{3}}{2} = \sin\alpha$.

$\alpha = 300^\circ$. or -60°

$\therefore 3\cos\theta + 3\sqrt{3}\sin\theta = 6\cos(\theta + 300^\circ)$

consider both equations to determine the quadrant.

A large number of errors here.

(ii) Hence $3\cos\theta + 3\sqrt{3}\sin\theta = 2$ is

$6\cos(\theta + 300^\circ) = 2$

$\cos(\theta + 300^\circ) = \frac{1}{3}$

3

$\theta + 300^\circ = 430^\circ 32'$, $649^\circ 28'$

[$300^\circ < \theta + 300^\circ < 660^\circ$]

$\theta = 130^\circ 32'$, $349^\circ 28'$.

This part generally well done.

Q9 5

8

Page 6 Suggested solutions	Marker's Feedback
Q.(9)(a)	

$$5\sin\theta - 3\cos\theta = 3$$

$$\frac{5\times 2t}{1+t^2} - \frac{3(1-t^2)}{1+t^2} = 3$$

$$\frac{10t-3+3t^2}{1+t^2} = 3.$$

$$10t-3+3t^2 \underset{+3-3t^2}{} = 3+3t^2 \underset{+3-3t^2}{}$$

$$10t = 6$$

$$t = \frac{3}{5}.$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{3}{5}.$$

4

$$\frac{\theta}{2} = 0.5404$$

$$\theta = 1.0808.$$

Test: $\theta = \pi$: $5\sin\pi - 3\cos\pi = 3$.

$\therefore \pi$ IS a solution.

$\theta = 1.0808$ (4DP), π.

A number of students worked and answered in degrees, hence lost a mark.
- if converted back to radians, marks were awarded.
- Small number did not test for $\theta = \pi$.

(b) LHS

$$\frac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1}$$

4

$$= \frac{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} - 1}{\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 1} \quad \begin{matrix}\times(1+t^2)\\ \times(1+t^2)\end{matrix}$$

$$= \frac{(1-t^2)+2t-(1+t^2)}{(1-t^2)-2t+(1+t^2)}$$

$$= \frac{2t-2t^2}{2-2t}$$

$$= \frac{2t(1-t)}{2(1-t)}$$

$$= t$$

$$= \tan\left(\frac{\theta}{2}\right)$$

= RHS QED.

Some students not starting with

LHS = $\boxed{\text{exactly what is on the LHS}}$

Some students showing very poor setting-out

⇒ must follow steps outlined here!

Q.10 5

Page 7 Suggested solutions	Marker's Feedback

Q.(10)(a) Step 1: Show true for $n=1$.

LHS	RHS
$= 4\times 1 + 2$	$= 2\times 1^2 + 4\times 1$
$= 6$	$= 6$

LHS = RHS $\Rightarrow$ True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $(4\times 1+2)+(4\times 2+2)+\ldots+(4k+2)=2k^2+4k$.

Step 3: Prove true for $n=k+1$

i.e. RTP $(4\times 1+2)+\ldots+(4k+2)+4(k+1)+2$
$= 2(k+1)^2+4(k+1)$

or $(4\times 1+2)+\ldots+(4k+2)+(4k+6)=2k^2+8k+6$

LHS

$(4\times 1+2)+\ldots+(4k+2)+4(k+1)+2$
$= 2k^2+4k \quad + 4k+6$ (by assumption)
$= 2k^2+8k+6$
$= 2(k+1)^2+4(k+1)$
$=$ RHS QED.

$\therefore$ If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for $n=1+1=2$ & so on for all positive integers n by the principle of mathematical induction.

Marker's Feedback:

Some students used summation notation during the question. This was not penalised, but students who wrote out the question in full i.e. $4\times 1+2+\ldots+4k+2$ etc. did better as they understood the question more clearly.

4

Some general comments: DESPITE being told this clearly,

→ a minority (about 20–30 students) did NOT write out the Required To Prove (RTP) step. This WAS penalised as it is VITAL so that you actually KNOW the Right Hand Side (RHS) you're aiming at.

→ a minority also did NOT go back and start the Left Hand Side (LHS) after stating Required To Prove. This is ALSO necessary (& WAS penalised) as the proof makes NO sense WITHOUT it.

→ A minority forgot to put "By Assumption" (or using 1 or ✱) in when they substituted the assumption in. You had to show WHAT you were substituting and WHY.

Page 8 Suggested solutions	Marker's Feedback
Q.(10)(b) Step 1: Show true for $n=1$. LHS $=5^1+23$ $=28$ which is divisible by 4. true for $n=1$. Step 2: Assume true for $n=k$ i.e. $5^k+23=4P$ P ... integer. or $5^k = 4P-23$ Step 3: Prove true for $n=k+1$ i.e. $5^{k+1}+23 = 4Q$, Q an integer. LHS $5^{k+1}+23$ $=5\times5^k+23$ $=5\times(4P-23)+23$ $=20P-115+23$ $=20P-92$ $=4(5P-23)$ $=4Q$, $Q=5P-23$, an integer as P is an integer. $=$ RHS QED. If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n.	Ditto the general comments about (a) for *RTP step *Start LHS step *Assumption step. Generally OK though. $\frac{4}{4}$

Page 9 Suggested solutions	Marker's Feedback
Q.(10)(c) Step 1: Show true for $n=2$ LHS $=2^2+6\times 2$ $=16$ which is divisible by 2 $\Rightarrow$ true for $n=2$. Step 2: Assume true for $n=2k$, k an integer. i.e. $(2k)^2+6\times 2k = 8P$, P an integer. or $4k^2+12k = 8P$, P an integer. Step 3: Prove true for $n=2k+2$, k an integer i.e. $(2k+2)^2+6(2k+2)=8Q$, Q an integer. LHS $(2k+2)^2 + 6(2k+2)$ $= 4k^2+8k+4+12k+12$ $=(4k^2+12k)+8k+16$ $= 8P + 8k+16$ [by assumption] $= 8[P+k+2]$ $= 8Q$, Q an integer as P & k integers $=$ RHS. If it is true for $n=2k$ it is true for $n=2k+2$. Hence as it is true for $n=2$ it will be true for all even positive integers n. 4	The best & most successful way to approach this question was to assume true for $n=2k$ & prove true for $n=2k+2$. Note: A <u>VERY</u> small minority [perhaps 3 or 4 students] assumed $k^2+6k=8P$ & then proved $(k+2)^2+6(k+2)$ $=8(P+\frac{k}{2}+2)$ which is an integer as k is even & got full marks, but most of the students who assumed for $n=k$ & proved for $n=k+2$ made mistakes.

Page 10 Suggested solutions	Marker's Feedback

Q.(10)(d) Step 1: Show true for $n=2$.

LHS $= 2^2-1 = 3.$

RHS $= \dfrac{(2!)^2(2+1)}{2\times 2} = 3.$

LHS $=$ RHS $\Rightarrow$ true for $n=2$.

Step 2: Assume true for $n=k$

i.e. $(2^2-1)(3^2-1)\ldots(k^2-1) = \dfrac{(k!)^2(k+1)}{2k}$

Step 3: Prove true for $n=k+1$

i.e. $(2^2-1)(3^2-1)\ldots(k^2-1)[(k+1)^2-1] = \dfrac{[(k+1)!]^2(k+2)}{2(k+1)}.$

LHS

$= (2^2-1)(3^2-1)\ldots(k^2-1)((k+1)^2-1]$

$= \dfrac{(k!)^2(k+1)}{2k} \times [(k+1)^2-1]$ (by assumption)

$\underline{4}$

$= \dfrac{(k!)^2(k+1)}{2k} \times (k^2+2k)$

$= \dfrac{(k!)^2(k+1) \times k(k+2)}{2k}$ $\times (k+1)$ / $\times (k+1)$

$= \dfrac{(k!)^2(k+1)^2 \times k(k+2)}{2k(k+1)}$

$= \dfrac{[(k+1)!]^2(k+2)}{2(k+1)}$

$=$ RHS QED.

If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=2$ it will be true for all integers $n \geq 2$ by the principle of mathematical induction.

Marker's Feedback:

Quite a few students could get down to

$(2^2-1)\ldots(k^2-1)[(k+1)^2-1]$

$= \dfrac{(k!)^2(k+1)}{2k} \times ((k+1)^2-1]$

but only a few saw the extra step to multiply both the numerator & denominator by $(k+1)$.

Page 11 Suggested solutions Q.11 = 8	Marker's Feedback
Q.(11)(a) By $\cos(A+B)+\cos(A-B)=2\cos A\cos B$ $\cos 80^\circ + \cos 40^\circ$ $= \cos(60^\circ+20^\circ)+\cos(60^\circ-20^\circ)$ * See (1) $= 2\cos 60^\circ \cos 20^\circ$ must show this step 2 $= 2 \times \frac{1}{2} \times \cos 20^\circ$ $= \cos 20^\circ$ $=$ RHS QED. (1): $A+B=80^\circ$ (1) $A-B=40^\circ$ (2) + $2A = 120^\circ$ $A = 60^\circ. \Rightarrow B = 20^\circ$. If needed.	Mostly done well.
(b) $2\sin 35^\circ \cos 25^\circ + 2\cos 20^\circ \sin 10^\circ$ $= [\sin(35^\circ+25^\circ)+\sin(35^\circ-25^\circ)+\sin(20^\circ+10^\circ)-\sin(20^\circ-10^\circ)$ [using $2\sin A\cos B=\sin(A+B)+\sin(A-B)$ & $2\cos A\sin B = \sin(A+B)-\sin(A-B)$] $= \sin 60^\circ + \sin 10^\circ + \sin 30^\circ - \sin 10^\circ$ $= \sin 60^\circ + \sin 30^\circ$ 2 $= \frac{\sqrt{3}+1}{2}$. $=$ RHS QED.	Mostly done well

p.12 Suggested solutions	Marker's Feedback
Q. 11(c)(i) LHS: $2\sin x(\cos 2x + \cos 4x + \cos 6x)$ $\underline{2}$ $= 2\cos 2x \sin x + 2\cos 4x \sin x + 2\cos 6x \sin x$ $= (\sin 3x - \sin x) + (\sin 5x - \sin 3x) + (\sin 7x - \sin 5x)$ [by $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$]. $= \sin 7x - \sin x$ = RHS QED.	Some students had difficulty answering this question correctly
(ii) Hence if $x = \frac{\pi}{7}$, $2\sin\frac{\pi}{7}\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) = \sin\pi - \sin\frac{\pi}{7}$ $2\sin\frac{\pi}{7}\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) = -\sin\frac{\pi}{7}$ $\div 2\sin\frac{\pi}{7}.$ $\underline{2}$ $\underline{\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}.}$ END OF SOLUTIONS!!!	students need to show proper working out.