GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC ASSESSMENT

TASK 1 2025

(December 2024)

Student Number:________________________________.

		ME12-1	ME12-2	ME12-3	ME12-4	ME12-5	ME12-6	ME12-7
1-5	/5			√				√
6(a)	/3							√
6(b)(c)	/6			√				√
7	/10			√				√
8	/12			√				√
9	/9			√				√
10	/13	√						√
11	/8			√				√
TOTALS	/66	/13		/50				/66

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1	applies techniques involving proof or calculus to model and solve problems
ME12-2	applies concepts and techniques involving vectors and projectiles to solve problems
ME12-3	applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations
ME12-4	uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
ME12-5	applies appropriate statistical processes to present, analyse and interpret data
ME12-6	chooses and uses appropriate technology to solve problems in a range of contexts
ME12-7	evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Select the alternative A, B, C or D that best answers the question.

1. A ○	B. ○	C. ○	D. ○
2. A ○	B. ○	C. ○	D. ○
3. A ○	B. ○	C. ○	D. ○
4. A ○	B. ○	C. ○	D. ○
5. A ○	B. ○	C. ○	D. ○

GIRRAWEEN HIGH SCHOOL

HSC TASK 1 2025 *(December 2024)*

MATHEMATICS

EXTENSION 1

Time allowed – 90 Minutes

(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- NESA-approved calculators may be used. Reference sheets will be provided.
- Each question attempted is to be marked clearly Question 6, Question 7 etc. Each question is to be answered on a new page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.
- For Questions 6-11: All necessary working should be shown in each question. Marks may be deducted for careless or badly arranged work.

Multiple Choice begins on the following page

For Multiple choice Questions 1-5: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.

Question 1

Daniel was solving $\sin 2x = cosx, 0 \leq x \leq 2\pi$. His working was:

$\sin 2x = \cos x$	Line 1
$2\sin x \cos x = \cos x$	Line 2
$\sin x = \frac{1}{2}$	Line 3
$x = \frac{\pi}{6}, \frac{5\pi}{6}$	Line 4

Daniel has made a mistake on one of the lines. The line he made a mistake on was

(A) Line 1

(B) Line 2

(C) Line 3

(D) Line 4

Question 2

$2\sin 2x \cos x$ can be rewritten using products to sums as

(A) $\sin 3x + \sin x$

(B) $\cos 3x + \cos x$

(C) $\sin 3x - \sin x$

(D) $\cos 3x - \cos x$

Question 3

If $2\sin\theta - 3\cos\theta$ is arranged in the form $Rcos(\theta + \alpha)$ then $\sin\alpha =$

(A) $\frac{2}{\sqrt{13}}$

(B) $-\frac{2}{\sqrt{13}}$

(C) $\frac{3}{\sqrt{13}}$

(D) $-\frac{3}{\sqrt{13}}$

Multiple choice continues on the following page

Multiple choice (continued)

Question 4

$2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)=$

(A) $\cos 2x$

(B) $\sin 2x$

(C) $2\cos x$

(D) $2\sin x$

Question 5

If $t=\tan\frac{x}{2}$ then $\tan x+\sec x=$

(A) $\frac{2t+1-t^2}{1+t^2}$

(B) $\frac{1}{t}$

(C) $\frac{1+t}{1-t}$

(D) $\frac{1+t^2}{1-t^2}$

Examination continues on the following page

Question 6 ***(9 marks) (Start solutions on a new page)*** **Marks**

(a) Sketch the region in the number plane where the inequalities $x^2 + y^2 \leq 25 \; and \; 4x - 3y > 0$ hold simultaneously. **3**

(b) Use compound angle formulae to find the exact value of

(i) $\sin 130^o \cos 70^o - \cos 130^o \sin 70^o$ **2**

(ii) $\frac{\tan 110^o - \tan 80^o}{1 + \tan 110^o \tan 80^o}$ **2**

(c) Show that the exact value of $cos\,\frac{\pi}{12}$ is $\frac{\sqrt{6}+\sqrt{2}}{4}$. **2**

Question 7 ***(10 Marks) (Start solutions on a new page)***

(a) (i) Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is in radians. **3**

(ii) Hence solve $\sqrt{3}\cos\theta - \sin\theta = 1 \; for \; 0 \leq \theta \leq 2\pi$. **2**

(b) Using the auxiliary angle method and choosing a substitution so that $0 < \alpha < 90^o$, solve $3\sin\theta - 2\cos\theta = 1$ for $0 \leq \theta \leq 360^o$. **5**

Examination continues on the following page

Question 8 ***(12 marks) (Start solutions on a new page)*** **Marks**

(a) Using the substitution $t = \tan\frac{\theta}{2}$, solve the equation **5**

$3\sin\theta + 4\cos\theta = 2 \; for \; 0 \leq \theta \leq 360^o$.

(b) (i) Express $\sqrt{2}\cos x + \sqrt{2}\sin x$ in the form $R\sin(x + \alpha)$. **4**

(ii) Hence sketch $y = \sqrt{2}\cos x + \sqrt{2}\sin x \;\; for \; 0 \leq x \leq 2\pi$. **3**

Question 9 ***(9 Marks) (Start solutions on a new page)***

(a) Solve for $x, 0 \leq x \leq 360^o$: **4**

$\cos 2x - \sin x = \frac{1}{2}$

(b) (i) Show using sums to products formulae that **2**

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 4\,cos\,5\theta\,sin\,2\theta\,cos\,\theta$

(ii) Hence solve for $\theta, 0 \leq \theta \leq 2\pi$ **3**

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 0$

Question 10 ***(13 marks) (Start solutions on a new page)***

(a) Prove by mathematical induction for all integers $n \geq 1$ **4**

$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$

(b) Prove by mathematical induction for all integers $n \geq 1$: **4**

$5^{2n} - 3^{2n}$ is divisible by 16.

(c) Prove by mathematical induction for all integers $n \geq 1$: **5**

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Examination continues on the following page

Question 11 ***(8 marks) (Start solutions on a new page)*** **Marks**

(i) Prove that $\cos 4\theta = 8\, cos^4\ \theta - 8\, cos^2\, \theta + 1$ **3**

(ii) Use sums to products formulae to show that **2**

$\cos\frac{4\pi}{9} + \cos\frac{2\pi}{9} = cos\,\frac{\pi}{9}$

(iii) Hence use (i) and (ii) to show that $cos\ \frac{\pi}{9}$ is a solution to **3**

the equation $8x^3 - 6x - 1 = 0$

END OF EXAMINATION

Mathematics Assessment Solutions

Page 1

Girraween High School

☐ Year 11	☑ Extension 1	Year: 2025
☐ Year 12	☐ Extension 2	Task Number 1

Suggested solutions	Marker's Feedback

Multiple Choice:

(1) C Line 3 was where the error was.
Dividing by $\cos x$ eliminated solutions where $\cos x = 0 \rightarrow$ namely $x = \frac{\pi}{2}$ & $x = \frac{3\pi}{2}$.

(2) A As $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$$2\sin 2x \cos x = \sin(2x+x) + \sin(2x-x)$$
$$= \sin 3x + \sin x.$$

(3) B $2\sin\theta - 3\cos\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

Equating parts

$2\sin\theta = -R\sin\theta\sin\alpha$ | $-3\cos\theta = R\cos\theta\cos\alpha$

$-2 = R\sin\alpha \ (1)$ | $-3 = R\cos\alpha \ (2)$

$(1)^2 + (2)^2: R^2(\cos^2\alpha + \sin^2\alpha) = 2^2 + 3^2.$

$$R = \sqrt{13}.$$

sub. $R = \sqrt{13}$ in (1) $-2 = \sqrt{13}\sin\alpha$

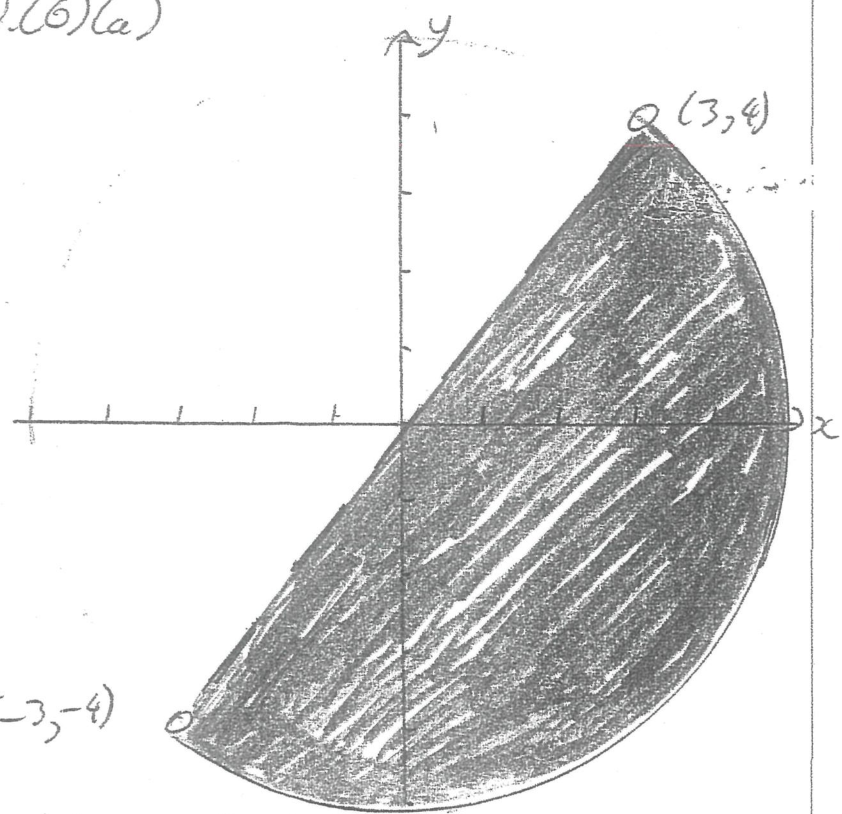
$$\frac{-2}{\sqrt{13}} = \sin\alpha.$$

(4) A $2\sin\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - x\right)$

$$= \sin\left(\frac{\pi}{2} - 2x\right)$$
$$= \cos 2x$$

(5) C $\tan x + \sec x$

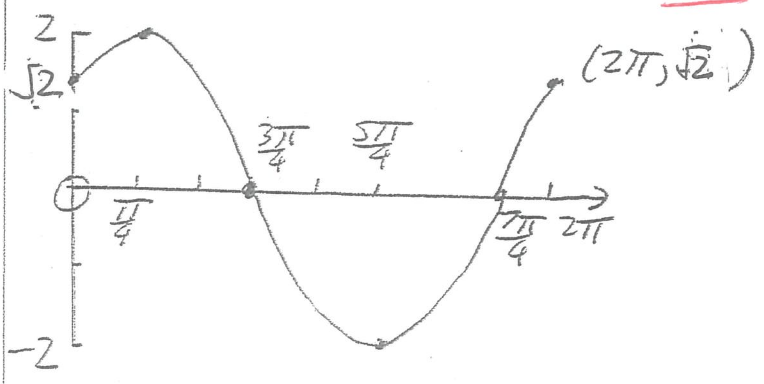
$$= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$$
$$= \frac{(1+t)^2}{(1-t)(1+t)}$$
$$= \frac{1+t}{1-t}.$$

Suggested solutions	Marker's Feedback
Q.(6)(a)  Note: $4x-3y>0$ $4x>3y$ $\frac{4}{3}>y$ or $y<\frac{4}{3}x$	1 — open circles 1 — Solid, dotted lines 1 — correct region Very few students are required to revise regions otherwise mostly done well. Some students forgot open circles and some dotted & solid lines
(b)(i) $\sin 130^\circ \cos 70^\circ - \cos 130^\circ \sin 70^\circ$ $= \sin(130^\circ - 70^\circ)$ $= \sin 60^\circ$ $= \frac{\sqrt{3}}{2}$	1 — $\sin(130^\circ - 70^\circ)$ 1 — $\frac{\sqrt{3}}{2}$. Mostly done well!
(ii) $\frac{\tan 110^\circ - \tan 80^\circ}{1+\tan 110^\circ \tan 80^\circ}$ $= \tan(110^\circ - 80^\circ)$ $= \tan 30^\circ$ $= \frac{1}{\sqrt{3}}$.	1 — $\tan(110^\circ - 80^\circ)$ 1 — $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{2}$ Mostly done well.
(c) $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ $= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$ $= \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2}$ $= \frac{\sqrt{6}+\sqrt{2}}{4}$ = RHS.	1 - mark for $\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ 1 - $\frac{\sqrt{6}+\sqrt{2}}{4}$ with $\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}$ as a preceding step. Any other method used then however student got a mark the second mark was only given if they could arrive to $\frac{\sqrt{6}+\sqrt{2}}{4}$ with correct working out.

Suggested solutions	Marker's Feedback
Q.(7)(a)(i) $\sqrt{3}\cos\theta - \sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$. (i) $R>0$ Equating parts, $\sqrt{3}\cos\theta = R\cos\theta\cos\alpha$ | $-\sin\theta = R\sin\theta\sin\alpha$ $\sqrt{3} = R\cos\alpha$ (1) | $-1 = R\sin\alpha$ (2) $(1)^2+(2)^2$: $R^2(\cos^2\alpha+\sin^2\alpha) = (\sqrt{3})^2+(-1)^2$ $R = 2$. 1 $\underline{3}$ Sub. $R=2$ in (1): $\sqrt{3} = 2\cos\alpha$, $\frac{\sqrt{3}}{2} = \cos\alpha$. Sub. $R=2$ in (2): $-1 = 2\sin\alpha$, $-\frac{1}{2} = \sin\alpha$ 1 $\alpha = \frac{11\pi}{6}$. 1 $\therefore \sqrt{3}\cos\theta - \sin\theta \equiv 2\cos(\theta - \frac{11\pi}{6})$	Some students couldn't find the correct value for α Common errors:- $\alpha = \frac{\pi}{6}$, $\alpha = \frac{5\pi}{6}$ * Mark deducted if they didn't justify α in 4th quadrant
(ii) Hence $\sqrt{3}\cos\theta - \sin\theta = 1$. $2\cos(\theta - \frac{11\pi}{6}) = 1$. $\cos(\theta - \frac{11\pi}{6}) = \frac{1}{2}$. Noting $-\frac{11\pi}{6} \le \theta - \frac{11\pi}{6} \le \frac{\pi}{6}$. $\underline{2}$ $\theta - \frac{11\pi}{6} = -\frac{5\pi}{3}, -\frac{\pi}{3}$. 1 $\theta = \frac{\pi}{6}, \frac{3\pi}{2}$. 1	* For part (i), if they got wrong value for α and as a result the domain change made part (ii) question easier, then no carry forward mark given for finding θ. * Some students didn't pay attention to domain when finding values of θ. * Mark deducted if θ not given in radians.

Suggested solutions	Marker's Feedback
Q.(7)(b) $3\sin\theta - 2\cos\theta \equiv R\sin(\theta - \alpha), R>0$ 1 for right pick $3\sin\theta - 2\cos\theta \equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ Equating parts $3\sin\theta = R\sin\theta\cos\alpha$ | $-2\cos\theta = -R\cos\theta\sin\alpha$ $3 = R\cos\alpha$ (1) | $2 = R\sin\alpha$ (2) 1 for R $(1)^2+(2)^2: R^2(\cos^2\alpha+\sin^2\alpha) = 3^2+2^2$ $R = \sqrt{13}$ Sub. $R=\sqrt{13}$ in (1): $3 = \sqrt{13}\cos\alpha$, $\frac{3}{\sqrt{13}} = \cos\alpha$ | Sub. $R=\sqrt{13}$ in (2): $2 = \sqrt{13}\sin\alpha$, $\frac{2}{\sqrt{13}} = \sin\alpha$ $\alpha = 33^\circ 41'$ 1 for α 5 Hence solving $3\sin\theta - 2\cos\theta = 1$ $\sqrt{13}\sin(\theta - 33^\circ 41') = 1$. $\sin(\theta - 33^\circ 41') = \frac{1}{\sqrt{13}}$ 1 $\theta - 33^\circ 41' = 16^\circ 6', 163^\circ 54'$ $\theta = 49^\circ 47'$ 1 $, 197^\circ 35'$	* Some students didn't choose the right form $R\sin(\theta-\alpha)$ * Need to pay attention to $0<\alpha<90^\circ$ * A few students didn't list all possible solutions. * Mark deducted when θ not given in degrees.

Suggested solutions	Marker's Feedback
Q.(8)(a) $3\sin\theta + 4\cos\theta = 2$ $\frac{3\times 2t}{1+t^2} + \frac{4(1-t^2)}{1+t^2} = 2.$ $6t + 4 - 4t^2 = 2 + 2t^2.$ $0 = 6t^2 - 6t - 2$ $0 = 3t^2 - 3t - 1.$ $t = \frac{3 \pm \sqrt{(-3)^2 - 4\times 3 \times -1}}{2\times 3}$ $= \frac{3\pm\sqrt{21}}{6}.$ $\tan\left(\frac{\theta}{2}\right) = \frac{3+\sqrt{21}}{6}$, $\frac{\theta}{2} = 51°39'$, $\theta = 103°18'.$ $\tan\left(\frac{\theta}{2}\right) = \frac{3-\sqrt{21}}{6}$, $\frac{\theta}{2} = 165°13'$, $\theta = 330°27'.$ Test: $\theta = 180°$ $3\sin 180° + 4\cos 180° = -4 \neq 2.$ $180°$ is not a solution $\therefore$ Solutions are $\theta = 103°18', 330°27'.$ 5	Lots of wrong answers for the solution of quadratic. Many students have difficulty in finding the correct solutions for $\tan\frac{\theta}{2} = \frac{3-\sqrt{21}}{6}$
	Some forgot to Test for 180°

Suggested solutions	Marker's Feedback
Q.(8)(b)(i) $\sqrt{2}\cos x + \sqrt{2}\sin x \equiv R\sin(x+\alpha)$	
$\sqrt{2}\cos x + \sqrt{2}\sin x \equiv R\sin x\cos\alpha + R\cos x\sin\alpha.$	
Equating parts,	
$\sqrt{2}\cos x = R\cos x\sin\alpha.$ | $\sqrt{2}\sin x = R\sin x\cos\alpha$	
$\sqrt{2} = R\sin\alpha\,(1)$ | $\sqrt{2} = R\cos\alpha\,(2)$	
$(1)^2 + (2)^2: R^2(\cos^2\alpha+\sin^2\alpha) = (\sqrt{2})^2+(\sqrt{2})^2$	Some wrote
$R = 2.$	$\tan\alpha = 1$
sub. $R=2$ in (1): $\sqrt{2} = 2\sin\alpha$ | sub. $R=2$ in (2):	$\alpha = \frac{\pi}{4}$
$\frac{\sqrt{2}}{2} = \sin\alpha$ | $\frac{\sqrt{2}}{2} = \cos\alpha.$ 4	They lost 1 mark.
$\alpha = \frac{\pi}{4}.$	
$\therefore \sqrt{2}\cos x + \sqrt{2}\sin x = 2\sin(x+\frac{\pi}{4}).$	
(ii) $\rightarrow$ Period $= 2\pi$. Amplitude $= 2$.	
Phase $= \frac{\pi}{4}.$	
$x = 0$ (or 2π) $y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$ 3	
	Common mistakes * wrong y intercept * not writing all the x values ie $0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

Suggested solutions	Marker's Feedback

Q.(9)(a)

$\cos 2x - \sin x = \frac{1}{2}$

$(1 - 2\sin^2 x)^{1} - \sin x = \frac{1}{2}$

$0 = 4\sin^2 x + 2\sin x - 1.$

$\sin x = \frac{-2 \pm \sqrt{(-2)^2 - 4 \times 4 \times -1}}{2 \times 4}$

$= \frac{-2 \pm 2\sqrt{5}}{8}$

$= \frac{-1 \pm \sqrt{5}}{4}$ 1

4

If $\sin x = \frac{-1+\sqrt{5}}{4}$, $x = 18^\circ, 162^\circ$. 1

If $\sin x = \frac{-1-\sqrt{5}}{4}$, $x = 234^\circ, 306^\circ$. 1

(9)(b)

(i) Either

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta$

$= [\sin(7\theta+\theta) + \sin(7\theta-\theta)] - [\sin(3\theta+\theta) + \sin(3\theta-\theta)]$

$= 2\sin 7\theta\cos\theta - 2\sin 3\theta\cos\theta$ 1

$= 2\cos\theta[\sin 7\theta - \sin 3\theta]$

$= 2\cos\theta[\sin(5\theta+2\theta) - \sin(5\theta-2\theta)]$

$= 2\cos\theta \times 2\cos 5\theta\sin 2\theta$ 1 2

$= 4\cos 5\theta\sin 2\theta\cos\theta$

$=$ RHS QED.

or

or

$\sin 8\theta - \sin 4\theta + \sin 6\theta - \sin 2\theta$

$= \sin(6\theta+2\theta) - \sin(6\theta-2\theta) + \sin(4\theta+2\theta) - \sin(4\theta-2\theta)$

$= 2\cos 6\theta\sin 2\theta + 2\cos 4\theta\sin 2\theta$ 1

$= 2\sin 2\theta[\cos 6\theta + \cos 4\theta]$

$= 2\sin 2\theta[\cos(5\theta+\theta) + \cos(5\theta-\theta)]$

$= 2\sin 2\theta[2\cos 5\theta\cos\theta]$ 1

$= 4\cos 5\theta\sin 2\theta\cos\theta.$

or

$\sin 8\theta - \sin 2\theta + \sin 6\theta - \sin 4\theta$

$= \sin(5\theta+3\theta) - \sin(5\theta-3\theta) + \sin(5\theta+\theta) - \sin(5\theta-\theta)$

$= 2\cos 5\theta\sin 3\theta + 2\cos 5\theta\sin\theta$ 1

$= 2\cos 5\theta[\sin 3\theta + \sin\theta]$

$= 2\cos 5\theta[\sin(2\theta+\theta) + \sin(2\theta-\theta)]$

$= 2\cos 5\theta \times 2\sin 2\theta\cos\theta$ 1

$= 4\cos 5\theta\sin 2\theta\cos\theta.$

Marker's Feedback

* The vast majority of students knew to use $\cos 2x = 1 - 2\sin^2 x$ & get a quadratic equation using $\sin x$.

* A very worrying group of about 10 students forgot the NULL FACTOR law when solving the equation.

* Many students didn't realise $\frac{-1+\sqrt{5}}{4}$ and $\frac{-1-\sqrt{5}}{4}$ are (a) NOT EQUAL and (b) NOT OPPOSITES so that there are actually 2 sets of 2 solutions [$\sin^{-1}\left(\frac{-1+\sqrt{5}}{4}\right) = 18^\circ, 162^\circ$ & $\sin^{-1}\left(\frac{-1-\sqrt{5}}{4}\right) = 234^\circ, 306^\circ$.

(9)(b)(i) was mostly well done.

Suggested solutions

Q(9)(b)(ii) Hence solving

$\sin 8\theta + \sin 6\theta - \sin 4\theta - \sin 2\theta = 0$

$4\cos 5\theta \quad \sin 2\theta \quad \cos\theta = 0$

$0 \le \theta \le 2\pi$

Either $\cos 5\theta = 0$ $\quad 0 \le 5\theta \le 10\pi$

$5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2},$

$\frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}, \frac{17\pi}{2}, \frac{19\pi}{2}$

$\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10},$

$\frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}$

$0 \le \theta \le 2\pi$

or $\sin 2\theta = 0$ $\quad 0 \le 5\theta \le 10\pi$. 3

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots 2\pi$

Already in $\cos 5\theta = 0$

or $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\rightarrow$ already in $\cos 5\theta = 0$.

Final solutions:

$\theta = 0, \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}, \pi,$

$\frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}, 2\pi.$

Marker's Feedback

A minority of students had no idea, but most students realised that there were 3 possibilities $\cos 5\theta = 0$, $\sin 2\theta = 0$ or $\cos\theta = 0$.

However, for $\cos 5\theta = 0$ MOST students forgot if $0 \le \theta \le 2\pi$ then $0 \le 5\theta \le 10\pi$ which meant they missed the solutions $\frac{7\pi}{10}, \frac{9\pi}{10}, \dots$ to $\frac{19\pi}{10}$.

1 for $\cos 5\theta = 0$ or $\sin 2\theta = 0$ or $\cos\theta = 0$

1 for getting some ~~one~~ of the answers

& 1 for ALL answers.

Suggested solutions	Marker's Feedback

Q.(10)(a) Step 1: Show true for $n=1$:

LHS $= 1\times 2 = 2.$

RHS $= \frac{1}{3}\times 1\times(1+1)(1+2) = 2.$

LHS $=$ RHS $\Rightarrow$ True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $1\times 2 + 2\times 3 + \dots + k(k+1) = \frac{1}{3}k(k+1)(k+2).$

Step 3: Prove true for $n=k+1$

i.e. RTP $1\times 2 + \dots + k(k+1) + (k+1)(k+2) = \frac{1}{3}(k+1)(k+2)(k+3).$ 1

including start

LHS

$1\times 2 + \dots + k(k+1) + (k+1)(k+2)$ 1 for assumption

$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$ [By assumption]

$= \frac{1k}{3}(k+1)(k+2) + \frac{1}{3}\times 3(k+1)(k+2)$

$= \frac{1}{3}(k+1)(k+2)[k+3]$

$=$ RHS QED 1 for finish. 4

If it is true for $n=k$ it is true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n by the principle of mathematical induction.

Mostly done well. (-1) mark if they did not mention 'by assumption' in question 10a only.

Suggested solutions	Marker's Feedback
Q.(10)(b) Step 1: Show true for $n=1$.	

LHS
$= 5^2 - 3^2$
$= 16$
which is divisible by 16.

Step 2: Assume true for $n=k$
i.e. $5^{2k} - 3^{2k} = 16P$, P an integer.
or $3^{2k} = 5^{2k} - 16P$.

Step 3: Prove true for $n=k+1$.
i.e. RTP: $5^{2k+2} - 3^{2k+2} = 16Q$, Q an integer.

1

Mostly done well.

LHS
$= 5^{2k+2} - 3^{2k+2}$ 1

1 for assumption.

$= 25 \times 5^{2k} - 9 \times (5^{2k} - 16P)$ (By assumption)
$= 25 \times 5^{2k} - 9 \times 5^{2k} + 9 \times 16P$
$= 16 \times 5^{2k} + 9 \times 16P$
$= 16[5^{2k} + 9P]$
$= 16Q$, $Q = 5^{2k} + 9P$, an integer as k, P integer.

4

If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n by the principle of mathematical induction.

1

Suggested solutions	Marker's Feedback

Q.(10)(c) Step 1: Show true for $n=1$.

LHS

$$= \frac{1}{1\times2\times3} = \frac{1}{6}.$$

RHS

$$= \frac{1(1+3)}{4\times(1+1)(1+2)} = \frac{4}{24} = \frac{1}{6}$$

LHS = RHS. True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $$\frac{1}{1\times2\times3} + \dots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$$

Step 3: Prove true for $n=k+1$

i.e. RTP

$$\frac{1}{1\times2\times3} + \dots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$ 1

LHS

$$= \frac{1}{1\times2\times3} + \dots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$ 1

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$ (By assumption) 1

$$= \frac{k(k+3)^2 + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}$$

MUST SHOW HOW from assumption to finish

Note: Students could also use factor theorem & polynomial division here.

$$= \frac{k^3+k^2+5k^2+5k+4k+4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{k^2(k+1)+5k(k+1)+4(k+1)}{4(k+1)(k+2)(k+3)}$$ (*)

$$= \frac{(k+1)(k^2+5k+4)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$ 1

= RHS QED.

If it is true for $n=k$, it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for all positive integers n by mathematical induction.

Marker's Feedback:

1

5

Majority got to this step.

Then some students wrote the final step without any explanation and lost one mark.

Others who did not finish only got 3.

Suggested solutions	Marker's Feedback
Q.(11)(i) $\cos 4\theta$ $= \cos 2(2\theta)$ $= 2\cos^2 2\theta - 1$ $= 2[2\cos^2\theta - 1]^2 - 1$ $= 2[4\cos^4\theta - 4\cos^2\theta + 1] - 1$ $= 8\cos^4\theta - 8\cos^2\theta + 2 - 1$ 3 $= 8\cos^4\theta - 8\cos^2\theta + 1$ $=$ RHS QED.	Generally well done using various methods.
(ii) $\cos\frac{4\pi}{9} + \cos\frac{2\pi}{9}$ $A = \cos\left(\frac{\pi}{3} + \frac{\pi}{9}\right) + \cos\left(\frac{\pi}{3} - \frac{\pi}{9}\right)$ $= 2\cos\frac{\pi}{3}\cos\frac{\pi}{9}$ 2 [By $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$.] $= 2 \times \frac{1}{2} \times \cos\frac{\pi}{9}$ $= \cos\frac{\pi}{9}$. $=$ RHS QED.	Generally well done.
(iii) Letting $\theta = \frac{\pi}{9}$, this means $\cos 4\theta + \cos 2\theta = \cos\theta$ $[8\cos^4\theta - 8\cos^2\theta + 1] + [2\cos^2\theta - 1] = \cos\theta$. $8\cos^4\theta - 6\cos^2\theta = \cos\theta$. $8\cos^4\theta - 6\cos^2\theta - \cos\theta = 0$ 3 $\therefore 8\cos^4\frac{\pi}{9} - 6\cos^2\frac{\pi}{9} - \cos\frac{\pi}{9} = 0$ $\cos\frac{\pi}{9}\left[8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} - 1\right] = 0$. As $\cos\frac{\pi}{9} \neq 0$, $8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} - 1 = 0$ $\therefore \cos\frac{\pi}{9}$ is a solution to $8x^3 - 6x - 1 = 0$. END OF SOLUTIONS!	Poorly done overall. Students must use (i) & (ii) to achieve any marks.