HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 2 Term 2 2022

STUDENT NUMBER: ______________________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 60 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 44

Section I Pages 3 – 4

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

Section II Pages 5 – 8

39 marks

Attempt Questions 6 – 9.

Question	***1-5***	***6***	***7***	***8***	***9***	***Total***
Total	*/5*	*/10*	*/11*	*/8*	*/10*	*/44*

This assessment task constitutes 30% of the Preliminary Higher School Certificate Course School Assessment

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Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. The range of $f\left(g\left(x\right)\right)$ if $f\left(x\right)=x+4$ and $g\left(x\right)=x^2$ is

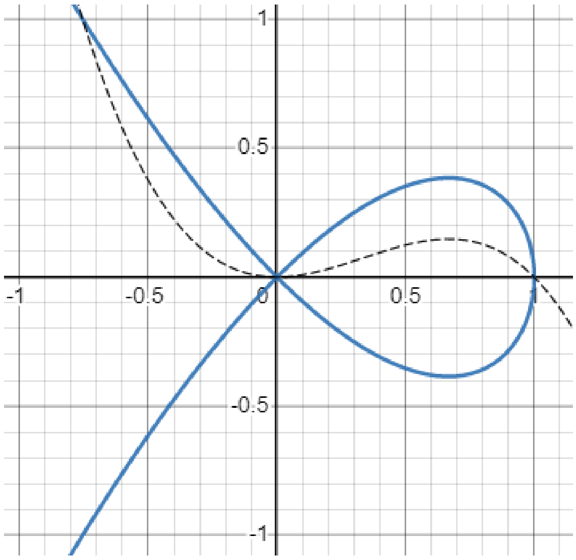
(A) $[0, \infty)$

(B) $(-\infty, \infty)$

(C) $(4, \infty)$

(D) $[4, \infty)$

Q2. The dotted curve represents $y = f(x)$.



Which of the following expressions best represents the solid curve?

(A) $y = |f(x)|$

(B) $y = \sqrt{f(x)}$

(C) $y^2 = f\left(x\right)$

(D) $y = \dfrac{1}{f(x)}$

Q3. If $f(x) = 3x^2 - 1$ for $x \geq 0$, then the domain of $f^{-1}(x)$ is

(A) $x \geq 1$

(B) $y \geq 0$

(C) $x \geq -1$

(D) $x \geq 0$

Q4. Which of the following is **NOT** a possible pair of parametric equations for the Cartesian equation $y = 2x + 4$?

(A) $x = t, y = 2t + 4$

(B) $x = t + 2, y = 2t$

(C) $x = \frac{t}{2}, y = t + 4$

(D) $x = 2t, y = 4t + 4$

Q5. If $f(x+1) = 2x^2 + x - 2$, find $f(x)$.

(A) $f(x) = 2x^2 - 3x + 1$

(B) $f(x) = 2x^2 - 3x - 4$

(C) $f(x) = 2x^2 + x - 3$

(D) $f(x) = 2x^2 - 3x - 1$

End of Section I

Section II

30 marks
Attempt Questions 6-8
Allow about 50 minutes for this section

Question 6 (10 marks) **Start a new answer booklet**

(a) Simplify $\dfrac{3ab^2}{x-2y} \div \dfrac{12ab-6a}{x^3-8y^3}$ **2**

(b) Find the values of x and y if $\dfrac{5\sqrt{3}+1}{2-\sqrt{3}} = x + y\sqrt{3}$. **2**

(c) Simplify $12^{2n} \div 3^n \times 48^{1-n}$ **3**

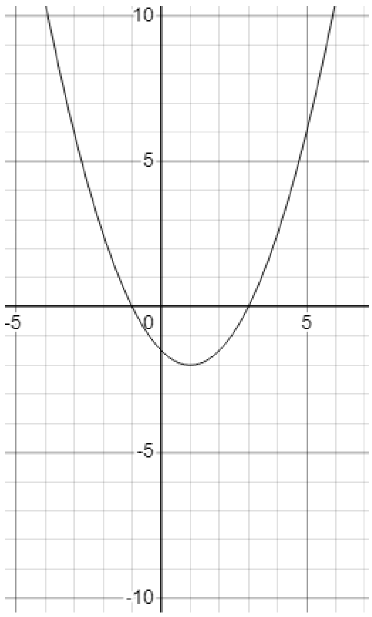
(d) Use the method of completing the square to find the exact values of x if $3x^2+6x-2=0$. **3**

Question 7 (11 marks) **Start a new answer booklet**

(a) Solve $\dfrac{x-2}{x-1} \le 2$. **3**

(b) Solve $|2x+1| + |x-2| \le 3$ **3**

(c) The diagram below is a sketch of the graph of the function $f(x) = \frac{1}{2}(x+1)(x-3)$



Use the graphs provided on the separate answer sheet, to graph the following transformations showing all asymptotes and intercepts.

(i) $y = f(|x|)$ **1**

(ii) $y = [f(x)]^2$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **2**

Question 8 (8 marks) **Start a new answer booklet**

(a) Solve $\cos\frac{\theta}{2} = \frac{-\sqrt{3}}{2}$ for $-360^o \leq \theta \leq 360^o$ **2**

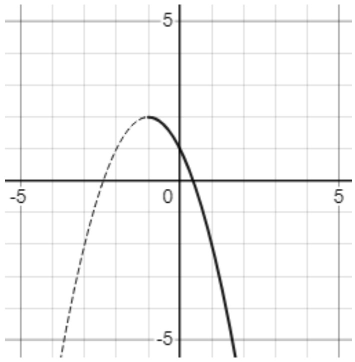
(b) Prove that $\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)^2 = 1 + \cot^2\theta$ **3**

(c) Solve $2\cos^2 x + \sin x + 1 = 0$ for $x \in [0, 2\pi]$. **3**

Question 9 (10 marks) **Start a new answer booklet**

(a) Find the Cartesian equation for the parametric equations $x = 3\sin\theta, y = 3\cos\theta, 0 \le \theta \le \pi$ **and** state the domain of the Cartesian equation. **3**

(b) Consider the curve $f(x) = 2 - (x+1)^2$ and its graph where $x \ge -1$



(i) The largest possible domain involving positive values of x for which $f(x)$ has an inverse function $f^{-1}(x)$ is $x \ge -1$. Explain how this restricted domain for the function $f(x)$ allows the inverse $f^{-1}(x)$ to be a function. **1**

(ii) State the domain of $y = f^{-1}(x)$. **1**

(iii) Find the equation of $y = f^{-1}(x)$. **3**

(iv) For the restricted domain, sketch the inverse function showing the main features. **2**

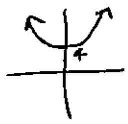
End of paper

Year 11 Maths Ext 1

Task 2, 2022

1) $f(x) = x + 4$, $g(x) = x^2$

$f(g(x)) = x^2 + 4$



R: $y \geq 4$

$[4, \infty)$ D

2) C

3) $f: y = 3x^2 - 1$ $D: x \geq 0$, $R: y \geq -1$

f^{-1}: $x = 3y^2 - 1$ $D: x \geq -1$ $R: y \geq 0$

C

4) A. $x = t$, $y = 2t + 4$

$y = 2x + 4$ ✓

B. $x = t + 2$, $y = 2t$

$t = x - 2$ $y = 2(x-2)$

$= 2x - 4$ × B

C. $x = \frac{t}{2}$, $y = t + 4$

$t = 2x$ $y = 2x + 4$ ✓

D. $x = 2t$, $y = 4t + 4$

$t = \frac{x}{2}$ $y = 4\left(\frac{x}{2}\right) + 4$

$= 2x + 4$ ✓

5) If $f(x+1) = 2x^2 + x - 2$

then $f((x-1)+1) = 2(x-1)^2 + (x-1) - 2$

$= 2(x^2 - 2x + 1) + x - 1 - 2$

$= 2x^2 - 4x + 2 + x - 3$

$= 2x^2 - 3x - 1$ D

or $f(x+1) = 2(x^2 + 2x + 1) - 3x - 4$

$f(x+1) = 2(x+1)^2 - 3(x+1) - 1$

$\therefore f(x) = 2x^2 - 3x - 1$

Markers feedback

6) a) $\frac{3ab^2}{x-2y} \div \frac{12ab-6a}{x^3-8y^3}$

$= \frac{3ab^2}{\cancel{x-2y}} \times \frac{(\cancel{x-2y})(x^2+2xy+4y^2)}{\underset{2}{\cancel{6a}}(2b-1)}$

$= \frac{b^2(x^2+2xy+4y^2)}{2(2b-1)}$

1 for factorising difference of 2 cubes
1 for reciprocal and simplifying

Poorly done. Many students didn't factorise the cubic expression correctly.

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b) $\frac{5\sqrt{3}+1}{2-\sqrt{3}} = \frac{5\sqrt{3}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{10\sqrt{3}+5\times 3+2+\sqrt{3}}{4-3}$

$= \frac{11\sqrt{3}+17}{1}$

$= 17+11\sqrt{3}$

$= x+y\sqrt{3}$

where $x=17$, $y=11$

1 for multiplying top and bottom by conjugate of denominator and expanding
1 for simplifying and equating

Generally done well. However some students brought the x & y terms into the rationalisation of the denominator

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c) $12^{2n} \div 3^n \times 48^{1-n}$

$= (3\times 2^2)^{2n} \times 3^{-n} \times (2\times 3\times 2^3)^{1-n}$

$= 3^{2n} \times 2^{4n} \times 3^{-n} \times 2^{4-4n} \times 3^{1-n}$

$= 3^{2n-n+1-n} \times 2^{4n+4-4n}$

$= 3\times 2^4$

$= 3\times 16$

$= 48$

1 for expressing with bases 2 and 3
1 for correct indices
1 for simplifying

Very poorly done! Many students did not use the rule

$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

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d) $3x^2+6x-2=0$

$3x^2+6x = 2$

$3(x^2+2x \quad) = 2$

$3(x^2+2x+1) = 2+3$ 1 for completing the square

$3(x+1)^2 = 5$

$(x+1)^2 = \frac{5}{3}$ 1.

$x+1 = \pm\sqrt{\frac{5}{3}}$

$x = -1 \pm \sqrt{\frac{5}{3}}$ 1.

Generally done well!

7) a) $\dfrac{x-2}{x-1} \leq 2$

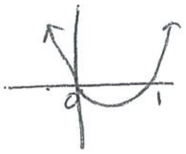
$\dfrac{x-2}{\cancel{x-1}} \times (x-1)^{\cancel{2}} \leq 2(x-1)^2$ 1.

$x^2 - x - 2x + 2 \leq 2(x^2 - 2x + 1)$

$x^2 - 3x + 2 \leq 2x^2 - 4x + 2$

$0 \leq x^2 - x$

$x(x-1) \geq 0$



$x \leq 0,\ x \geq 1$ 1.

but $x \neq 1$

$x \leq 0,\ x > 1$ 1.

<u>OR</u>

<u>if $x > 1$</u>	<u>if $x < 1$</u>
then $x - 2 \leq 2(x-1)$	then $x - 2 \geq 2(x-1)$
$x - 2 \leq 2x - 2$	$x - 2 \geq 2x - 2$
$0 \leq x$	$0 \geq x$
$x \geq 0$ 1.	$x \leq 0$
but $x > 1$	which satisfies $x < 1$
$\therefore x > 1$ 1.	$\therefore x \leq 0$ 1.

$x \leq 0,\ x > 1$

generally well done but a lot of students missed that $x \neq 1$

- Those who didn't graph their quadratic inequality found difficulty.

Those who tried to use this method generally missed the respective domains that affected their answer.

b) $|2x+1| + |x-2| \leq 3$

<u>if $x \leq -\frac{1}{2}$</u>	<u>if $-\frac{1}{2} < x < 2$</u>	<u>if $x \geq 2$</u>
$-(2x+1) + -(x-2) \leq 3$	$2x+1 + -(x-2) \leq 3$	$2x + 1 + x - 2 \leq 3$
$-2x - 1 - x + 2 \leq 3$	$2x + 1 - x + 2 \leq 3$	$3x - 1 \leq 3$
$-3x + 1 \leq 3$	$x + 3 \leq 3$	$3x \leq 4$
$-3x \leq 2$	$x \leq 0$	$x \leq 1\frac{1}{3}$
$x \geq -\frac{2}{3}$	but $-\frac{1}{2} < x < 2$	but $x \geq 2$
but $x \leq -\frac{1}{2}$		$\therefore$ no solution
$\therefore -\frac{2}{3} \leq x \leq -\frac{1}{2}$	$\therefore -\frac{1}{2} < x \leq 0$	

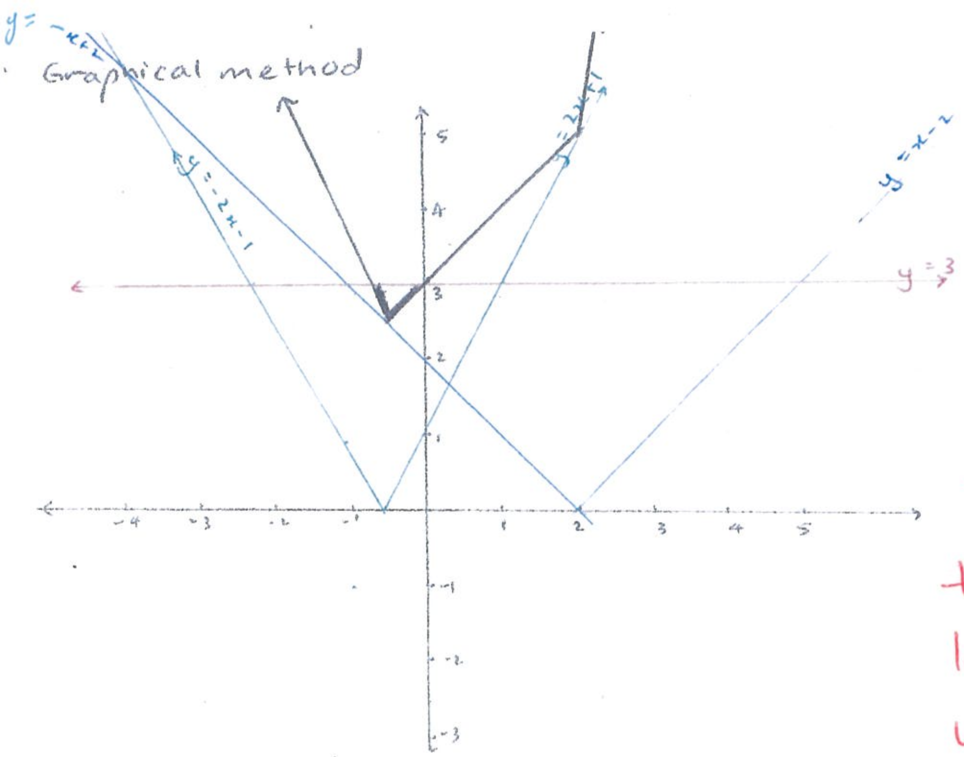
$-\frac{2}{3} \leq x \leq 0$

<u>If using Algebra</u>.

- Poorly done question in <u>general</u>, mainly again due to not taking into account the <u>domains</u> of each part properly

OR ...

7 b). Graphical method



$-2x-1+-x+2=3$
$-3x+1=3$
$-3x=2$
$x=-\frac{2}{3}$

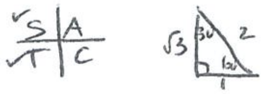
$2x+1+-x+2=3$
$x+3=3$
$x=0$

$-\frac{2}{3} \leq x \leq 0$

- when graphing there is a need to make it large enough to work effectively with and
- labelling the branches helps when doing the sum of the absolutes correctly.

8) a) $\cos\frac{\theta}{2} = -\frac{\sqrt{3}}{2}$ for $-360^\circ \le \theta \le 360^\circ$

$-180^\circ \le \frac{\theta}{2} \le 180^\circ$



1 related angle is 30°

1

$$\frac{\theta}{2} = -180^\circ + 30^\circ, \quad 180^\circ - 30^\circ$$

$$\frac{\theta}{2} = -150^\circ, \quad 150^\circ$$

$$\theta = -300^\circ, \quad 300^\circ$$

Markers feedback

some students didn't use degree

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b) Prove that $\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)^2 = 1 + \cot^2\theta$

$$LHS = \left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)^2$$

$$= \sin^2\theta + 2\cancel{\sin\theta} \times \frac{\cos^2\theta}{\cancel{\sin\theta}} + \frac{\cos^4\theta}{\sin^2\theta}$$

$$= \sin^2\theta + 2\cos^2\theta + \frac{\cos^4\theta}{\sin^2\theta} \qquad 1. \quad 1$$

$$= \sin^2\theta + \cos^2\theta + \cos^2\theta + \frac{\cos^2\theta \cdot \cos^2\theta}{\sin^2\theta}$$

$$= 1 + \cos^2\theta\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right) \qquad (\sin^2\theta + \cos^2\theta = 1) \quad 1. \quad 1$$

$$= 1 + \cos^2\theta\,(1 + \cot^2\theta)$$

$$= 1 + \cos^2\theta \times \operatorname{cosec}^2\theta \qquad (1 + \cot^2\theta = \operatorname{cosec}^2\theta)$$

$$= 1 + \cos^2\theta \times \frac{1}{\sin^2\theta} \qquad 1. \quad 1$$

$$= 1 + \frac{\cos^2\theta}{\sin^2\theta}$$

$$= 1 + \cot^2\theta$$

LHS = RHS

or:

$$LHS = \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta}\right)^2$$

$$= \operatorname{cosec}^2\theta$$

$$= 1 + \cot^2\theta.$$

Mostly well done. some students need to be more familiar with Trig identities.

or

$$LHS = \sin^2\theta + 2\cos^2\theta + \frac{\cos^4\theta}{\sin^2\theta} \qquad 1.$$

$$= \frac{\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta}{\sin^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2}{\sin^2\theta} \qquad 1. \quad (\sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1^2}{\sin^2\theta}$$

$$= \operatorname{cosec}^2\theta \qquad (1 + \cot^2\theta = \operatorname{cosec}^2\theta)$$

$$= 1 + \cot^2\theta \qquad 1.$$

LHS = RHS

Markers feedback

8) c) $2\cos^2 x + \sin x + 1 = 0$

$2(1-\sin^2 x) + \sin x + 1 = 0$ — 1. for quadratic in terms of $\sin x$

$2 - 2\sin^2 x + \sin x + 1 = 0$

$-2\sin^2 x + \sin x + 3 = 0$

$2\sin^2 x - \sin x - 3 = 0$ 1

$(2\sin x - 3)(\sin x + 1) = 0$

$2\sin x = 3$ or $\sin x = -1$

$\sin x = \frac{3}{2}$ 1 $x = \frac{3\pi}{2}$ 1

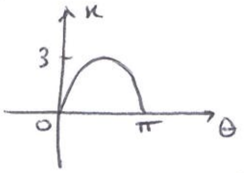
but $-1 \le \sin x \le 1$ ← need to justify why this solution is rejected

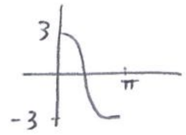
$\therefore$ no solution

$\therefore x = \frac{3\pi}{2}$

some students use degree instead of radian

9) a) $x = 3\sin\theta$, $y = 3\cos\theta$, $0 \le \theta \le \pi$





$\therefore 0 \le x \le 3$ $\qquad -3 \le y \le 3$

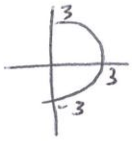
$\sin\theta = \frac{x}{3}$ $\qquad \cos\theta = \frac{y}{3}$

but $\sin^2\theta + \cos^2\theta = 1$ 1.

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$ 1.

$$x^2 + y^2 = 9 \quad \text{for} \quad 0 \le x \le 3$$ 1.



Most students were able to find the Cartesian equation $\frac{x^2}{9} + \frac{y^2}{9} = 1$ but a surprisingly large number then made errors after that e.g. $x^2+y^2 \neq 1$

Very few students (approx 10) were able to find the domain correctly. Most incorrectly stated $-3 \le x \le 3$.

b) (i) The restricted domain makes $y = f(x)$ a one-to-one function so its inverse is also a one-to-one function.

or

The restricted domain makes every y value correspond to only one x value so that the inverse has only one y value for every x value, hence making the inverse a function. 1.

Horizontal/vertical line test is a visual technique that we may use but it is not a reason and should not be given as the only explanation.

(ii) Domain of $f(x)$ is $x \ge -1$
Range of $f(x)$ is $y \le 2$

$\therefore$ Domain of $f^{-1}(x)$ is $x \le 2$ 1.

(iii) f: $y = 2 - (x+1)^2$

f^{-1}: $x = 2 - (y+1)^2$ 1.

$x - 2 = -(y+1)^2$

$2 - x = (y+1)^2$

$y + 1 = \pm\sqrt{2-x}$

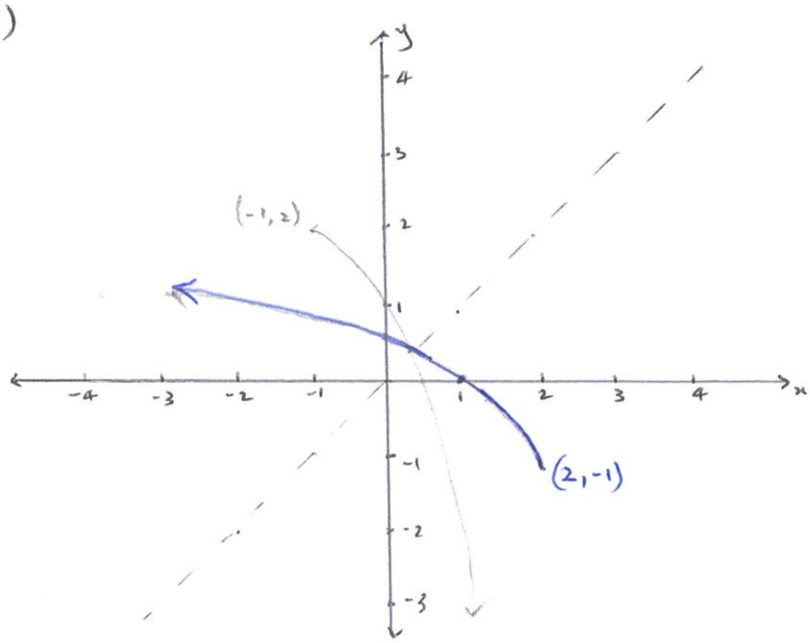
$y = -1 \pm \sqrt{2-x}$ 1. but D: $x \le 2$
R: $y \ge -1$

$\therefore y = -1 + \sqrt{2-x}$ 1.

Most students were able to find $y = -1 \pm \sqrt{2-x}$ but then many stated $x \ge 2$ instead of $y \ge -1$ to justify why the negative case is discarded to give $y = -1 + \sqrt{2-x}$

9) b)

(iv)



1 for passing through (2, -1), (1, 0) and basic shape

1 for accuracy

Question 7

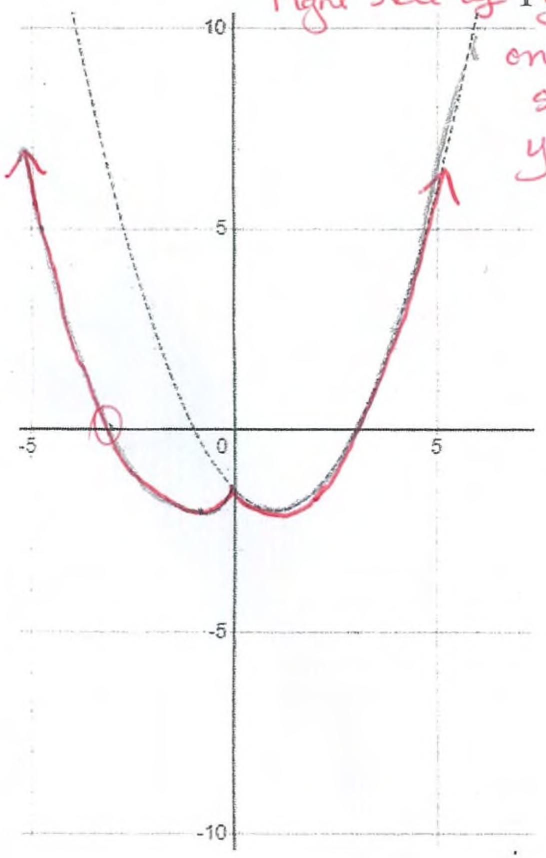
The diagrams below are graphs of the function $f(x) = \frac{1}{2}(x+1)(x-3)$.

These questions produced a mixed bag of results mainly because of
1. Recognition of what was asked was not known
2. Inaccuracy and not going through basic points to produce an accurate transformation

Use the graphs provided to sketch each of the following, showing all asymptotes and intercepts.

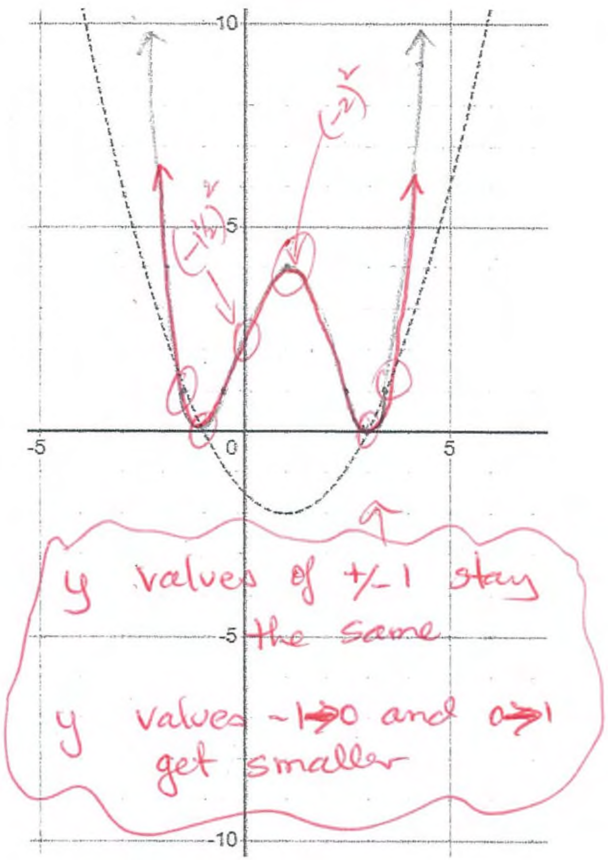
(i) $y = f(|x|)$ **1**

$-x$ values match $+x$ values i.e mirror image of right side of y axis on left side of y axis



(ii) $y = [f(x)]^2$ **2**

y values are squared



(iii) $y = \dfrac{1}{f(x)}$ **2**

y values are recipricated