HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 2 Term 2 2023

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 60 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 44

Section I Pages 2 – 3
5 marks
Attempt Questions 1 – 5
Answer on the Objective Response Answer Sheet provided

Section II Pages 4 – 7
42 marks
Attempt Questions 6 – 9.

Question	***1-5***	***6***	***7***	***8***	***9***	***Total***
Total	/5	/10	/11	/10	/11	/47

This assessment task constitutes 35% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. $\cos\left(\frac{3\pi}{2}-x\right)$ is equivalent to:

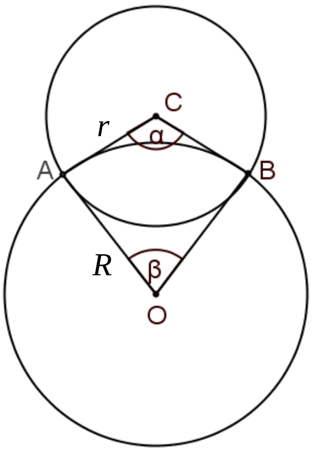
(A) $\cos x$

(B) $\sin x$

(C) $-\sin x$

(D) $-\cos x$

Q2. The perimeter of the figure is



(A) $(r\alpha + R\beta)$ units

(B) $\left(r(2\pi-\alpha)+R(2\pi-\beta)\right)$ units

(C) $\left(r\alpha^\circ + R\beta^\circ\right)$ units

(D) $\left(r(360-\alpha)^\circ + R(360-\beta)^\circ\right)$ units

Q3. $\sec^2\left(2\alpha^\circ-\beta^\circ\right)-\dfrac{\sin^2\left(2\alpha^\circ-\beta^\circ\right)}{\cos^2\left(2\alpha^\circ-\beta^\circ\right)}=$

(A) 0

(B) 1

(C) -1

(D) $\tan^2\left(2\alpha^\circ-\beta^\circ\right)$

Q4. Using the proof of $f\left(g\left(x\right)\right)=g\left(f\left(x\right)\right)=x$, or otherwise, which of pair of $f\left(x\right)$ and $g\left(x\right)$ are inverse functions of each other:

(A) $f\left(x\right)=x^2-4,\ x\ge 0$ and $g\left(x\right)=-\sqrt{x+2},\ x\ge -2$

(B) $f\left(x\right)=3x-1$ and $g\left(x\right)=\frac{1}{3}x+1$

(C) $f\left(x\right)=\left(x-2\right)^2,\ x\ge 2$ and $g\left(x\right)=2+\sqrt{x},\ x\ge 2$

(D) $f\left(x\right)=\sqrt{x-2},\ x\ge 2$ and $g\left(x\right)=2+x^2,\ x\ge 0$

Q5. The Cartesian equation $x^2=9\left(y-1\right)$ can be written in parametric form:

(A) $x=3t,\ y=t^2+1$

(B) $x=t,\ y=3t^2+1$

(C) $x=\frac{t}{3},\ y=t^2+1$

(D) $x=t,\ y=3\left(t^2+1\right)$

End of Section I

Section II

30 marks
Attempt Questions 6-8
Allow about 50 minutes for this section

Question 6 (10 marks) **Start a new answer booklet**

(a) Rationalise the denominator $\dfrac{3+\sqrt{8}}{3-\sqrt{8}}$ **2**

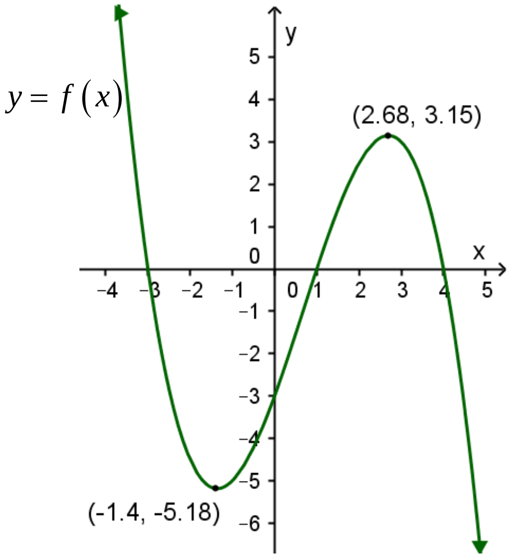
(b) Find $\cot\theta$ if $\cos\theta = -\dfrac{7}{10}$, $\sin\theta > 0$, in exact form. **2**

(c) Solve $\dfrac{2}{(x-2)(x-3)} \geq 1$ **3**

(d) Rearrange the equation to make a the subject: $B = \sqrt{\dfrac{3a(b-c)}{2a+1}}$ **3**

Question 7 (11 marks) **Start a new answer booklet**

(a) The diagram below is a sketch of the graph $y = f(x)$.



Use the graphs provided on the separate answer sheet, to graph the following transformations showing all asymptotes and intercepts.

(i) $y = \left| f(x) \right|$ **1**

(ii) $y = \sqrt{f(x)}$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **2**

(iv) $y = f^{-1}(x)$, where domain of $f(x): -1.4 \le x \le 2.68$ **2**
(Include the line $y = x$)

(v) If $g(x) = \dfrac{x}{2}$, sketch $y = g\left[f(x)\right]$ **1**

(b) Solve $\sin 3\theta = \dfrac{1}{4}$ for $0^\circ \le \theta \le 180^\circ$, to the nearest degree. **3**

Question 8 (10 marks) **Start a new answer booklet**

(a) Simplify $\dfrac{48^{\frac{1}{4}}}{18\times 24^{-\frac{1}{2}}}$ **2**

(b) Prove that $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=\dfrac{2}{\sin\theta}$ **2**

(c) (i) Find the domain of the function $f\left(x\right)=-\sqrt{2x+3}$. **1**

(ii) Find the equation of its inverse $f^{-1}\left(x\right)$. **3**

(iii) Find the intersecting point of $f\left(x\right)$ and $f^{-1}\left(x\right)$. **2**

Question 9 (11 marks) **Start a new answer booklet**

(a) Solve for x, $\sec^2 x - \tan x = 3$, $0 \le x \le 2\pi$ **3**

(b) Solve the inequality $|2x-1| > |x-3|$ graphically, or otherwise. **3**

(c) (i) Complete the table and sketch the Cartesian graph of **2**

$$x = 1 - 2\sin^2 t,\ \ y = \sin t, \quad 0 \le t \le \frac{\pi}{2}.$$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x					
y					

(ii) Find the Cartesian equation for the parametric equations **3**

$x = 1 - 2\sin^2 t$, $y = \sin t$, $0 \le t \le \frac{\pi}{2}$ **<u>and</u>** state the domain and range of the Cartesian equation.

End of paper

Solutions of Mathematics YR 11 Extension 1 Assessment Task 2, Term 2 2023/24

SOLUTION	COMMENT

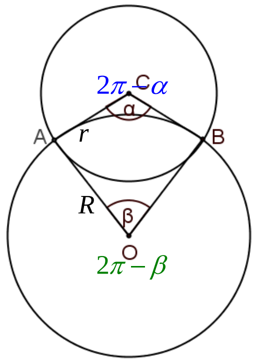
Question 1

$$\cos\left(\frac{3\pi}{2}-x\right)=-\sin x$$

$\uparrow$

cos in (QIII) is a negative value **(C)**

Question 2



$$P=r\theta_{major1}+R\theta_{major2}$$
$$=r(2\pi-\alpha)+R(2\pi-\beta)$$ **(B)**

Question 3

$$\sec^2(2\alpha^\circ-\beta^\circ)-\frac{\sin^2(2\alpha^\circ-\beta^\circ)}{\cos^2(2\alpha^\circ-\beta^\circ)}=\sec^2(2\alpha^\circ-\beta^\circ)-\tan^2(2\alpha^\circ-\beta^\circ)$$
$$=1$$ **(B)**

Question 4

(A) $f(x)=x^2-4,\ x\ge 0$ and $g(x)=-\sqrt{x+2},\ x\ge -2$

$y\ge -4$

Range of $f(x)\ne$ Domain of $g(x)$ $\therefore$ not (A)

(B) $f(x)=3x-1$ and $g(x)=\frac{1}{3}x+1$

$$f(g(x))=3g(x)-1$$
$$f(g(x))=3\left[\frac{1}{3}x+1\right]-1$$
$$=x+3-1$$
$$=x+2$$ $\therefore$ not (B)

(C) $f(x)=(x-2)^2,\ x\ge 2$ and $g(x)=2+\sqrt{x},\ x\ge 2$

$y\ge 0$

Range of $f(x)\not\subset$ Domain of $g(x)$ $\therefore$ not (C)

(D) $f(x)=\sqrt{x-2},\ x\ge 2$ and $g(x)=2+x^2,\ x\ge 0$

$y\ge 0$

Range of $f(x)\subseteq$ Domain of $g(x)$ **(D)**

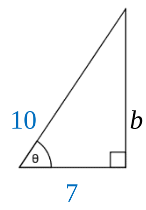
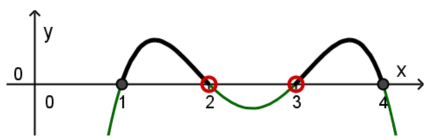
Question 5

Cartesian equation $x^2=9(y-1)$

(A) $x=3t$ and $y=t^2+1$

$t=\frac{x}{3}$ $\quad$ $y=\left(\frac{x}{3}\right)^2+1$

SOLUTION	COMMENT
Question 5 cont. $$y=\left(\frac{x}{3}\right)^2+1$$ $$y-1=\frac{x^2}{9}$$ $$\therefore 9(y-1)=x^2$$ **(A)**	

SOLUTION	COMMENT
Question 6(a) $\dfrac{3+\sqrt{8}}{3-\sqrt{8}}=\dfrac{3+\sqrt{8}}{3-\sqrt{8}}\times\dfrac{3+\sqrt{8}}{3+\sqrt{8}}$ $=\dfrac{\left(3+\sqrt{8}\right)^2}{9-8}$ $=9+6\sqrt{8}+8$ $=17+6\left(2\sqrt{2}\right)$ $=17+12\sqrt{2}$	Generally well done.
Question 6(b) $\cos\theta=-\dfrac{7}{10}$, $\sin\theta>0$, i.e. (QII) where $b=\sqrt{10^2-7^2}$ $=\sqrt{51}$  $\therefore \cot\theta=-\dfrac{7}{\sqrt{51}}$	Few students did not retain the information that θ is an angle in quadrant 2, hence $\cot\theta<0$. Many students did not know how to approach this question.
Question 6(c) $\dfrac{2}{(x-2)(x-3)}\geq 1,\quad x\neq 2,\ 3$ $\dfrac{2}{(x-2)(x-3)}\times(x-2)^2(x-3)^2\geq 1\times(x-2)^2(x-3)^2$ $2(x-2)(x-3)\geq(x-2)^2(x-3)^2$ $2(x-2)(x-3)-(x-2)^2(x-3)^2\geq 0$ $(x-2)(x-3)\left[2-(x-2)(x-3)\right]\geq 0$ $(x-2)(x-3)\left[2-\left(x^2-5x+6\right)\right]\geq 0$ $(x-2)(x-3)\left[2-x^2+5x-6\right]\geq 0$ $(x-2)(x-3)\left(-x^2+5x-4\right)\geq 0$ $-(x-2)(x-3)\left(x^2-5x+4\right)\geq 0$ $-(x-2)(x-3)(x-1)(x-4)\geq 0$  $\therefore [1,\ 2)\cup(3,\ 4]$	Some students did not know what to do at all. Many students who square both sides but expanded the factors and got lost as what to do. The students who factorized the expressions after gathering the terms to LHS or RHS continue to solve with ease. Some students did not draw the graph hence was not able to get answer. Some students drew the wrong graph. Many students forgot that $x\neq 2$ or 3.
Question 6(d) $B=\sqrt{\dfrac{3a(b-c)}{2a+1}}$ $B^2=\dfrac{3a(b-c)}{2a+1}$ $B^2(2a+1)=3a(b-c)$ $2aB^2+B^2=3a(b-c)$ $B^2=3a(b-c)-2aB^2$ $\therefore a=\dfrac{B^2}{3(b-c)-2B^2}$	Poorly done. Many successfully squared both sides but got totally lost as to how to make a the subject.

SOLUTION	COMMENT
Question 7(b) $\sin 3\theta = \frac{1}{4}$, **(QI, II)** $0^\circ \le \theta \le 180^\circ$ $0^\circ \le 3\theta \le 540^\circ$, 1.5 revolutions $3\theta = 14.4775^\circ,\ 180^\circ - 14.4775^\circ,\ 360^\circ + 14.4775^\circ,\ 540^\circ - 14.4775^\circ$ $3\theta = 14.4775^\circ,\ 165.5225^\circ,\ 374.4775^\circ,\ 525.5225^\circ$ \\ $\therefore \theta = 4.8258^\circ,\ 55.1742^\circ,\ 124.8258^\circ,\ 175.1742^\circ$ $\therefore \theta = 5^\circ,\ 55^\circ,\ 125^\circ,\ 175^\circ$ (to nearest degree)	
Question 8(a) $\frac{48^{\frac{1}{4}}}{18\times 24^{-\frac{1}{2}}} = \frac{\left(3\times 2^4\right)^{\frac{1}{4}}}{\left(3^2\times 2\right)\times\left(3\times 2^3\right)^{-\frac{1}{2}}}$ $= \frac{\left(3\times 2^4\right)^{\frac{1}{4}}\times\left(3\times 2^3\right)^{\frac{1}{2}}}{\left(3^2\times 2\right)}$ $= \frac{3^{\frac{1}{4}}\times 2\times 3^{\frac{1}{2}}\times 2^{\frac{3}{2}}}{3^2\times 2}$ $= \frac{3^{\frac{3}{4}}\times 2^{\frac{3}{2}}}{3^2}$ $= \frac{2^{\frac{3}{2}}}{3^{2-\frac{3}{4}}}$ $= \frac{2^{\frac{3}{2}}}{3^{\frac{5}{4}}}$	A lot of students did not realise the best way to simplify is to reduce to prime bases. 1 For making some reasonable progress towards simplifying. 1 For correct answer
Question 8(b) To prove: $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{2}{\sin\theta}$ Proof: LHS $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$ $= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$ $= \frac{\left(\sin^2\theta + \cos^2\theta\right) + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$ $= \frac{(1) + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$ $= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$ $= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$ $= \frac{2}{\sin\theta}$ $\therefore$LHS = RHS	This was mostly well done 1 for working at this stage 1 for correct final simplification

SOLUTION	COMMENT
Question 8(c) (i) $f(x) = -\sqrt{2x+3}$ Domain: $2x+3 \geq 0$ $2x \geq -3$ $\therefore x \geq -\frac{3}{2}$	Mostly well done 1 for correct domain
(ii) $f(x) = -\sqrt{2x+3}$ Range: $f(x) \leq 0$ i.e. domain of $f^{-1}(x)$: $x \leq 0$ Let $y = -\sqrt{2x+3}$ Its inverse: $x = -\sqrt{2y+3}$ $-x = \sqrt{2y+3}$ $(-x)^2 = 2y+3$ $x^2 = 2y+3$ $x^2 - 3 = 2y$ $y = \frac{1}{2}(x^2-3)$ $\therefore f^{-1}(x) = \frac{1}{2}(x^2-3), \quad x \leq 0$	1 for interchanging x and y and attempting to get the inverse fn. 1 for correct inverse fn 1 for the correct domain (very important)
(iii) $f(x) = -\sqrt{2x+3}$ — ① $f^{-1}(x) = \frac{1}{2}(x^2-3), \quad x \leq 0$ — ② $y = x$ — ③ Sub ③ into ② $x = \frac{1}{2}(x^2-3)$ $2x = x^2 - 3$ $x^2 - 2x - 3 = 0$ $(x+1)(x-3) = 0$ $\therefore x = -1, \ x \leq 0$ Hence the intersecting point is (-1, -1)	A lot of students did not make either the fn or the inv fn equal to x. Making them equal to each other led to a mess. 1 for getting solutions of -1 and 3 1 for disregarding 3 as it is not in the domain of fn AND inv fn.

SOLUTION	COMMENT

Question 9(a)

$$\sec^2 x - \tan x = 3, \quad 0 \le x \le 2\pi$$
$$1 + \tan^2 x - \tan x = 3$$
$$\tan^2 x - \tan x - 2 = 0$$
$$(\tan x + 1)(\tan x - 2) = 0$$

$\tan x = -1$ **(QII, IV)** or $\tan x = 2$ **(QI, III)**

$$x = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4} \qquad\qquad x = 1.1071,\ \pi + 1.1071$$
$$x = \frac{3\pi}{4},\ \frac{7\pi}{4} \qquad\qquad x = 1.1071,\ 4.2487$$

$$\therefore x = 1.1071,\ \frac{3\pi}{4},\ 4.2487,\ \frac{7\pi}{4}$$

COMMENT:

NOTE: Radians, not degrees

1 for making use of identity and factorizing correctly.

1 for solutions of $\tan x = -1$. Solutions should be exact, not 2.356 and 5.498.

1 for solutions of $\tan x = 2$. Some students discarded these solutions because they incorrectly thought $-1 \le \tan x \le 1$. This is true for $\sin x$ and $\cos x$, but not $\tan x$.

Some students used degrees rather than radians

$1.1071^c \cong 63^o 26',\ 4.2487^c \cong 243^o 26'$

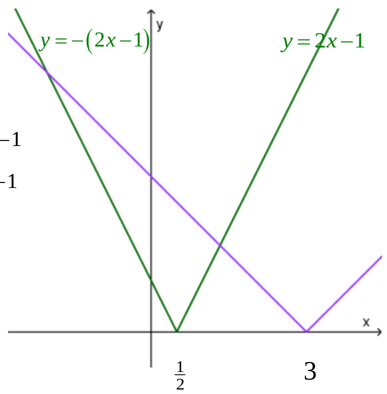
Question 9(b) $|2x-1| > |x-3|$

Method 1: Graphical method

As graphically shown,
At the intersecting points:

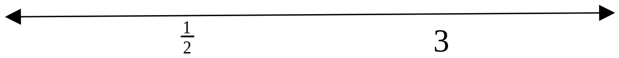
$$-(x-3) = -(2x-1) \quad \text{or} \quad -(x-3) = 2x-1$$
$$x - 3 = 2x - 1 \qquad\qquad -x + 3 = 2x - 1$$
$$\therefore x = -2 \qquad\qquad 4 = 3x$$
$$\therefore x = \frac{4}{3}$$

$$\therefore |2x-1| > |x-3|$$
$$\therefore x < -2 \quad \text{or} \quad x > \frac{4}{3}$$



Method 2: Algebraic method/ Critical point method

$|2x-1| > |x-3|$



If $x \le \frac{1}{2}$ or If $\frac{1}{2} < x < 3$ or If $x \ge 3$

If $x \le \frac{1}{2}$	If $\frac{1}{2} < x < 3$	If $x \ge 3$
$-(2x-1) > -(x-3)$	$2x-1 > -(x-3)$ or	$2x-1 > x-3$
$2x-1 < x-3$	$2x-1 > -x+3$	$x > -2$
$x < -2$	$3x > 4$	
but $x \le \frac{1}{2}$	$\therefore x > \frac{4}{3}$	but $x \ge 3$
$\therefore x < -2$	but $\frac{1}{2} < x < 3$	$\therefore x \ge 3$
	$\therefore \frac{4}{3} < x < 3$	

$$\therefore x < -2,\ \tfrac{4}{3} < x < 3,\ x \ge 3$$
$$\therefore x < -2,\ x > \tfrac{4}{3}$$

COMMENT:

1 for correct method
1 for correct values of x
1 for interpreting correctly

Some students incorrectly used
$$x - 3 = 2x - 1$$
$$-x = 2$$
$$x = -2$$
Instead of $-(x-3) = -(2x-1)$

Several numerical errors

Many students using the graphical method did not extend their graphs far enough to get the two sets of solutions.

Method 4: Squaring method

$$|2x-1| > |x-3|$$
$$(2x-1)^2 > (x-3)^2$$
$$4x^2 - 4x + 1 > x^2 - 6x + 9$$
$$3x^2 + 2x - 8 > 0$$
$$3x + 6x - 4x - 8 > 0$$
$$(x+2)(3x-4) > 0$$

Which is a concave up parabola with x intercepts at $x = -2, \frac{4}{3}$

$\therefore x < -2,\ x > \frac{4}{3}$

Question 9(c)

(i) $x = 1 - 2\sin^2 t, \ \ y = \sin t, \quad 0 \le t \le \frac{\pi}{2}$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x	1	0.5	0	–0.5	-1
y	0	0.5	0.71	0.87	1

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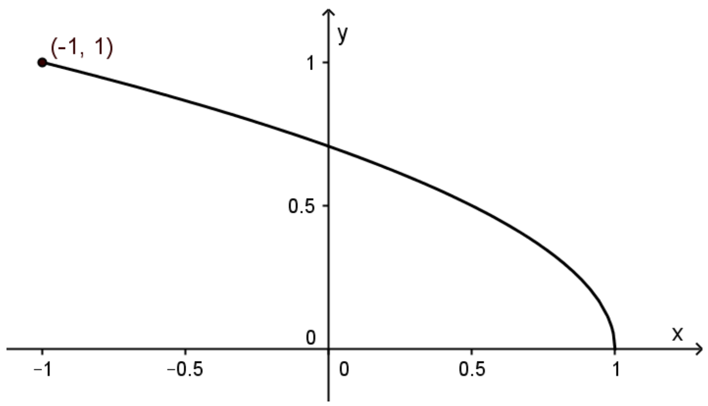
Students need to be aware that any working done on the question paper is not seen by markers and will not be awarded any marks.

SOLUTION	COMMENT

Question 9(c)(i)cont.



Many students did not provide a table of values. If the graph showed appropriate values, the 2 marks were awarded.
The graph needed to look like a curve, not a straight line.

(ii) $x = 1 - 2\sin^2 t, \ 0 \le t \le \frac{\pi}{2}$ —— ①

$y = \sin t, \quad 0 \le t \le \frac{\pi}{2}$ —— ②

Sub ② into ① $x = 1 - 2y^2$ $2y^2 = 1 - x$ $y^2 = \frac{1}{2}(1-x)$ $y = \pm\sqrt{\frac{1}{2}(1-x)}$ but $0 \le y \le 1$ $\therefore y = \frac{1}{2}(1-x), \quad -1 \le x \le 1$ Domain: $[-1,\ 1]$ Range: $[0,\ 1]$ **END OF ASSESSMENT**	Many students did not deal with discarding the $-\sqrt{\ }$ It looked like students were rushed for time at the end of the exam.