HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 2 Term 2 2024

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 60 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination.

Total marks – 45

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

Section II Pages 4 – 7

40 marks

Attempt Questions 6 – 9.

Question	***1-5***	***6***	***7***	***8***	***9***	***Total***
Total	*/5*	*/10*	*/10*	*/10*	*/10*	*/45*

This assessment task constitutes 35% of the Preliminary Higher School Certificate Course School Assessment.

Outcomes Examined ME11-1, ME11-2, ME11-3, MA11-1, MA11-2, MA11-3, MA11-4, MA11-9, ME11-7

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. If $\tan\theta = \dfrac{3}{4}$ and $\cos\theta < 0$ then $\sin\theta$ is:

(A) $\dfrac{3}{5}$

(B) $\dfrac{4}{5}$

(C) $-\dfrac{3}{5}$

(D) $-\dfrac{4}{5}$

Q2. The solution of $x^2 - 3x < 4$ is:

(A) $\{x : 1 < x < -4\}$

(B) $\{x : -1 < x < 4\}$

(C) $\{x : x < -1, x > 4\}$

(D) $\{x : x < -4, x > 1\}$

Q3. $\dfrac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta}$ is equivalent to:

(A) 1

(B) $\sec^2\theta + \cos^2\theta$

(C) $\tan^2\theta + \sin^2\theta$

(D) $\tan^2\theta + \cos^2\theta$

Q4. Which of the following pairs of functions are NOT inverse functions?

(A) $f(x)=2x-1$ and $g(x)=\frac{1}{2}(x+1)$

(B) $f(x)=\frac{1}{2x-1}, x>\frac{1}{2}$ and $g(x)=\frac{2x+1}{x}, x>0$

(C) $f(x)=x^2-1, x\geq 0$ and $g(x)=\sqrt{x+1}, x\geq -1$

(D) $f(x)=x^2-1, x\leq 0$ and $g(x)=-\sqrt{x+1}, x\geq -1$

Q5. The Parametric equations $x=6\cos\theta$ and $y=6\sin\theta$ when written in Cartesian form as:

(A) $x^2+y^2=6$

(B) $x^2-y^2=6$

(C) $x^2-y^2=36$

(D) $x^2+y^2=36$

End of Section I

Section II

40 marks
Attempt Questions 6-8
Allow about 50 minutes for this section

Question 6 (10 marks) **Start a new answer booklet**

(a) Showing full working, rationalise the denominator of $\dfrac{3+2\sqrt{5}}{2\sqrt{5}-1}$ **2**

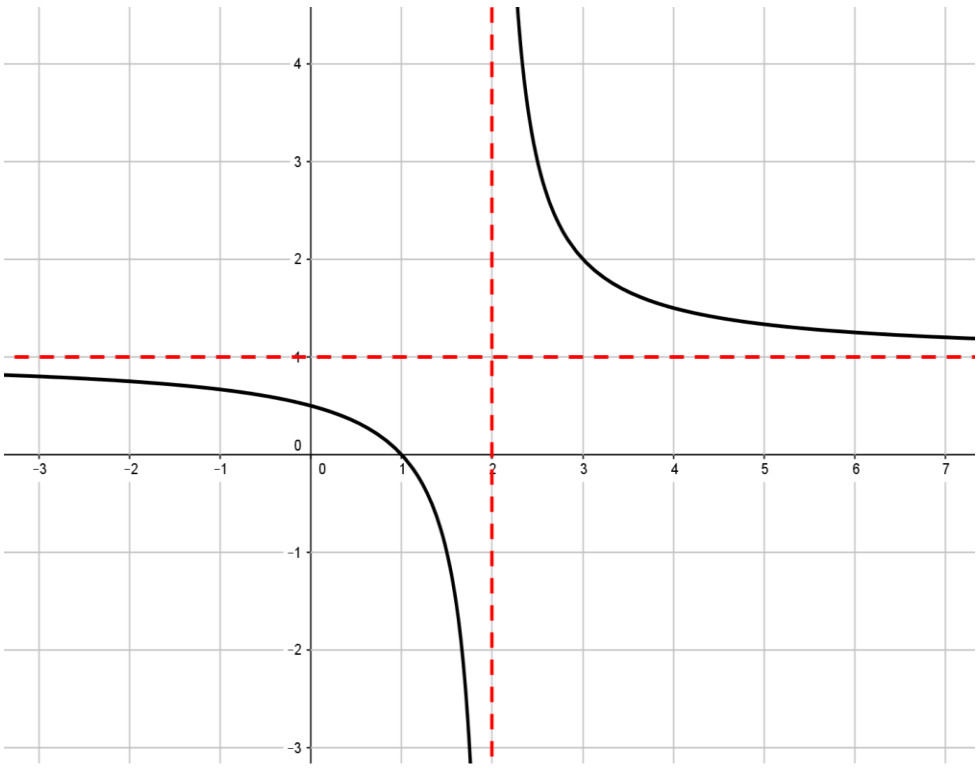
(b) Simplify $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}$ **2**

(c) Solve $\left|x^2-1\right|\le 4$ **3**

(d) Simplify $\dfrac{x^3-1}{x^2-1}\div\dfrac{3x^2+3x+3}{x^2-4x-5}$ **3**

Question 7 (10 marks) **Start a new answer booklet**

(a) The diagram below is a sketch of the graph $y = f(x)$.



Use the graphs provided on the separate answer sheet, to graph the following transformations showing all asymptotes and intercepts.

(i) $y = |f(x)|$ **1**

(ii) $y^2 = f(x)$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **2**

(iv) If $g(x) = 2x$, sketch $y = g[f(x)]$ **2**

(b) Solve $\cos 2\theta = \dfrac{1}{2}$ for $0 \le \theta \le 2\pi$. **3**

Question 8 (10 marks) **Start a new answer booklet**

(a) Find the value of $\dfrac{a}{b^3c^2}$ in index form if: **2**

$$a=\left(\frac{2}{5}\right)^4,\ b=\left(-\frac{1}{3}\right)^3 \text{ and } c=\left(\frac{3}{5}\right)^2$$

(b) Prove that $\sec\theta+\tan\theta=\dfrac{1+\sin\theta}{\cos\theta}$ **2**

(c) (i) Find the domain and range of the function $f(x)=\sqrt{5-x}-1$. **2**

(ii) Find the equation of its inverse $f^{-1}(x)$. **2**

(iii) Find the intersecting point of $f(x)$ and $f^{-1}(x)$. **2**

Question 9 (10 marks) **Start a new answer booklet**

(a) Solve $5\cos^2 x + 2\sin x = 2$ for $0^\circ \le x \le 180^\circ$ **3**

(b) Find the cartesian equation of $x = 2p + 3, y = p^2$ and describe any important features of the equation **2**

(c) Write $x^2 + y^2 - 6x + 2y - 26 = 0$ in parametric form with parameter θ. **2**

(d) Find the values of x for which the inequalities are simultaneously satisfied: **3**

$$\frac{7}{(3-x)(x+3)} > -1 \qquad \text{and} \qquad x^2 + 3x - 10 < 0$$

End of Assessment

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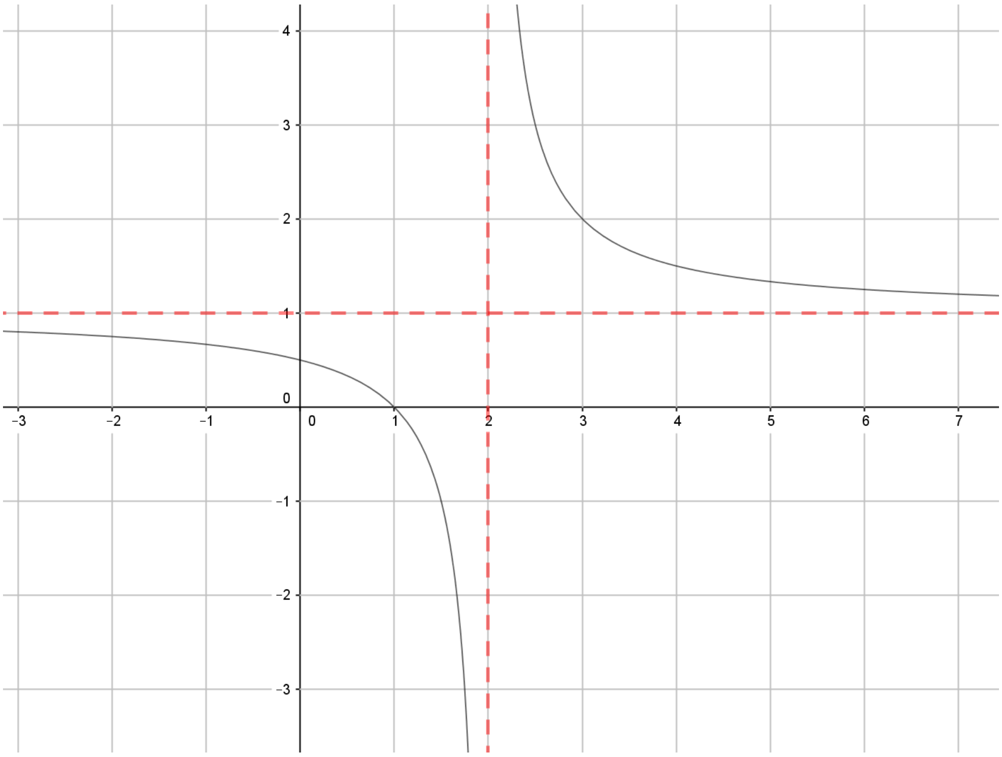
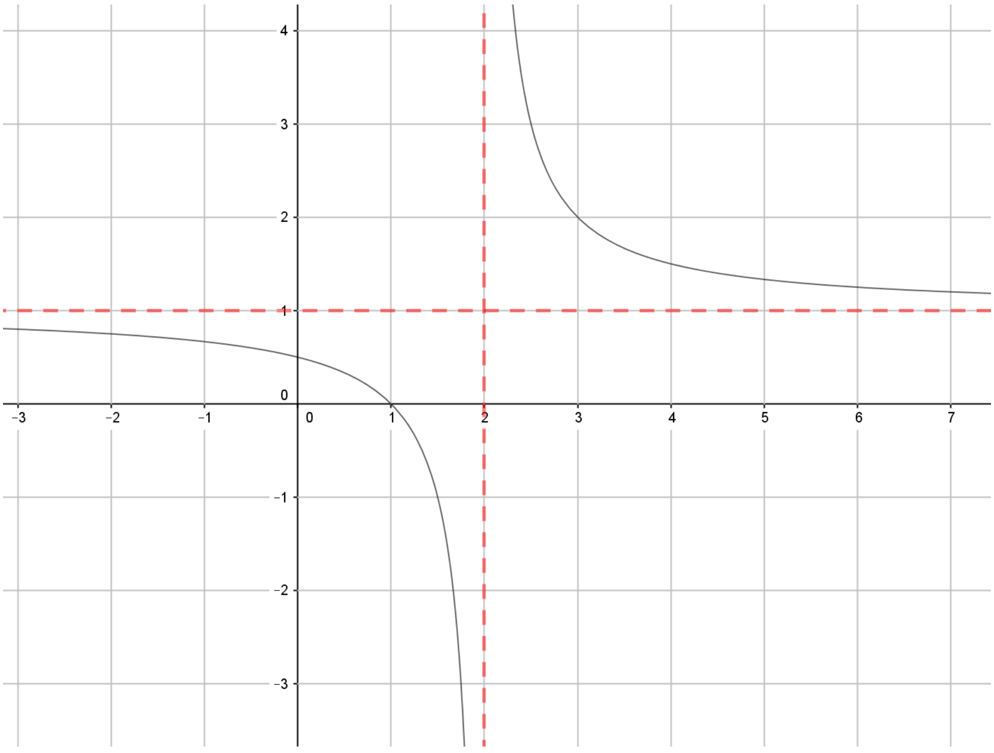
Question 7(a) Place this sheet inside the Question 7 Answer Booklet

(i) $y = \left| f(x) \right|$

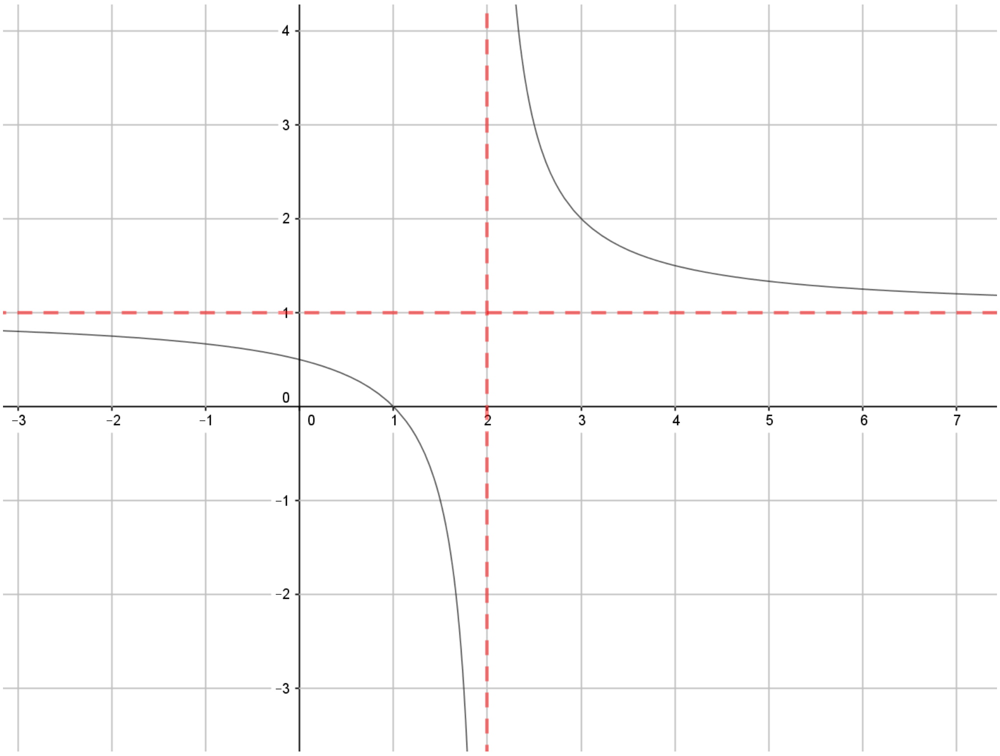


(ii) $y^2 = f(x)$

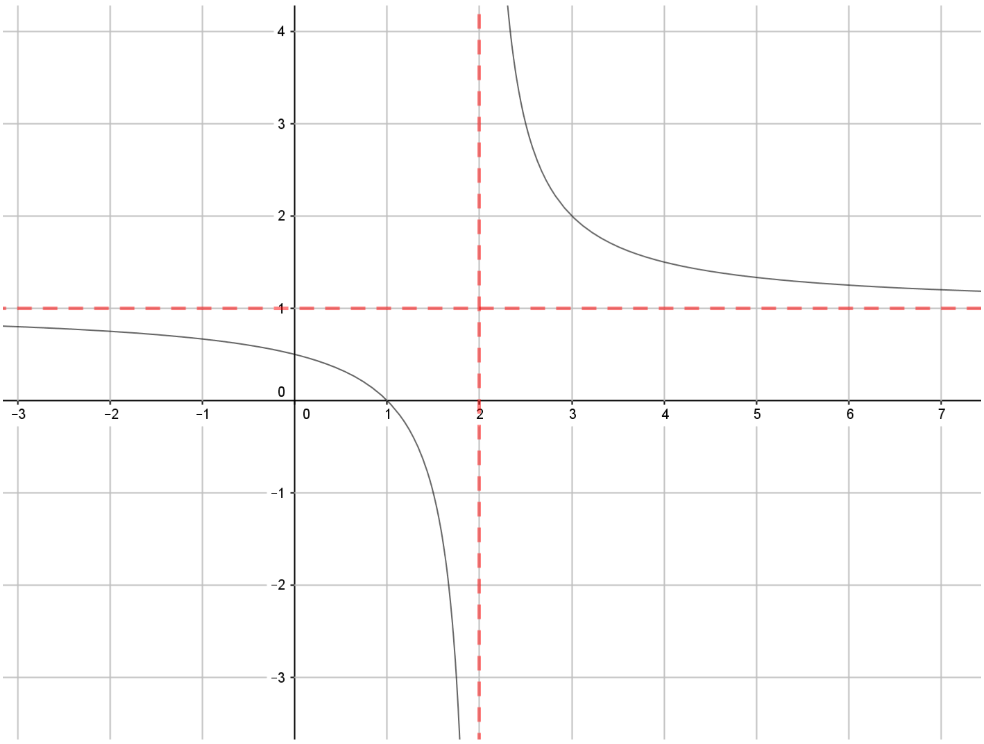


Question 7(a)

(iii) $y = \dfrac{1}{f(x)}$



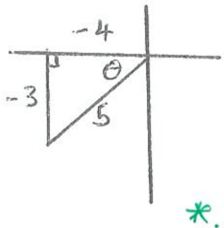
(iv) If $g(x) = 2x$, sketch $y = g[f(x)]$



YEAR 11 Ext 1 Task 2 Solutions and Feedback

Q1. $\tan\frac{3}{4} > 0$ and $\cos\theta < 0$ $\therefore$ third quad

$$\sin\theta = -\frac{3}{5}$$



(C)

Mostly well done.

*. Incorrect choice of A when forgotten it is in QIII

Q2. $x^2 - 3x < 4$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

$\{x : -1 < x < 4\}$

(B)

Mostly well done.

Incorrect choice of (C) student did not use a graph or number line to obtain answer.

Q3. $\dfrac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$ (D)

$= \sec^2\theta - \sin^2\theta$ Not an option

$= 1 + \tan^2\theta - \sin^2\theta$ " " "

$= \tan^2\theta + \cos^2\theta$ ✓

50% obtained the correct answer.

25% chose incorrectly the answer (C)

Q4. (A) $f(x) = 2x - 1$

$y = 2x - 1$

$\therefore$ inv $x = 2y - 1$

$2y = x + 1$

$y = \frac{1}{2}(x+1)$

$g(x) = \frac{1}{2}(x+1)$ ✓

$\therefore$ they are a pair of inverse fns.

(B) $f(x) = \dfrac{1}{2x-1}$, $x > \frac{1}{2}$ $f(x) > 0$

$y = \dfrac{1}{2x-1}$

$\therefore$ inv $x = \dfrac{1}{2y-1}$

$2y - 1 = \dfrac{1}{x}$

$2y = \dfrac{1}{x} + 1$

$y = \dfrac{1}{2x} + \dfrac{1}{2}$

$= \dfrac{1+x}{2x}$

$\neq \dfrac{2x+1}{x}$ $\therefore$ Not an inverse fn.

(B)

66% obtained correct answer.

approx. 25% obtained incorrectly (D)

No need to look at (C) & (D) but the working is over the page to confirm they are pairs of inverse functions.

Q5. $x = 6\cos\theta \Rightarrow \cos\theta = \dfrac{x}{6}$ $y = 6\sin\theta \Rightarrow \sin\theta = \dfrac{y}{6}$

Mostly done well.

using $\cos^2\theta + \sin^2\theta = 1$

$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$

$$x^2 + y^2 = 36$$

$\therefore$ (D)

Q4 (C) $f(x) = x^2 - 1, \ x \geq 0, \ y \geq -1$

$y = x^2 - 1$

$\therefore$ inv: $x = y^2 - 1$

$y^2 = x + 1$

$y = \pm\sqrt{x+1}$ but $x \geq -1$ & $y \geq 0$

$\therefore y = \sqrt{x+1}$

$\therefore g(x) = \sqrt{x+1}$

$\therefore f(x)$ & $g(x)$ are inverse fns

(D) $f(x) = x^2 - 1, \ x \leq 0, \ y \geq -1$

$y = x^2 - 1$

$\therefore$ inv: $x = y^2 - 1$

$y^2 = x + 1$

$y = \pm\sqrt{x+1}$ but $x \geq -1$ & $y \leq 0$

$\therefore y = -\sqrt{x+1}$

$\therefore g(x) = -\sqrt{x+1}$

$\therefore f(x)$ & $g(x)$ are inverse fns.

* For Q4 you could also show that

$f(g(x)) = g(f(x)) = x$ for (A), (C) & (D)

but not for (B)

Question 6

(a) $\dfrac{(3+2\sqrt{5})}{(2\sqrt{5}-1)} \times \dfrac{(2\sqrt{5}+1)}{(2\sqrt{5}+1)} = \dfrac{6\sqrt{5}+3+20+2\sqrt{5}}{20-1}$

$= \dfrac{8\sqrt{5}+23}{19}$

Mostly done well. Just reminding everyone conjugate of $(2\sqrt{5}-1)$ is $(2\sqrt{5}+1)$, not $(\sqrt{5}+1)$

(b) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$

$= \dfrac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$ $(\sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

Mostly done well.

$= \dfrac{2}{\sin\theta}$

$= 2\operatorname{cosec}\theta$

(c) $|x^2-1| \le 4$

$-4 \le x^2-1 \le 4$

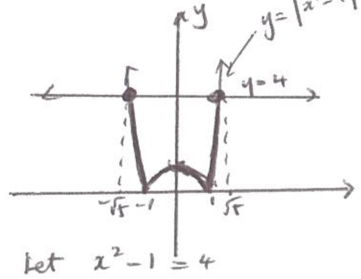
$-4 \le x^2-1$ and $x^2-1 \le 4$

$-3 \le x^2$ $\quad x^2 \le 5$

all x and $-\sqrt{5} \le x \le \sqrt{5}$

$\therefore \{x : -\sqrt{5} \le x \le \sqrt{5}\}$

OR



Let $x^2-1=4$

$x^2=5$

$x = \pm\sqrt{5}$

$\therefore$ As graphically shown, $\{x : -\sqrt{5} \le x \le \sqrt{5}\}$

Mostly done well. If a graph is used as a tool, then expected to sketch $y=|x^2-4|$

(d) $\dfrac{x^3-1}{x^2-1} \div \dfrac{3x^2+3x+3}{x^2-4x-5} = \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} \times \dfrac{(x-5)(x+1)}{3(x^2+x+1)}$

Mostly done well.

$= \dfrac{x-5}{3}$

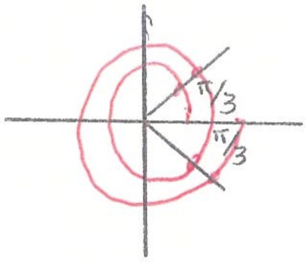
Question 7

Generally well done. Some students didn't adjust the domain for 2θ

(a) (i) – (iv) See Seperate Sheet.

(b) $\cos 2\theta = \frac{1}{2}$ $\qquad 0^c \leq \theta \leq 2\pi^c$

$0^c \leq 2\theta \leq 4\pi^c$



$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 8

See Next page for markers comments + extra solutions.

(a) $\frac{a}{b^3c^2} = \frac{\left(\frac{2}{5}\right)^4}{\left[\left(-\frac{1}{3}\right)^3\right]^3\left[\left(\frac{3}{5}\right)^2\right]^2}$

$= \frac{\frac{2^4}{5^4}}{\frac{-1}{3^{\cancel{9}\,5}} \cdot \frac{\cancel{3^4}}{5^4}}$

$= \frac{2^4}{5^4} \cdot \frac{3^5 \cdot 5^4}{-1^9}$

$= -2^4 \cdot 3^5$

(b) RTP $\sec\theta + \tan\theta = \frac{1+\sin\theta}{\cos\theta}$

LHS $= \sec\theta + \tan\theta$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

$=$ RHS

(c) (i) $f(x) = \sqrt{5-x} - 1$ $\qquad$ D: $5 - x \geq 0$ $\qquad$ R: $y \geq -1$

$-x \geq -5$

$x \leq 5$

(ii) let $y = f(x)$

$\therefore y = \sqrt{5-x} - 1$

$\therefore$ inv $x = \sqrt{5-y} - 1$

$x + 1 = \sqrt{5-y}$

$(x+1)^2 = 5 - y$

$y = 5 - (x+1)^2$ $\qquad$ D: $x \geq -1$

$f^{-1}(x) = 5 - (x+1)^2$ $\qquad$ R: $y \leq 5$

(iii) $f(x)$ & $f^{-1}(x)$ will intersect on $y = x$ if they do intersect.

$\therefore \sqrt{5-x} - 1 = x$

$\sqrt{5-x} = x + 1$

$5 - x = x^2 + 2x + 1$

$0 = x^2 + 3x - 4$

$0 = (x+4)(x-1)$

$\therefore x = -4$ & $x = 1$

but only if $-1 \leq x \leq 5$ (See Domains)

$\therefore$ when $x = 1, y = 1$ $\quad (1, 1)$

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Question 8

a) If $a = \left(\frac{2}{5}\right)^4$, $b = \left(-\frac{1}{3}\right)^3$, $c = \left(\frac{3}{5}\right)^2$

then $\dfrac{a}{b^3 c^2} = \dfrac{\left(\frac{2}{5}\right)^4}{\left(\left(-\frac{1}{3}\right)^3\right)^3 \left(\left(\frac{3}{5}\right)^2\right)^2}$

$= \dfrac{\left(\frac{2}{5}\right)^4}{\left(-\frac{1}{3}\right)^9 \left(\frac{3}{5}\right)^4}$

1 for correct substitution in index form and some correct manipulation in index form

$= \dfrac{\frac{2^4}{5^4}}{\frac{(-1)^9}{3^9} \times \frac{3^4}{5^4}} \begin{matrix} \div 5^4 \\ \div 5^4 \end{matrix}$

$= \dfrac{2^4}{\frac{-1}{3^9} \times 3^4}$

$= \dfrac{2^4}{-1 \times 3^{-5}}$

1 for correct answer in index form

$= -2^4 \times 3^5$

must be in index form

$(= -3888)$

Common error

NOT $\dfrac{a}{bc}$

$= \dfrac{\left(\frac{2}{5}\right)^4}{\left(-\frac{1}{3}\right)^3 \left(\frac{3}{5}\right)^2}$

$= \dfrac{2^4}{5^4} \div \left(\dfrac{(-1)^3}{3^3} \times \dfrac{3^2}{5^2}\right)$

$= \dfrac{2^4}{5^4} \times \dfrac{3^{\cancel{3}}}{-1} \times \dfrac{5^2}{\cancel{3^2}}$

$= \dfrac{2^4 \times 3 \times 5^{\cancel{2}}}{-5^{\cancel{4}2}}$

$= -\dfrac{2^4 \times 3}{5^2}$

$= \dfrac{48}{-25}$

1 mark only

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b) Prove that $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$

LHS $= \sec\theta + \tan\theta$ ← Start here, rewriting LHS of Q or RHS → or RHS $= \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ ← NOT here →

1 for interpretting $\sec\theta$ and $\tan\theta$

$= \dfrac{1+\sin\theta}{\cos\theta}$

1 for expressing as a single fraction

LHS = RHS

RHS $= \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ 1 for separating fraction

$= \sec\theta + \tan\theta$ 1

RHS = LHS

8 c) i) $f(x) = \sqrt{5-x} - 1$

D: $5 - x \geq 0$
$-x \geq -5$
$x \leq 5$ or $(-\infty, 5]$ 1

R: $y \geq -1$ or $[-1, \infty)$ 1

ii) $f: \quad y = \sqrt{5-x} - 1$

$f^{-1}: \quad x = \sqrt{5-y} - 1$

$x+1 = \sqrt{5-y}$

NOT $x^2 = 5-y-1$ or $5-y+1$ ✗

$(x+1)^2 = 5-y$

$y = 5-(x+1)^2$

2 marks awarded for $y = 5-(x+1)^2$

$= 5 - (x^2+2x+1)$

$y = -x^2 - 2x + 4$

Very few students correctly stated

$f^{-1}(x) = 5-(x+1)^2$

D: $x \geq -1$
R: $y \leq 5$

$f^{-1}(x) = 5-(x+1)^2$
$= -x^2 - 2x + 4$

iii) A function and its inverse intersect on $y = x$ so solve

$\sqrt{5-x} - 1 = x$

$\sqrt{5-x} = x+1$

$5 - x = (x+1)^2$

$5 - x = x^2 + 2x + 1$

$0 = x^2 + 3x - 4$

$0 = (x+4)(x-1)$

$x = -4, 1$

1 for finding $x = -4, 1$

but D for $f^{-1}(x)$ is $x \geq -1$
$\therefore x \neq -4$

The only point of intersection is $(1, 1)$

1 for giving reasons to discard $x = -4$.

or

$5 - (x+1)^2 = x$

$5 - (x^2 + 2x + 1) = x$

$5 - x^2 - 2x - 1 = x$

$-x^2 - 3x + 4 = 0$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4, 1$

but D for $f^{-1}(x)$ is $x \geq -1$
$\therefore x \neq -4$

The only point of intersection is $(1, 1)$

Solving $f(x) = f^{-1}(x)$

$\sqrt{5-x} - 1 = 5 - (x+1)^2$ is inefficient and requires factorising involved polynomials.

Only one student was successful by this method.

Question 9

(a) $5\cos^2 x + 2\sin x = 2 \qquad 0° \le x \le 180°$

$5(1-\sin^2 x) + 2\sin x - 2 = 0$ first mark

$5 - 5\sin^2 x + 2\sin x - 2 = 0$

$5\sin^2 x - 2\sin x - 3 = 0$

$(5\sin x + 3)(\sin\theta - 1) = 0$ 2nd mark

$\therefore \sin x = -\frac{3}{5}$ or $\sin\theta = 1$

No solutions in domain $\qquad \theta = 90°$

$\underline{\underline{\theta = 90°}}$ 3rd mark

Generally not too bad
Some issues with factorising correctly (i.e. the signs)
A lot gave more solutions than the correct because they didn't take into account the domain $0° \le x \le 180°$

(b) $x = 2p + 3 \Rightarrow 2p = x - 3$

$p = \frac{x-3}{2}$

$y = p^2$

$y = \left(\frac{x-3}{2}\right)^2$

$y = \frac{1}{4}(x-3)^2$ which is a parabola with vertex $(3,0)$ with vertical dilation of $1/4$.

Most got the equation correct.
A lot didn't describe it as a parabola with at least one important feature

(c) $x^2 + y^2 - 6x + 2y - 26 = 0$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 26 + 10$

$(x-3)^2 + (y+1)^2 = 36$

first mark
Most were able to get to here other than a few arithmetic errors.

same

using $\cos^2\theta + \sin^2\theta = 1$
and

$\left(\frac{x-3}{6}\right)^2 + \left(\frac{y+1}{6}\right)^2 = 1$

* $\therefore \frac{x-3}{6} = \cos\theta \quad \& \quad \frac{y+1}{6} = \sin\theta$ second mark

$x - 3 = 6\cos\theta \qquad y + 1 = 6\sin\theta$

$\underline{x = 6\cos\theta + 3}, \qquad \underline{y = 6\sin\theta - 1}$

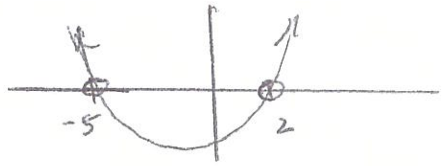
These or These should be final answer

$\underline{\underline{\text{OR}}}$

$\underline{x = 6\sin\theta + 3} \quad \underline{y = 6\cos\theta - 1}$ if you set up * other way for $\sin\theta$ & $\cos\theta$

(d) $\dfrac{7}{(3-x)(x+3)} > -1$ $\underline{\underline{\text{and}}}$ $x^2 + 3x - 10 < 0$

$(x+5)(x-2) < 0$



$\{x : -5 < x < 2\}$ — $\underline{1 \text{ mark}}$

This one was generally well done

$7(3-x)(x+3) > -(3-x)^2(x+3)^2$

$7(3-x)(x+3) + (3-x)^2(x+3)^2 > 0$ ← Do not expand! ← factorise!

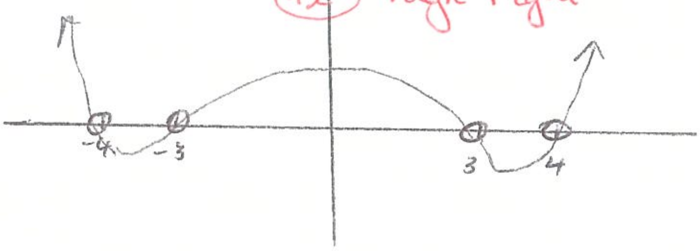
$(3-x)(x+3)\left[7 + (3-x)(x+3)\right] > 0$

$(3-x)(x+3)\left[7 + 9 - x^2\right] > 0$

$(3-x)(x+3)(16 - x^2) > 0$

$(3-x)(x+3)(4-x)(4+x) > 0$ — $\underline{1 \text{ mark}}$

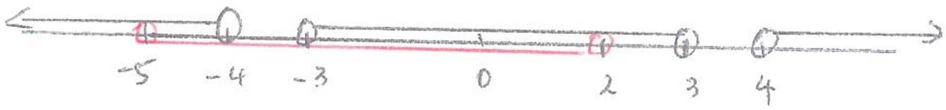
$+x^4$ high right



$\{x : x < -4, -3 < x < 3, x > 4\}$

This was a mixed bag. Some poor attempts, some excellent. common errors where mixing up the signs of the factors, leaving out the + underlined above. Graphing incorrectly (ie. upside down)

Now look for intersection (and → ∩) of both solutions.



$\{x : -5 < x < -4, -3 < x < 2\}$

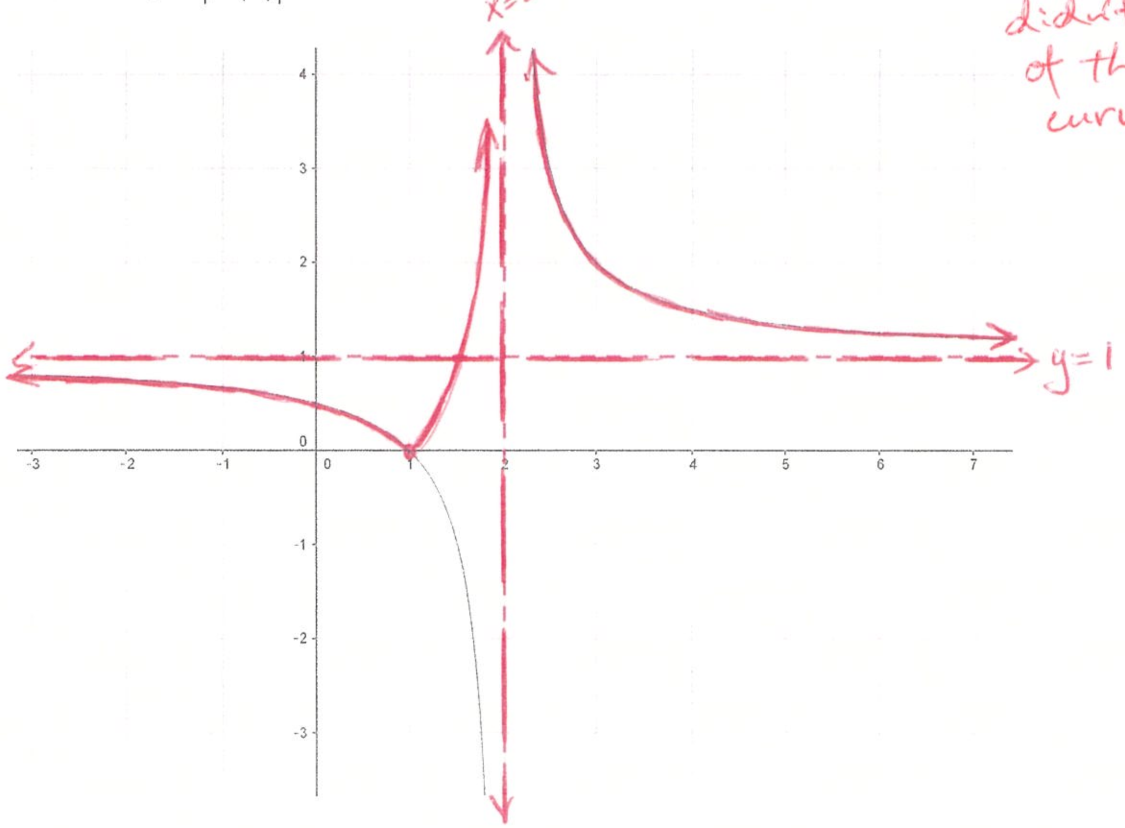
OR $(-5, -4) \cup (-3, 2)$.

$\underline{1 \text{ mark}}$. Correct from previous error was taken into account

Question 7(a) **Place this sheet inside the Question 7 Answer Booklet**

mostly well done. Some students didn't draw on top of the parts of the curve included.

(i) $y = |f(x)|$

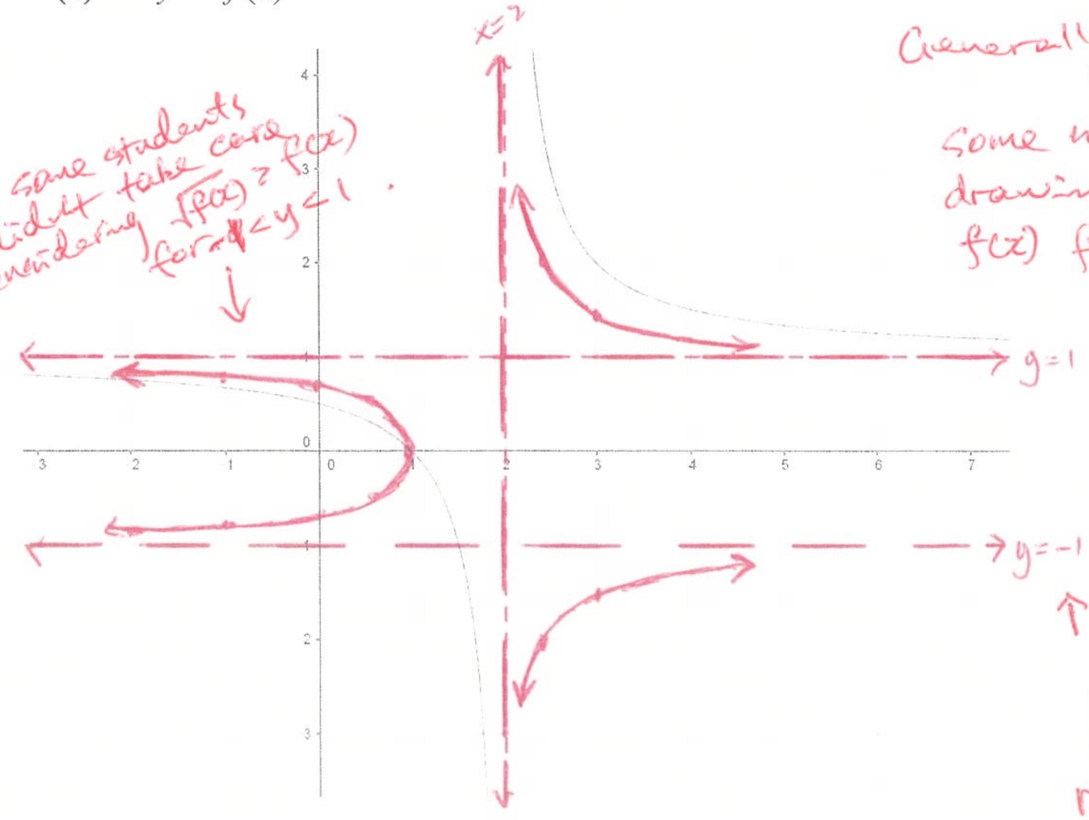


(ii) $y^2 = f(x)$

Generally well done.

Some mistakes drawing $\sqrt{f(x)}$ above $f(x)$ for $y>1$.

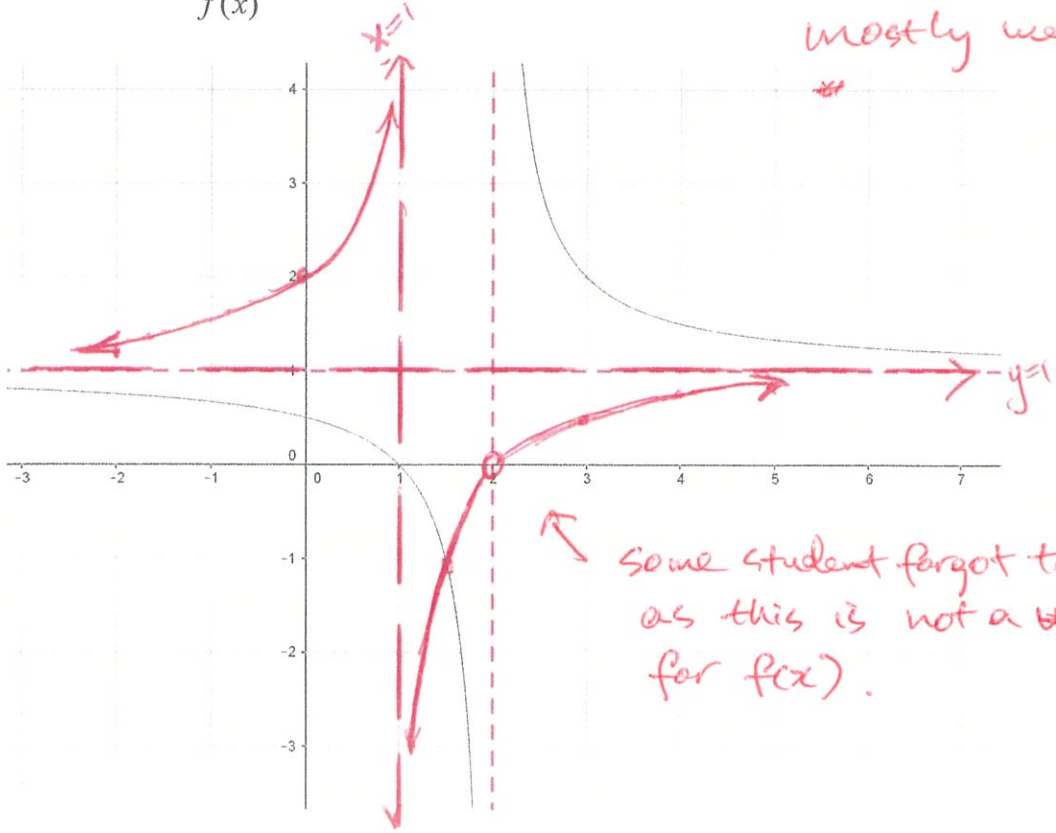
Some students didn't take care considering $\sqrt{f(x)} > f(x)$ for $0<y<1$.



Some students didn't draw $y=-\sqrt{f(x)}$ by reflecting the curves and asymptotes

Question 7(a)

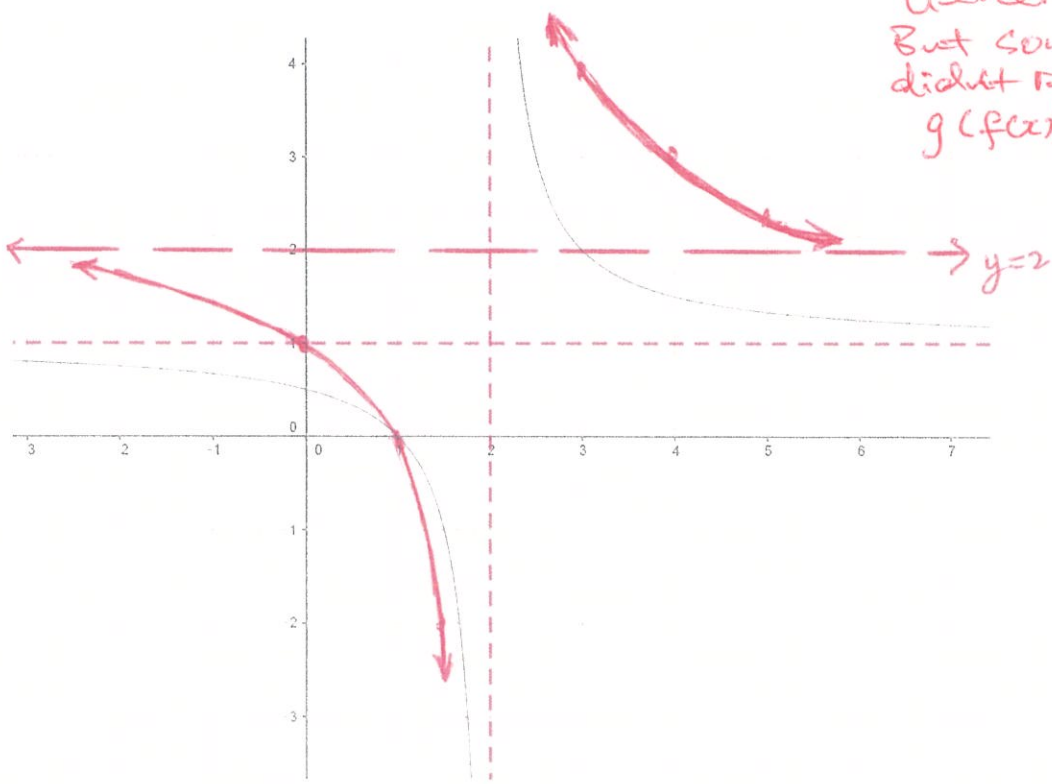
(iii) $y = \dfrac{1}{f(x)}$



mostly well done.

some student forgot to exclude $x=2$ as this is not a ~~value~~ valid value for $f(x)$.

(iv) If $g(x) = 2x$, sketch $y = g[f(x)]$



Generally well done. But some students didn't recognise $g(f(x)) = 2f(x)$.

many student forgot to move the asymptote up ~~to~~ to $y=2$.