Student Name:______________________________

Class Teacher:______________________________

Hurlstone Agricultural High School

2021 **HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 1**

Mathematics Extension 1

General Instructions

- Reading time – 3 minutes
- Working time – 40 minutes
- Write using black or blue pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For Questions in Section 2, show relevant mathematical reasoning and/or calculations

Total marks: 33

Section 1 – 3 marks

- Attempt Questions 1-3
- Allow about 5 minutes for this section

Section 2 – 30 marks

- Attempt Questions 4-6
- Allow about 35 minutes for this section

Written by 2021 HAHS Maths Faculty, rewritten by Ariq Abdullah

Section 1

Attempt Questions 1-3

Allow about 5 minutes from this section.

Question 1

The exact value of $\cos\left(\frac{\pi}{12}\right)$ is equal to the which of the following expressions?

a. $\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

b. $\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

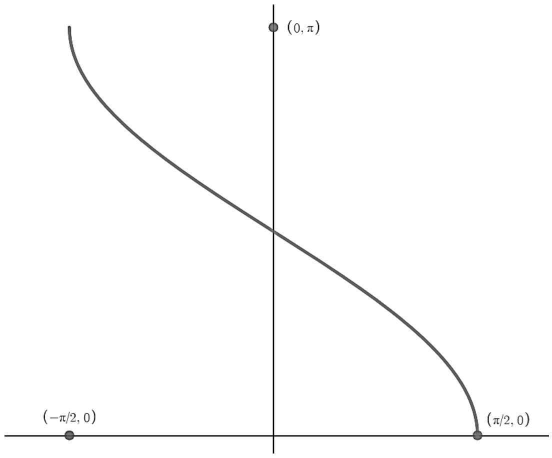
c. $\cos\frac{\pi}{4}\sin\frac{\pi}{6}+\sin\frac{\pi}{4}\cos\frac{\pi}{6}$

d. $\cos\frac{\pi}{4}\sin\frac{\pi}{6}-\sin\frac{\pi}{4}\cos\frac{\pi}{6}$

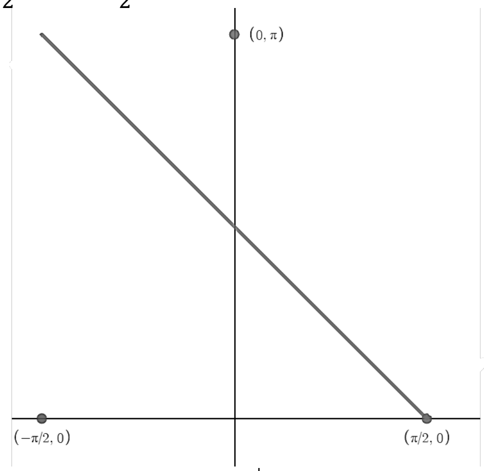
Question 2

Which graph best represents $y=\cos^{-1}(-sinx)$, for $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$?

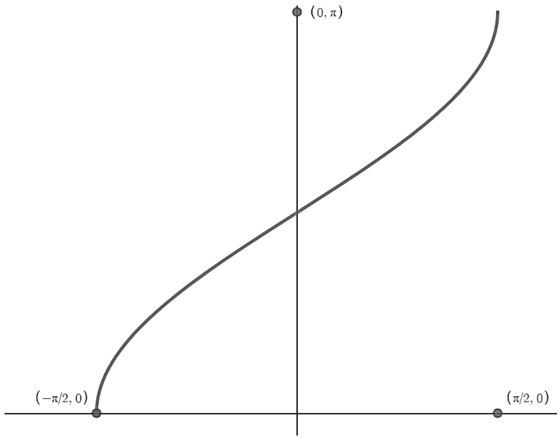
a.



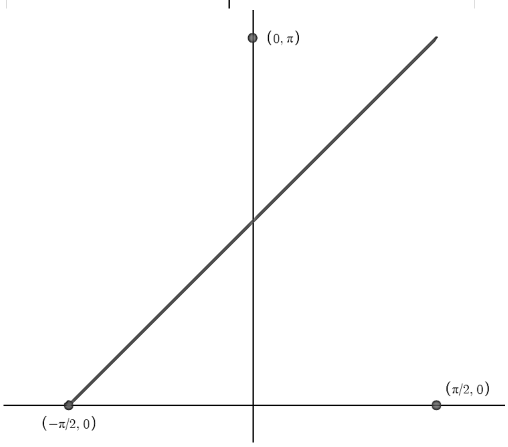
c.



b.



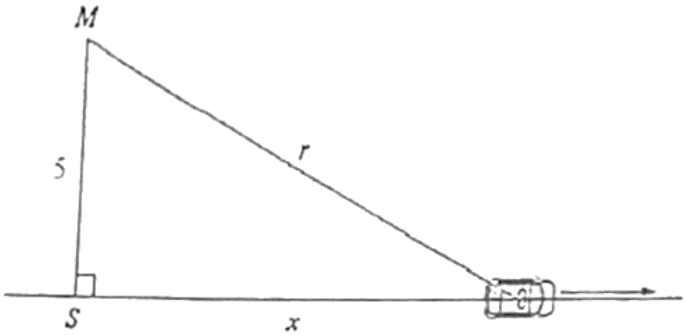
d.



Question 3

A radio transmitter M is situated 5 km from a straight road. The closest point on the road to the transmitter is S.

The distance from the car to S is x km and from the other car M is r km.



Which of the following expressions is equal to $\frac{dr}{dt}$, given that $\frac{dx}{dt} = 50$ km/h?

a. $\dfrac{x}{\sqrt{x^2+25}}$

b. $\dfrac{100x}{\sqrt{x^2+25}}$

c. $\dfrac{50x}{\sqrt{x^2+25}}$

d. $\dfrac{25x}{\sqrt{x^2+25}}$

Section 2

30 marks
Attempt Questions 4-6
Allow about 35 minutes for this section

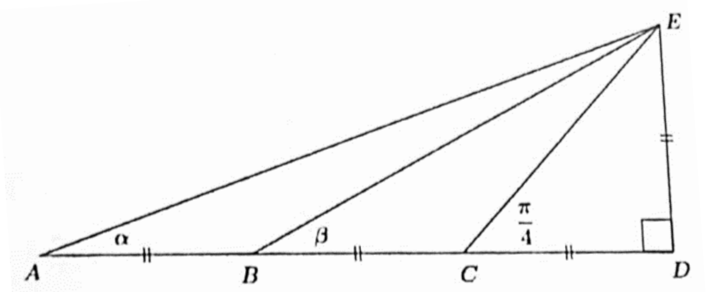
Question 4 (10 Marks) **Marks**

(a) Prove $\cos A - \sin 2A \sin A = \cos A \cos 2A$ **2**

(b) Rewrite the product $\sin 7x \cos 3x$ as a sum or difference of trigonometric functions. **2**

(c) Using the substitution $t = \tan\frac{\theta}{2}$, prove that $\sin\theta + \sin\theta\cot^2\theta = cosec\theta$ for $\theta \neq n\pi, n \in \mathbb{Z}$ **3**

(d) In the given figure below, $AB = BC = CD = DE$. Prove that $a + \beta = \frac{\pi}{4}$. **3**



NOT TO SCALE

Question 5 (10 marks) **Marls**

(a) A particle is moving along the x-axis. Its velocity v at time t is given by $v = \sqrt{20t - 2t^2}$ metres per second. **3**
Find the acceleration of the particle when $t = 8$.
Express your answer as an exact value in its simplest form with a rational denominator.

(b) A cup of coffee with an initial temperature $90°C$ is placed in a room with a constant temperature, A, of $22°C$. **4**
The temperature, $T°C$, of the coffee after t minutes is given by $T = A + Be^{-kt}$, where A, B and k are positive constants
The temperature of the coffee drops to $60°C$ after 10 minutes.
How long does it take for the temperature of the coffee to drop to $30°C$?
Give your answer to the nearest minute.

(c) A spherical balloon is being inflated at a constant rate of $200\ cm^3s^{-1}$.

(i) Show that $\frac{dr}{dt} = \frac{1}{4\pi r^2}$, where v is the volume and r is the radius of the sphere. **1**

(ii) At what rate of the radius of the balloon increasing when the radius is 10 cm? **2**

Question 6 (10 marks) Marks

(a) Let $f(x) = \frac{1}{2}\cos^{-1}(x)$ 2

(i) Sketch the graph of $y = f(x)$, indicating clearly the coordinates of the endpoints of the graph

(ii) State the range of $f(x)$. 1

(b) Prove that $f(x) = \tan(\cos^{-1} x)$ is an odd function. 2

(c) Write $\sin\left(2\cos^{-1}\left(\frac{1}{5}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational. 2

(d) Find the exact values of x and y which satisfy the simultaneous equations. 3

$$sin^{-1}x + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3}$$
$$3\sin^{-1} x - \frac{1}{2}\cos^{-1} y = \frac{2\pi}{3}$$

End of Exam

ME11-3 applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

<table>
<tr><th>Part</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)</td><td>$$\begin{aligned}\text{LHS} &= \cos A - \sin 2A \sin A\\ &= \cos A - (2\cos A \sin A)\sin A \quad \begin{bmatrix}\text{Using the double-angle formulae:}\\ \sin 2A = 2\sin A\cos A\end{bmatrix}\\ &= \cos A - 2\cos A\sin^2 A\\ &= \cos A(1-2\sin^2 A)\\ &= \cos A\cos 2A \quad \begin{bmatrix}\text{Using the double-angle formulae:}\\ \cos 2A = 1-2\sin^2 A\end{bmatrix}\\ &= \text{RHS}\end{aligned}$$
$$\therefore \cos A - \sin 2A\sin A = \cos A\cos 2A$$</td><td>2 marks
Correct solution

1 mark
Substantially correct</td></tr>
<tr><td>(b)</td><td>$$\begin{aligned}\sin 7x\cos 3x &= \frac{1}{2}\left(\sin(7x+3x)+\sin(7x-3x)\right)\\ &= \frac{1}{2}(\sin 10x + \sin 4x)\end{aligned}$$</td><td>2 marks
Correct solution

1 mark
Substantially correct</td></tr>
<tr><td>(c)</td><td>Let $t=\tan\frac{\theta}{2}$, then $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\cot\theta = \frac{1-t^2}{2t}$

Now, $$\begin{aligned}\text{LHS} &= \sin\theta + \sin\theta\cot^2\theta\\ &= \frac{2t}{1+t^2}+\frac{2t}{1+t^2}\left(\frac{1-t^2}{2t}\right)^2\\ &= \frac{2t}{1+t^2}+\frac{2t}{1+t^2}\left(\frac{(1-t^2)^2}{4t^2}\right)\\ &= \frac{2t}{1+t^2}+\frac{(1-t^2)^2}{2t(1+t^2)}\\ &= \frac{4t^2+(1-t^2)^2}{2t(1+t^2)}\\ &= \frac{4t^2+t^4-2t^2+1}{2t(1+t^2)}\end{aligned}$$</td><td>3 marks
Correct solution

2 marks
Substantially correct

1 mark
Correctly writes the t-formula for cotangent</td></tr>
</table>

	$$=\frac{t^4+2t^2+1}{2t\left(1+t^2\right)}$$ $$=\frac{\left(1+t^2\right)^2}{2t\left(1+t^2\right)}$$ $$=\frac{1+t^2}{2t}$$ $$=\frac{1}{\sin\theta}$$ $$=\operatorname{cosec}\theta$$ $$=\text{RHS}$$ $$\therefore \sin\theta+\sin\theta\cot^2\theta=\operatorname{cosec}\theta \quad \text{for} \quad \theta\neq n\pi,\ n\in\mathbb{Z}$$	
(d)	$ED=DC=CB=BA$ (given) let $ED=x$ $\tan\frac{\pi}{4}=1$ $\tan\beta=\frac{x}{2x}=\frac{1}{2}$ $\tan\alpha=\frac{x}{3x}=\frac{1}{3}$ $$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$ $$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$$ $$=1$$ $$=\tan\frac{\pi}{4}$$ $$\alpha+\beta=\frac{\pi}{4}$$	**3 marks** Correct solution **2 marks** Substantially correct **1 mark** Establishes correct expression for α and β.

Question No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

ME11-4 - applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change.

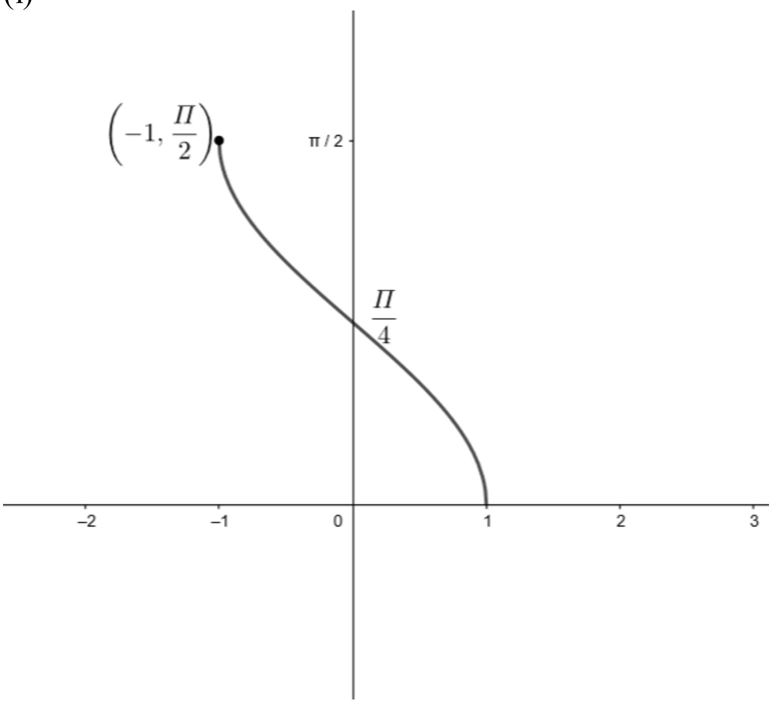
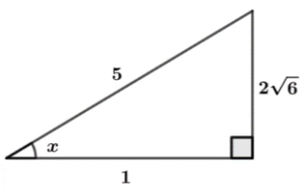
	Solutions	Marking Guidelines
(a)	$v=\sqrt{20t-2t^2}=\left(20t-2t^2\right)^{\frac{1}{2}}$ $a=\frac{dv}{dt}=\frac{1}{2}\left(20t-2t^2\right)^{\frac{-1}{2}}\times\left(20-4t\right)$ $=\frac{2\left(5-t\right)}{\sqrt{20t-2t^2}}$ $t=8:\ a=\frac{2\left(5-8\right)}{\sqrt{20\times8-2\left(8^2\right)}}=\to\frac{-3\sqrt{2}}{4}$	**(a)3 marks:** Correct solution, showing differentiation, substitution and simplification. **2 marks:** Considerable progress. **1 mark:** Some relevant progress.
(b)	$T=A+Be^{-kt}$ $\quad A=22;\ t=0\to T=90$ $\therefore B=68$ $\quad t=10\to T=60$ $\therefore 60=22+68e^{-10k}$ $38=68e^{-10k}$ $e^{-10k}=\frac{19}{34}$ $\therefore k=\frac{-1}{10}\ln\frac{19}{34}$ When $T=30$: $30=22+68e^{-kt}$ $\to\frac{2}{17}=e^{-kt}$ $t=\frac{-1}{k}\ln\frac{2}{17}\approx 37\,\text{min}.$	**(b) 4 marks:** Complete solution illustrating *B, k,* relevant equations and solution. **3 marks:** Almost every factor of solution complete. **2 marks:** More than 1 aspect towards solution illustrated. **1 mark:** Some relevant progress. .***Note:*** *k* must be a positive value for it to fit the given equation involving *e^(-kt)*. If a negative sign has "gone missing" in your working out, you did not earn full marks despite giving the correct final answer.
(c)	**(i)** $V=\frac{4}{3}\pi r^2$ $\frac{dV}{dr}=4\pi r^2 \quad\to\quad \frac{dr}{dV}=\frac{1}{4\pi r^2}$ **(ii)** $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{1}{4\pi r^2}\times200$ $=\frac{1}{4\pi(10)^2}\times200$ $=\frac{1}{2\pi}\text{cms}^{-1}\quad(\approx0\cdot1591549)$	**(c) (i) 1 mark:** Correct solution justified. **(ii) 2 marks:** Correct answer with working. **1 mark:** Correct expression and substitution into chain rule, or equivalent.

Year 12 Mathematics Extension 1 Assessment Task 1 2021

Question No. 6 Solutions and Marking Guidelines

Outcomes Addressed in this Question

ME11-3 - applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

Solutions	**Marking Guidelines**
(a) (i) 	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
(ii) For, $f\left(x\right)=\frac{1}{2}\cos^{-1}\left(x\right)$ Range is $0\le y\le\frac{\pi}{2}$	**Award 1** mark for the correct answer.
(b) $f\left(-x\right)=\tan\left(\cos^{-1}\left(-x\right)\right)$ $=\tan\left(\pi-\cos^{-1}x\right)$ $=-\tan\left(\cos^{-1}x\right)$ $=-f\left(x\right)$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
(c) Let $x=\cos^{-1}\left(\frac{1}{5}\right)$ $\therefore\cos x=\frac{1}{5}$ $\sin x=\frac{2\sqrt{6}}{5}$ 	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution

$\sin\left(2\cos^{-1}\left(\frac{1}{5}\right)\right)=\sin 2x$ $=2\sin x\cos x$ $=2\times\frac{2\sqrt{6}}{5}\times\frac{1}{5}=\frac{4}{25}\sqrt{6}$ **(d)** $\sin^{-1}x+\frac{1}{2}\cos^{-1}y=\frac{\pi}{3}$ ....(1) $3\sin^{-1}x-\frac{1}{2}\cos^{-1}y=\frac{2\pi}{3}$ ....(2) *Add* $(1)+(2)$ $4\sin^{-1}x=\pi$ $\sin^{-1}x=\frac{\pi}{4}$ $\therefore x=\frac{1}{\sqrt{2}}$ Substitute $x=\frac{1}{\sqrt{2}}$ into (1) $\sin^{-1}\frac{1}{\sqrt{2}}+\frac{1}{2}\cos^{-1}y=\frac{\pi}{3}$ $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}y=\frac{\pi}{3}$ $\frac{1}{2}\cos^{-1}y=\frac{\pi}{3}-\frac{\pi}{4}$ $\frac{1}{2}\cos^{-1}y=\frac{\pi}{12}$ $\cos^{-1}y=\frac{\pi}{6}$ $\therefore y=\frac{\sqrt{3}}{2}$	**Award 3** marks for the correct solution. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

Multiple Choice Answers	
1.	A
2.	D
3.	C