Student Name:________________________________
Class Teacher:________________________________

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2021 **Year 12**
HSC Task 2

Mathematics Extension 1

Examiners Mr G Rawson, Mrs S Faulds, Mr D Potaczala

General Instructions
- Reading time – 3 minutes
- Working time – 40 minutes
- Write using black or blue pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- An A4 hand written summary sheet may be used.
- In Questions 4-6, show relevant mathematical reasoning and/or calculations.

Total marks: 33

Section 1 – 3 marks
- Attempt Questions 1-3
- Allow about 4 minutes for this section

Section 2 – 30 marks
- Attempt Questions 4-6
- Allow about 36 minutes for this section

Written by 2021 HAHS Maths Faculty, rewritten by Ariq Abdullah

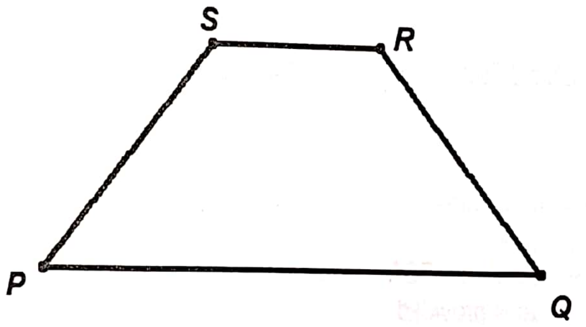
Section 1

Attempt Questions 1-3

Allow about 4 minutes from this section.

Question 1

PQRS is a trapezium where $\overrightarrow{PS} = \vec{p}, \overrightarrow{SR} = \vec{s}, \overrightarrow{PQ} = 2\overrightarrow{SR}$



Which of the following is equivalent to $\overrightarrow{QS}$?

a. $2\vec{s} + \vec{p}$

b. $2\vec{s} - \vec{p}$

c. $\vec{p} - 2\vec{s}$

d. $-\vec{p} - 2\vec{s}$

Question 2

If A and B are points defined by position vectors $\vec{a} = \vec{\imath} + \vec{\jmath}$ and $\vec{b} = 5\vec{\imath} - 2\vec{\jmath}$ respectively, then $|\overrightarrow{AB}|$ is equal to which of the following?

a. $\sqrt{2}$

b. 5

c. $\sqrt{29}$

d. 10

Question 3

What is the number of solutions for the equation $sin2x - sinx = 0$ over the domain $[0,2\pi]$?

a. 2

b. 3

c. 4

d. 5

Section 2

30 marks
Attempt Questions 4-6
Allow about 36 minutes for this section

Question 4 (10 Marks) **Marks**

(a) Use the principle of Mathematical Induction to show that for all integers $n \geq 1$, **3**

$$1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$$

(b)

i) Show that $(k+3)^3 = k^3 + 9k^2 + 27k + 27$ **1**

ii) Using the result in part (i), or otherwise, prove by the principle of Mathematical Induction that the sum of the cubes of three consecutive positive integers is divisible by 3. **3**

(c) Prove the following identity: **3**

$$\frac{sinA}{cosA + sinA} + \frac{sinA}{cosA - sinA} = tan2A$$

Question 5 (10 marks) **Marls**

(a) Suppose $|\vec{v}| = 3$ and $|\vec{w}| = 2$. **2**
If $\vec{v}$ is parallel to $\vec{w}$, what values might $\vec{v}.\vec{w}$ take?
Give reasons for your answers(s)

(b) Given the position vectors $\vec{u} = \binom{-4}{3}$ and $\vec{v} = \binom{2}{5}$ **3**
Find the vector projection of $\vec{u}$ onto $\vec{v}$.

(c) A small plane can fly at 350 km/h in still conditions. The plane needs to fly due north but has to deal with 70km/h winds from due east. Find the direction and speed that the plane needs to fly in order to travel due north (both correct to two decimal places). **2**

(d) Prove using vector methods that the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides. **3**

Question 6 (10 marks) Marks

(a) Use the compound angle results to prove the identity $sin3\theta - 3sin\theta - 4\sin^3\theta$ 2

(b)

(i) Express $cosx + \sqrt{2}sinx$ in the form $Rcos(x - \alpha)$. 2

Question 6 continued on next page

(ii) Hence, solve the equation $cosx + \sqrt{2}sinx = \sqrt{3}, 0 \leq x \leq 2\pi.$ 2

(c) Use the substitution $t = tanx$ to find all angles x, where $0 \leq x \leq 2\pi$, for which $sin2x + cos2x = 1$. 2

(d) Show that the exact value of $\cos\left(\frac{7\pi}{12}\right) = -\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$. 2

End of Exam

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Part / Outcome	Solutions	Marking Guidelines
	Multiple Choice (Q1-3) **1 – C** **2 – B** **3 – D**	
(a) ME12-1	<u>Show true for $n = 1$</u> $\text{LHS} = 1\times 1! = 1$ $\text{RHS} = (1+1)! - 1 = 1$ $\therefore$ true for $n = 1$ <u>Assume true for $n = k$</u> ie $1\times 1! + 2\times 2! + 3\times 3! + \ldots + k\times k! = (k+1)! - 1$ and <u>Prove true for $n = k + 1$</u> ie $1\times 1! + 2\times 2! + 3\times 3! + \ldots + k\times k! + (k+1)\times(k+1)! = (k+2)! - 1$ $\text{LHS} = \underbrace{1\times 1! + 2\times 2! + 3\times 3! + \ldots + k\times k!}_{assumption} + (k+1)\times(k+1)!$ $= (k+1)! - 1 + (k+1)\times(k+1)!$ $= (k+1)!\left[1+(k+1)\right] - 1$ ie $= (k+1)!(k+2) - 1$ $= (k+2)! - 1$ $=$ RHS $\therefore$ true by mathematical induction for all $n \geq 1$	**3 marks** – Correct solution **2 mark** – Substantially correct **1 mark** – Partial progress towards correct solution
(b) (i) ME12-1	$(k+3)^3 = (k+3)(k+3)^2$ $= (k+3)(k^2+6k+9)$ $= k^3+6k^2+9k+3k^2+18k+27$ $= k^3+9k^2+27k+27$	**1 mark** – Correct solution

	Question 6 continued…	
(b) (ii) ME12-1	Let the 3 consecutive numbers be n, $n+1$, $n+2$ So, prove $n^3+(n+1)^3+(n+2)^3$ is divisible by 3. <u>Show true for $n = 1$</u> $1^3+(1+1)^3+(1+2)^3=1+8+27$ $=36=3(12)$ ie, divisible by 3 <u>Assume true for $n = k$</u> ie $k^3+(k+1)^3+(k+2)^3=3M$, for integer M and $\therefore (k+1)^3+(k+2)^3=3M-k^3$ and <u>Prove true for $n = k + 1$</u> $(k+1)^3+(k+2)^3+(k+3)^3=3N$, for integer N $\text{LHS}=(k+1)^3+(k+2)^3+(k+3)^3$ $=3M-k^3+(k+3)^3$ *from assumption* $=3M-k^3+k^3+9k^2+27k+27$ *from (i)* $=3M+9k^2+27k+27$ $=3(M+3k^2+9k+9)$ $=3N$, where N is an integer $\therefore$ true for $n\ge 1$ by Mathematical Induction	**3 marks** – Correct solution **2 mark** – Substantially correct **1 mark** – Partial progress towards correct solution
(c) ME12-3	$\text{LHS}=\dfrac{\sin A}{\cos A+\sin A}+\dfrac{\sin A}{\cos A-\sin A}$ $=\dfrac{\sin A(\cos A-\sin A)+\sin A(\cos A+\sin A)}{\cos^2 A-\sin^2 A}$ $=\dfrac{\sin A\cos A-\sin^2 A+\sin A\cos A+\sin^2 A}{\cos^2 A-\sin^2 A}$ $=\dfrac{2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\dfrac{\sin 2A}{\cos 2A}$ $=\tan 2A$ $=\text{RHS}$	**3 marks** – Correct solution **2 mark** – Substantially correct **1 mark** – Partial progress towards correct solution

on 1 Task 2 2021

Question No. 5 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

Outcome	Solutions	Marking Guidelines

ME12-2

(a) If parallel the angle between vectors is 0° or 180°

$$\underset{\sim}{v}\bullet\underset{\sim}{u} = 3\times 2\cos 0 = 6 \quad \text{or} \quad \underset{\sim}{v}\bullet\underset{\sim}{u} = 3\times 2\cos 180 = -6$$

2 marks
Correct solution
1 mark
Substantial progress towards correct solution

ME12-2

(b)

$$\underset{\sim}{u} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}; \quad \underset{\sim}{v} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

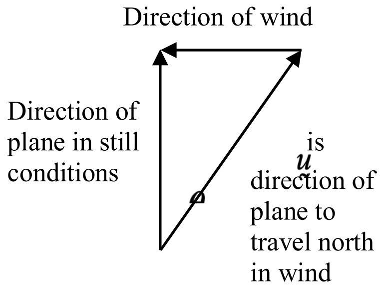
$$\left|\underset{\sim}{v}\right|^2 = 2^2 + 5^2 = 5 \qquad \underset{\sim}{v}\bullet\underset{\sim}{u} = (-4)\times 2 + 3\times 5 = 7$$

$$\frac{\underset{\sim}{u}\bullet\underset{\sim}{v}}{\left|\underset{\sim}{v}\right|^2}\bullet\underset{\sim}{v} = \frac{7}{29}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{14}{29} \\ \frac{35}{29} \end{pmatrix}$$

3 marks
Correct solution
2 marks
Single error
1 mark
Substantial progress towards correct solution

ME12-2

(c)



$$\left|\underset{\sim}{u}\right| = \sqrt{350^2 + 70^2} = 356.93 \text{ km/h}$$

$$\tan\theta = \frac{70}{350}$$
$$\theta = 11.31$$

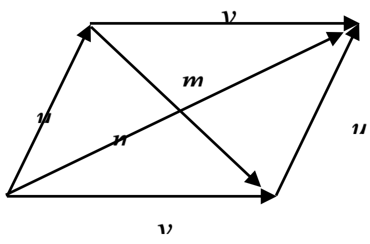
N 11.31°E or 011° as a true bearing

2 marks
Correct solution
1 mark
Substantial progress towards correct solution

Note: marks were still awarded if students interpreted the wind blowing in the easterly direction

ME12-2

(d)



To prove: $2\left|\underset{\sim}{v}\right|^2 + 2\left|\underset{\sim}{u}\right|^2 = \left|\underset{\sim}{m}\right|^2 + \left|\underset{\sim}{n}\right|^2$

$$\underset{\sim}{m} = \underset{\sim}{v} - \underset{\sim}{u} \text{ and } \underset{\sim}{n} = \underset{\sim}{v} + \underset{\sim}{u}$$

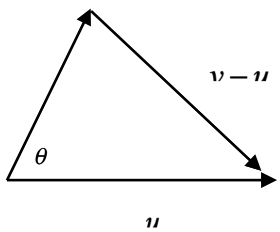
3 marks
Correct solution, including correct use of the definition $\left|\underset{\sim}{a}\right|^2 = \underset{\sim}{a}\bullet\underset{\sim}{a}$
2 marks
Substantial progress towards correct solution
1 mark
Establishes worded question in terms of algebraic vector notation

$$\begin{aligned} RHS &= |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 \\ &= |\underset{\sim}{v} - \underset{\sim}{u}|^2 + |\underset{\sim}{v} + \underset{\sim}{u}|^2 \\ &= (\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u}) + (\underset{\sim}{v} + \underset{\sim}{u}) \bullet (\underset{\sim}{v} + \underset{\sim}{u}) \\ &= |\underset{\sim}{v}|^2 - 2\underset{\sim}{v} \bullet \underset{\sim}{u} + |\underset{\sim}{u}|^2 + |\underset{\sim}{v}|^2 + 2\underset{\sim}{v} \bullet \underset{\sim}{u} + |\underset{\sim}{u}|^2 \\ &= 2|\underset{\sim}{v}|^2 + 2|\underset{\sim}{u}|^2 \\ &= LHS \end{aligned}$$

Note: a large portion of students misunderstood how to rewrite $|\underset{\sim}{v} - \underset{\sim}{u}|^2 + |\underset{\sim}{v} + \underset{\sim}{u}|^2$. Students used binomial expansion to expand the prior expression in all manners of wrong ways to get something similar to $|\underset{\sim}{v}|^2 - 2\underset{\sim}{v} \bullet \underset{\sim}{u} + |\underset{\sim}{u}|^2 + |\underset{\sim}{v}|^2 + 2\underset{\sim}{v} \bullet \underset{\sim}{u} + |\underset{\sim}{u}|^2$.

The pike symbols (| |) do not behave like brackets and thus binomial expansion does not apply. Nor does it really apply to $(\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u})$ (just try $(\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u})$).

What should have been used is the definition $|\underset{\sim}{a}|^2 = \underset{\sim}{a} \bullet \underset{\sim}{a}$. From there you get $|\underset{\sim}{v} - \underset{\sim}{u}|^2 = (\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u})$. To understand how the dot product can be "expanded" you can use the cosine rule.



It is clear that $|\underset{\sim}{v} - \underset{\sim}{u}|^2 = |\underset{\sim}{v}|^2 + |\underset{\sim}{u}|^2 - 2|\underset{\sim}{v}||\underset{\sim}{u}|\cos\theta$

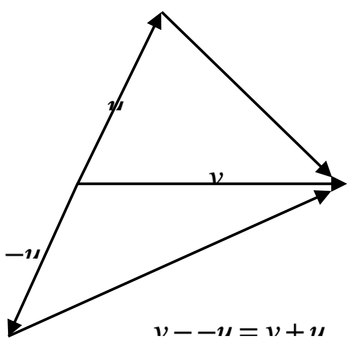
$= |\underset{\sim}{v}|^2 + |\underset{\sim}{u}|^2 - 2\underset{\sim}{v} \bullet \underset{\sim}{u}$ (dot product)

This means $(\underset{\sim}{v} - \underset{\sim}{u}) \bullet (\underset{\sim}{v} - \underset{\sim}{u}) = |\underset{\sim}{v}|^2 + |\underset{\sim}{u}|^2 - 2\underset{\sim}{v} \bullet \underset{\sim}{u}$

Similarly, $|\underset{\sim}{v} + \underset{\sim}{u}|^2 = |\underset{\sim}{v}|^2 + |-\underset{\sim}{u}|^2 - 2|\underset{\sim}{v}||-\underset{\sim}{u}|\cos(180 - \theta)$

$= |\underset{\sim}{v}|^2 + |\underset{\sim}{u}|^2 + 2\underset{\sim}{v} \bullet \underset{\sim}{u}$

$= (\underset{\sim}{v} + \underset{\sim}{u}) \bullet (\underset{\sim}{v} + \underset{\sim}{u})$



Outcomes Addressed in this Question

ME12.3 – applies advamced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations/

Outcome	Solutions	Marking Guidelines
ME12.3	**(a)** $\text{LHS} = \sin 3\theta$ $= \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= (2\sin\theta\cos\theta)\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$ $= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$ $= 3\sin\theta\cos^2\theta - \sin^3\theta$ $= 3\sin\theta(1-\sin^2\theta) - \sin^3\theta$ $= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= \text{RHS}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution showing knowledge of some of the appropriate trig, identities.
ME12.3	**(b) (i)** $R = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$ $\cos x + \sqrt{2}\sin x = \sqrt{3}\left(\dfrac{1}{\sqrt{3}}\cos x + \dfrac{\sqrt{2}}{\sqrt{3}}\sin x\right)$ $= \sqrt{3}(\cos(x-\alpha))$ where $\alpha = \tan^{-1}\sqrt{2}$ $= 0.955$ $\therefore \cos x + \sqrt{2}\sin x = \sqrt{3}(\cos(x - 0.955))$	**2 marks** Correct solution giving values for both R and α. **1 mark** Substantial progress towards correct solution. **Note:** Answer for α is more correctly stated in radians. Marks were also awarded for equivalent answer stated in degrees/
ME12.3	**(ii)** $\cos x + \sqrt{2}\sin x = \sqrt{3} \qquad 0 \le x \le 2\pi$ $\sqrt{3}(\cos(x-0.955)) = \sqrt{3}$ $\cos(x-0.955) = 1 \qquad -0.995 \le x - 0.995 \le 2\pi - 0.995$ $(x-0.955) = \cos^{-1} 1$ $= 0$ $\therefore x = 0.995 \qquad 0 \le x \le 2\pi$	**2 marks** Correct solution giving a single value for x in radians. **1 mark** Substantial progress towards correct solution.

ME12.3	**(c)** $\sin 2x + \cos 2x = 1$ Let $t = \tan\frac{2x}{2} = \tan x$ $\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1 \qquad 0° \le x \le 360°$ $2t + 1 - t^2 = 1 + t^2$ $= 2t^2 - 2t$ $= 2t(t-1)$ $t = 0,1$ $\text{ie.} \tan x = 0,1$ $x = 0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$	**2 marks** Correct solution giving all correct values for x in radians. **1 mark** Substantial progress towards correct solution.
ME12.3	**(d)** $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$ $= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ $= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$ $= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$ $= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{1-\sqrt{3}}{2\sqrt{2}}$ $= \frac{\sqrt{2}-\sqrt{6}}{4}$ $= -\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ as required	**2 marks** Correct solution showing all required steps. **1 marks** Substantial progress towards correct solution, showing most of the required steps including knowledge of relevant trig. identities and exact ratios.