Student Name: _____

Teacher: _____

2023

Hurlstone Agricultural High School
Year 11 Assessment Task 1

Mathematics Extension 1

Examiners

- **Mrs P Biczo**
- **Mr G Huxley**
- **Mr B Hoang**

General Instructions

- Reading time – 3 minutes
- Working time – 40 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use
- In Questions 7 to 8, show relevant mathematical reasoning and/or calculations

Total marks: 25

Section I – 6 marks (page 2–4)

- Attempt Questions 1–6
- Allow about 8 minutes for this section.

Section II – 19 marks (pages 5–8)

- Attempt Questions 7–9
- Allow about 32 minutes for this section.
- For each question use a separate answer booklet.

Section I

6 marks
Attempt Questions 1 to 6
Allow about 6 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 6

---

1. A curve is described by the parametric equations $x = \frac{t}{2}$ and $y = 3t^2$

   What is the cartesian equation of the curve?

   A. $y = \frac{3x^2}{4}$ **B.** $y = 12y^2$

   C. $y = 12x^2$ **D.** $y = \frac{2}{x}$

2. What are the values of x for which $(x + 3)(x - 1)(2 - x) > 0$?

   A. $x < -3$ or $1 < x < 2$ **B.** $x > 3$ or $1 < x < 2$

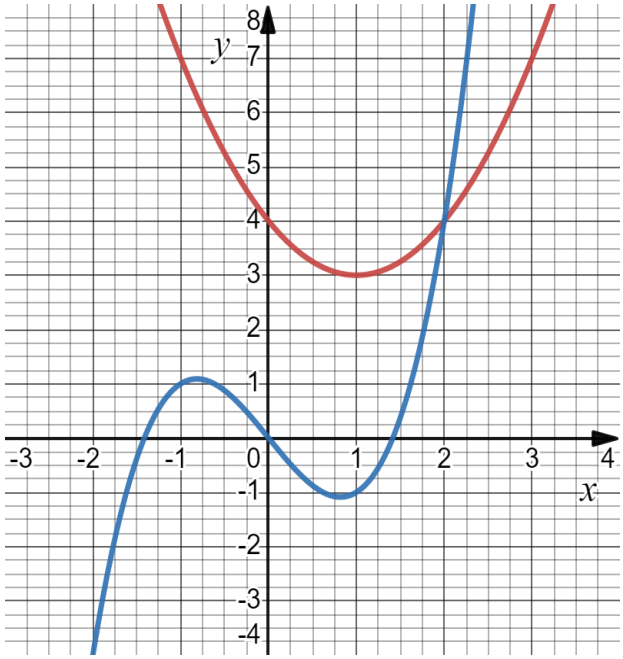
   C. $-3 < x < 1$ or $x > 2$ **D.** $-3 < x < 1$

3. When is the sum of two functions $f(x)$ and $g(x)$ always equal to zero?

   A. $f(x) = g(x)$ **B.** $\frac{1}{f(x)} = g(x)$

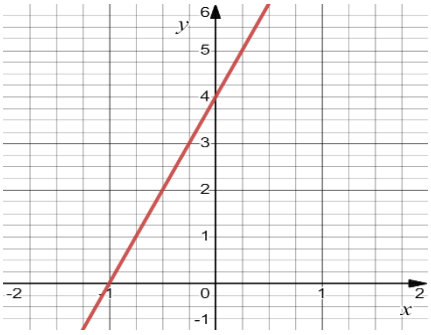
   C. $f(x) = g(-x)$ **D.** $f(x) = -g(x)$

4. The graphs of $y = x^2 - 2x + 4$ and $y = x^3 - 2x$ are shown.

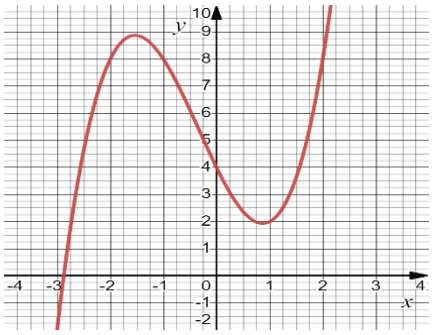


Which diagram represents the graph of $y = x^3 + x^2 - 4x + 4$?

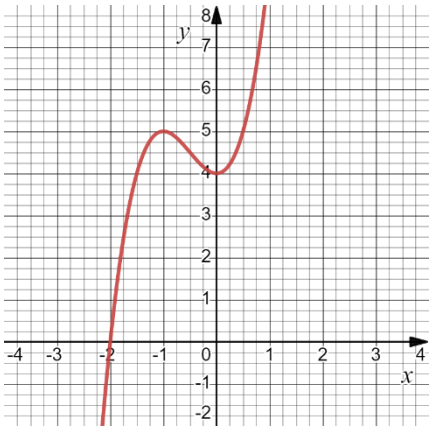
A.



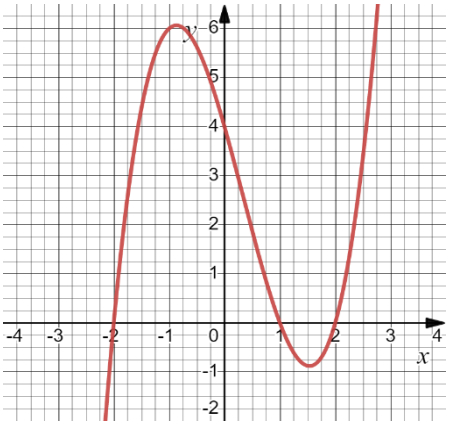
B.



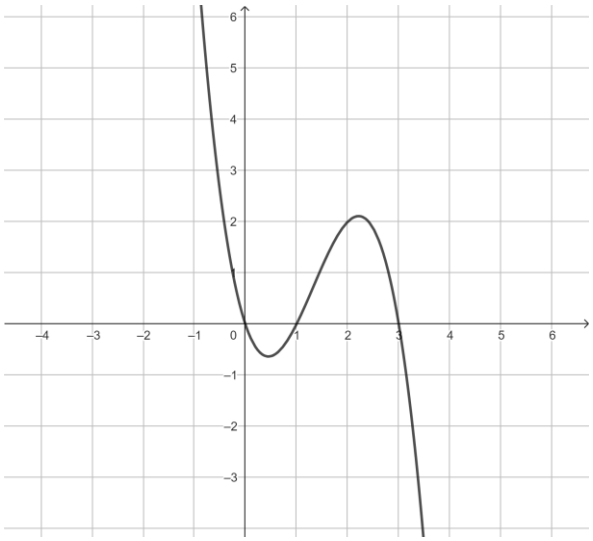
C.

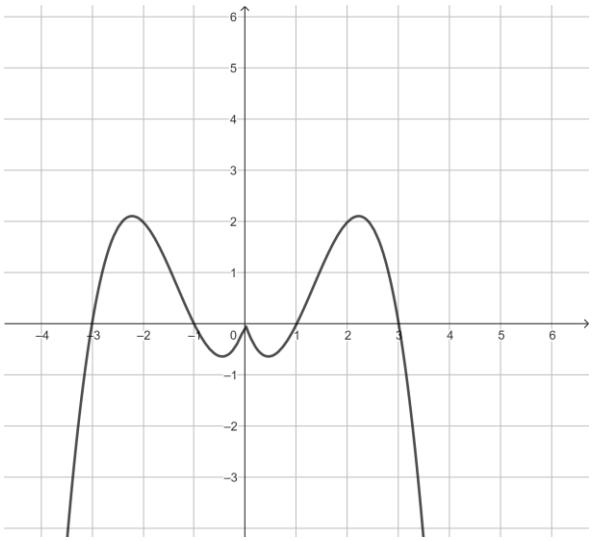


D.



5. The graph of $y = f(x)$ is shown below on the left.





The graph to the right of $y = f(x)$ can be described by which of the following graphs?

A. $y = |f(x)|$ **B.** $y = |f(|x|)|$

C. $|y| = f(x)$ **D.** $y = f(|x|)$

6. The solution for $|2x - 5| \geq 3$ is given by which of the following?

A. $x \geq 4$ **B.** $x \leq 1$ or $x \geq 4$

C. $1 \leq x \leq 4$ **D.** $x \leq 1$

~ END OF SECTION I ~

Section II

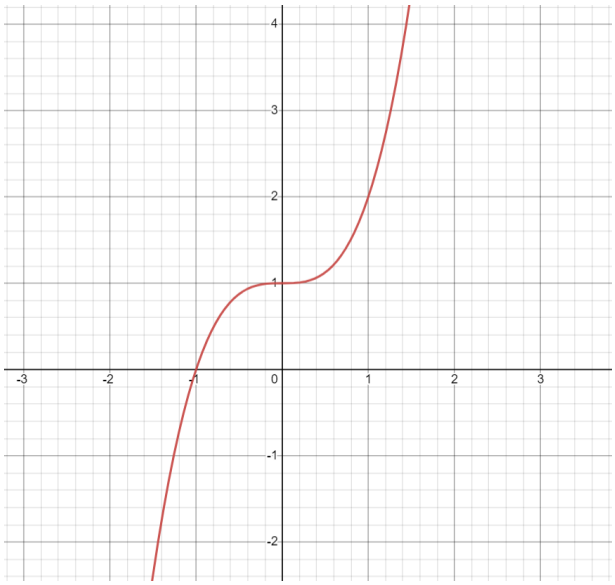
19 marks
Attempt Questions 7 to 9
Allow about 32 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 7 (7 marks) **Marks**

(a) The graph $y = f(x)$ is shown below. **2**



Sketch the graph of $y = \frac{1}{f(x)}$ on the graph provided in the Question 7 answer booklet.

Clearly show the behaviour of the curves including any relevant intercepts and asymptotes.

(b) For what values of x is $\frac{x}{x-1} \geq -2$? **3**

(c) Find the radius and centre of the circle given in parametric form. **2**

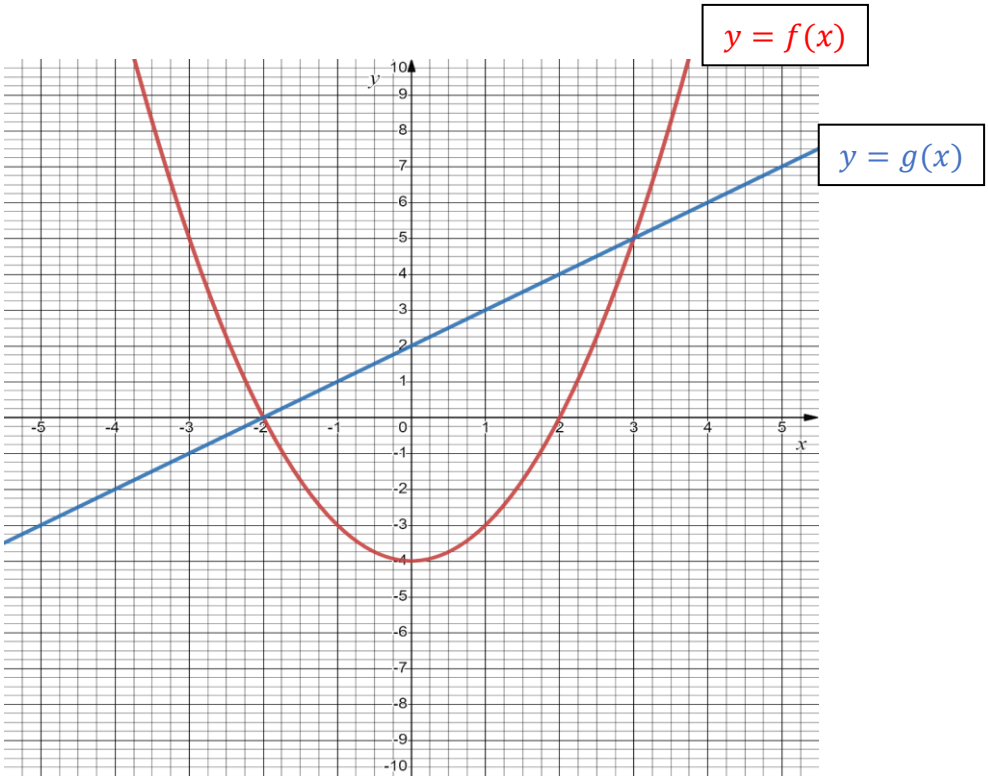
$$x = 1 + \cos\alpha$$

$$y = \sin\alpha - 2$$

~ End of question 7 ~

Question 8 (6 marks) **Marks**

The graphs of two functions are drawn on the co-ordinate number plane below.
The two functions are:
A quadratic function $y = f(x)$ and a linear function $y = g(x)$.



For parts (i) and (ii), you will need to make sketches on the copies of this number plane provided in your Question 8 answer booklet.
Both of your responses will need to be accurate according to the scale provided.

i. On the number plane provided in your Q8 answer booklet, draw a sketch of: **3**

$$y = f(x) - g(x)$$

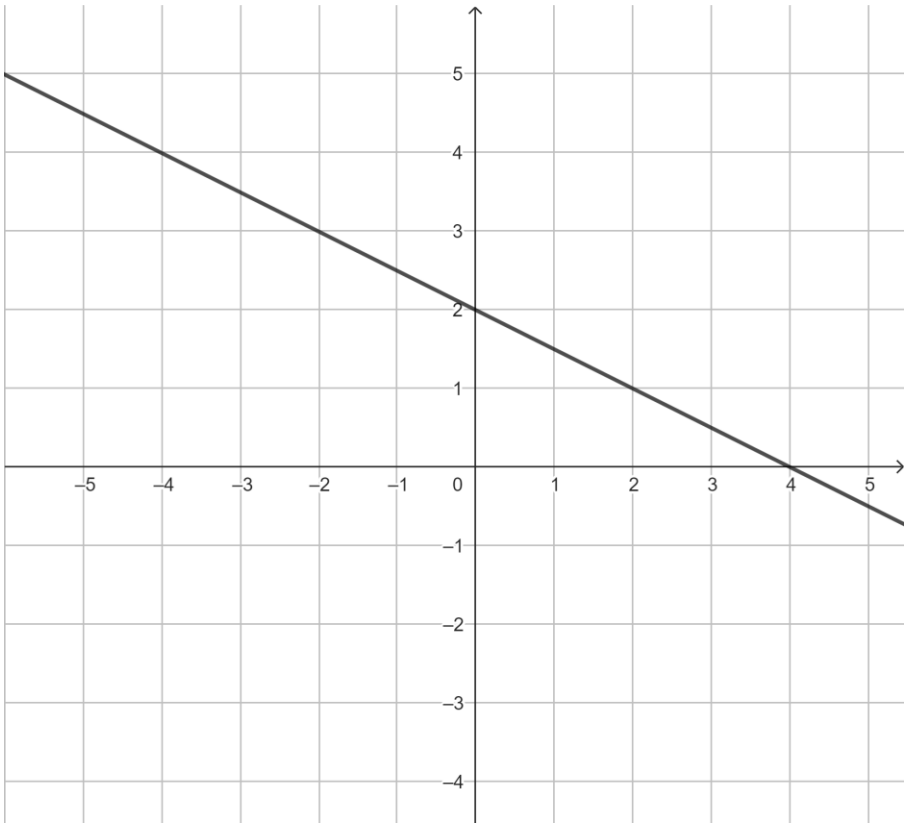
ii. On the number plane provided in your Q8 answer booklet, draw a sketch of: **3**

$$y = f(x)\, g(x)$$

~ End of question 8 ~

Question 9 (6 marks) **Marks**

(a) The graph of $y = f(x)$ is shown below. **2**

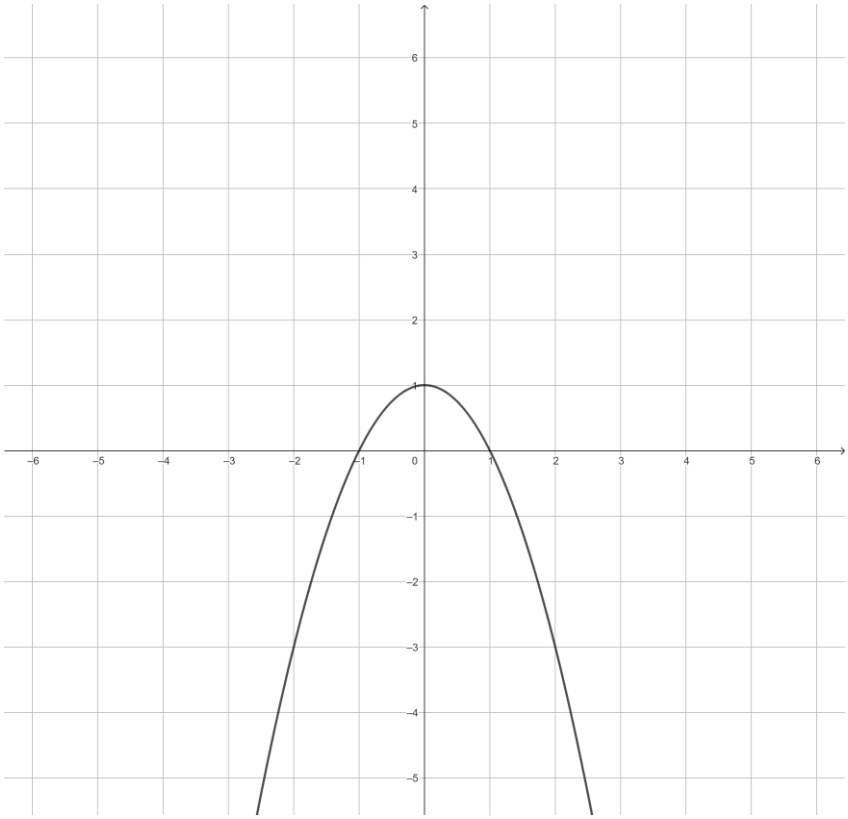


Sketch the graph of $y^2 = f(x)$ on the graph provided in your Q9 answer booklet.

Clearly show the behaviour of the curves including any relevant intercepts and asymptotes.

Part (c) continues over the next page.

(b) The graph $f(x) = 1 - x^2$ is shown below. **2**



Sketch the graph of $y = \frac{1}{\sqrt{1-x^2}}$ on the graph provided in your Q9 answer booklet.

Clearly show the behaviour of the curves including any relevant intercepts and asymptotes.

(c) It is given that $g(x) = \frac{|x-1|}{x}$. Sketch the graph of $y = g(x)$ in your Q9 **2**

answer booklet.

Clearly show the behaviour of the curves including any relevant intercepts and asymptotes.

~ End of question 9 ~

~ END OF SECTION II ~

~ END OF PAPER ~

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

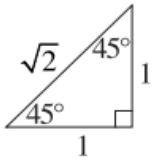
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

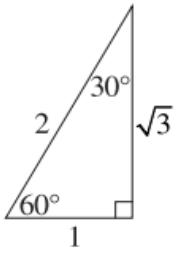
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

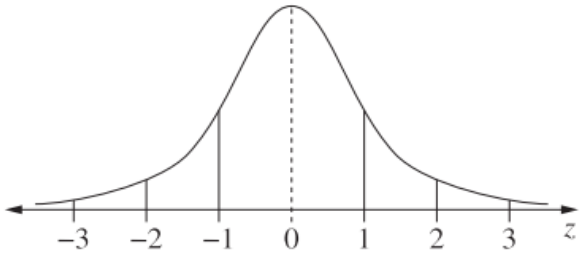
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

This Page Is Intentionally Left Blank

Student Name: _____

Teacher: _____

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Start Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

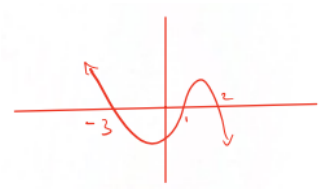
6. A ○ B ○ C ○ D ○

Year 11 Mathematics Extension 1

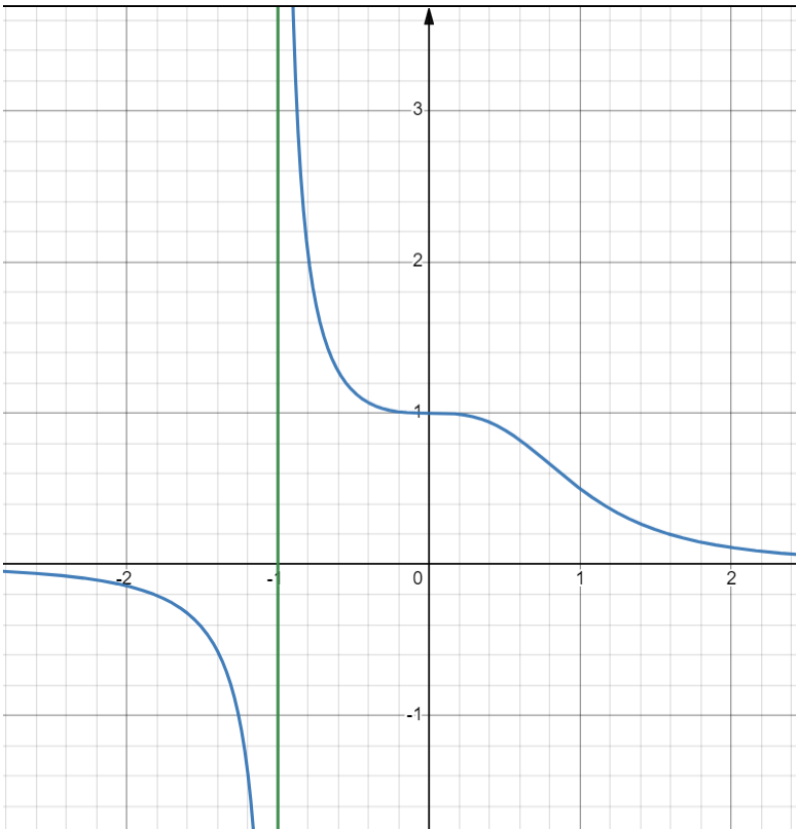
Multiple Choice Questions Solutions and Marking Guidelines

Outcomes Addressed in this Question

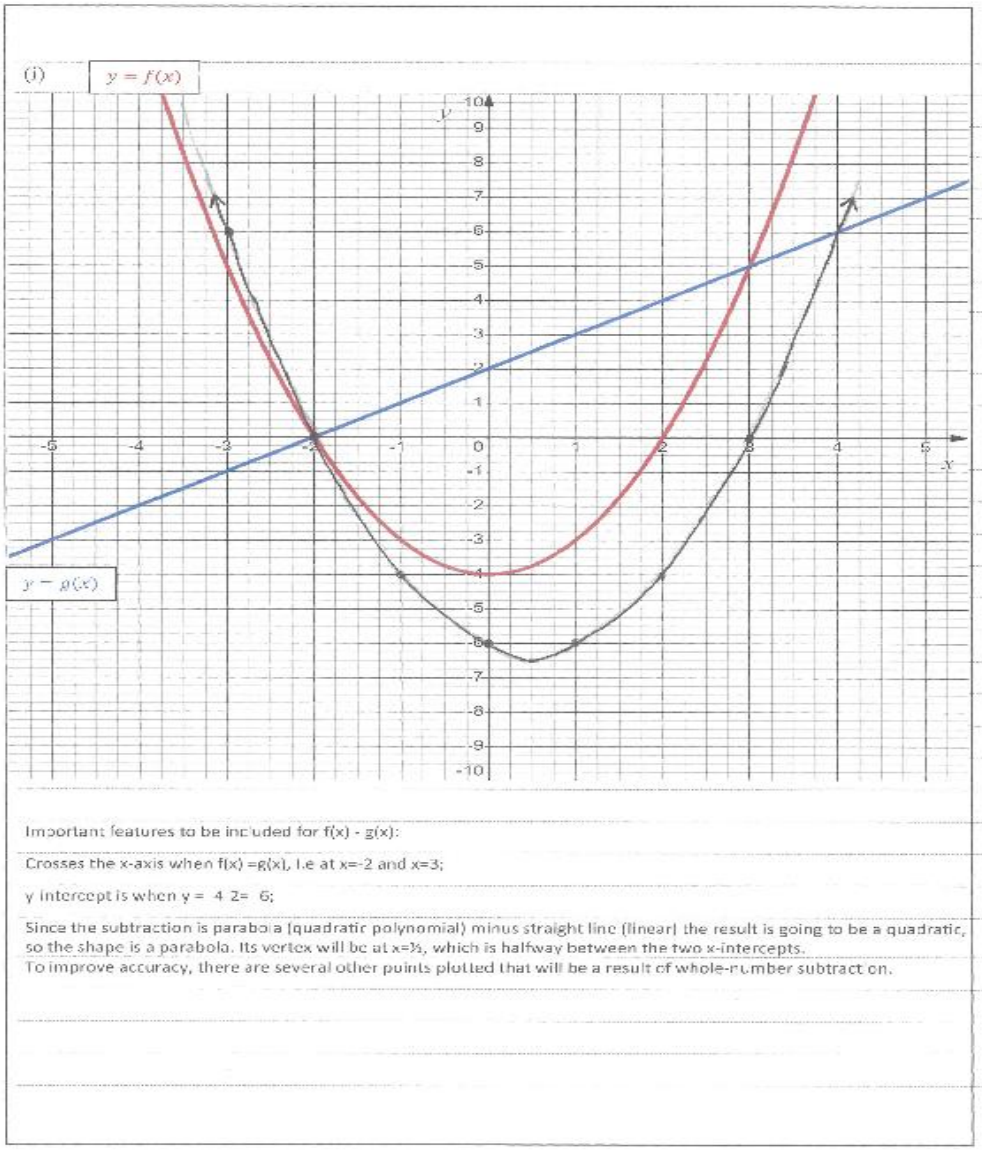
ME11-2 Manipulates algebraic expressions and graphical functions to solve problems.
ME11-7 Communicates making comprehensive use of mathematical language, notation, diagrams and graphs.

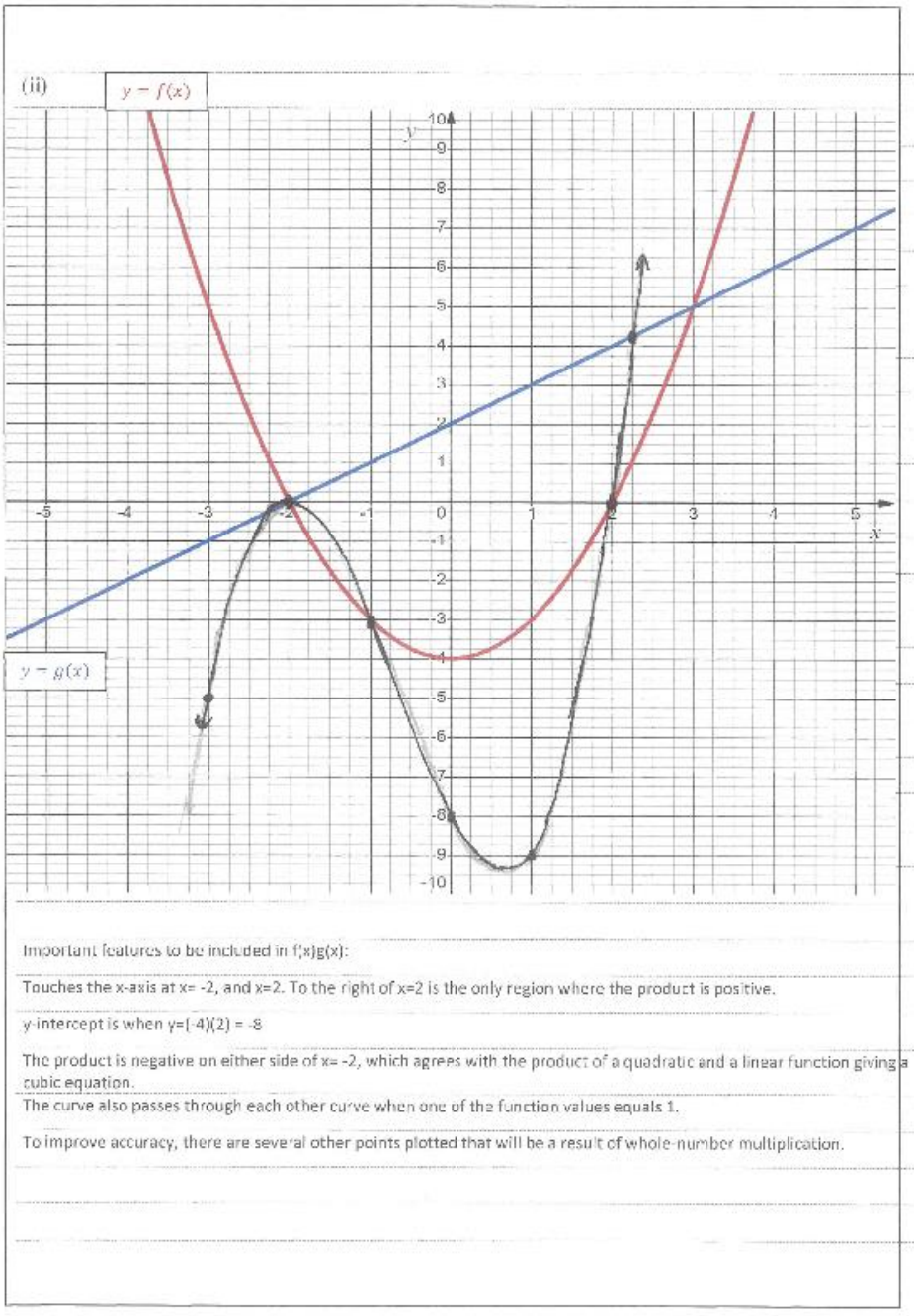
Question	Solutions	Marking Guidelines
(1) **ME11-2**	**Answer: C** $2x = t$ Substitute into $y = 3t^2$ $y = 12x^2$	**1 mark**
(2) **ME11-2**	**Answer: A** Plotting the inequality:  The values that lie above the x-axis must be $x < -3$ or $1 < x < 2$	**1 mark**
(3) **ME11-2**	**Answer: D** A sum is always equal to zero if one quantity is equal to the other quantity multiplied by negative 1.	**1 mark**
(4) **ME11-7**	**Answer: B** The graph we are looking for is the sum of the two given graphs in the question. The point (2, 8) is a point of difference for the 3 cubic graphs in the responses. The sum must pass through this point.	**1 mark**

(5) **ME11-7**	**Answer: D** As the graph given on the right is the same as the one on the left for $x > 0$ (graphs same to right of y-axis), and the reflection of this is graphed, this is $y = f(\|x\|)$. $\therefore$ correct answer is D.	**1 mark**
(6) **ME11-2**	**Answer: B** $\|2x - 5\| \geq 3$ $\therefore 2x - 5 \leq -3$ or $2x - 5 \geq 3$ $\therefore 2x \leq 2$ or $2x \geq 8$ $\therefore x \leq 1$ or $x \geq 4$ $\therefore$ correct answer is B.	**1 mark**

Year 11 Mathematics Extension 1 Task 1 2023		
Question No. 7	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
ME11-2 manipulates algebraic expressions and graphical functions to solve problems.		
Question	**Solutions**	**Marking Guidelines**
(a)		**2 marks** Correctly sketches the curve, identifying all relevant features and correct shape. At $x = 1$, there should be a reflection. **1 mark** Correctly identifies the asymptote at $x = -1$ and makes substantial progress towards the correct solution.
(b)	$\frac{x}{x-1} \geq -2$ $\frac{x(x-1)^2}{(x-1)} \geq -2(x-1)^2$ $x(x-1) \geq -2(x^2-2x+1)$ $x^2 - x \geq -2x^2 + 4x - 2$ $3x^2 - 5x + 2 \geq 0$ $(3x-2)(x-1) \geq 0$ Since $x \neq 1$	**3 Marks** Both solutions are correct and relevant working out is provided. **2 Marks** Makes substantial progress towards the solution and/or provides one correct solution. **1 Mark** Correctly multiples the inequality by the square of the denominator or makes

	$x \leq \frac{2}{3}$ or $x > 1$	some progress towards the correct solution.
(c)	$x = 1 + \cos\alpha$ $\cos\alpha = x - 1$ $y = \sin\alpha - 2$ $\sin\alpha = y + 2$ Using the trigonometric identity $cos^2\alpha + sin^2\alpha = 1$ $(x-1)^2 + (y+2)^2 = 1$ The circle has a radius of 1 and a centre of $(1, -2)$	**2 marks** Correctly identifies the radius and centre from the Cartesian equation. **1 mark** Correctly writes the parametric equations in Cartesian form.

Year 11 Mathematics Extension 1 Task 1 2023	
Question No. 8 Solutions and Marking Guidelines	
Outcomes Addressed in this Question. **ME11-7** Communicates making comprehensive use of mathematical language, notation, diagrams and graphs.	
Solutions	**Marking Guidelines**
 Important features to be included for f(x) - g(x): Crosses the x-axis when f(x) =g(x), i.e at x=-2 and x=3; y intercept is when y = 4 2= 6; Since the subtraction is parabola (quadratic polynomial) minus straight line (linear) the result is going to be a quadratic, so the shape is a parabola. Its vertex will be at x=½, which is halfway between the two x-intercepts. To improve accuracy, there are several other points plotted that will be a result of whole-number subtract on.	**Explanation of features of this graph are included below the sample solution.** **3 marks:** All components of graph included. **2 marks:** One required component incomplete. **1 mark:** Significant relevant progress towards solution.



Explanation of features of this graph are included below the sample solution.

3 marks:
All components of graph included.

2 marks:
One required component incomplete.

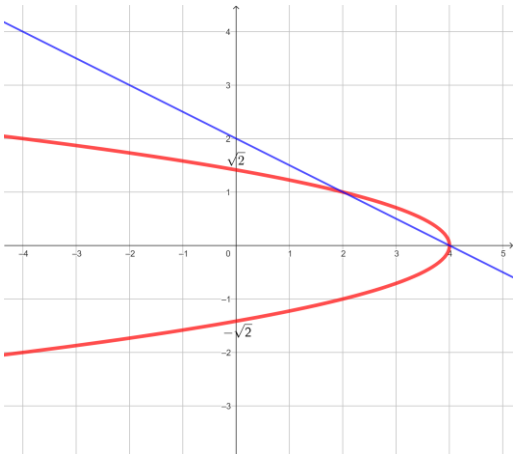
1 mark:
Significant relevant progress towards solution.

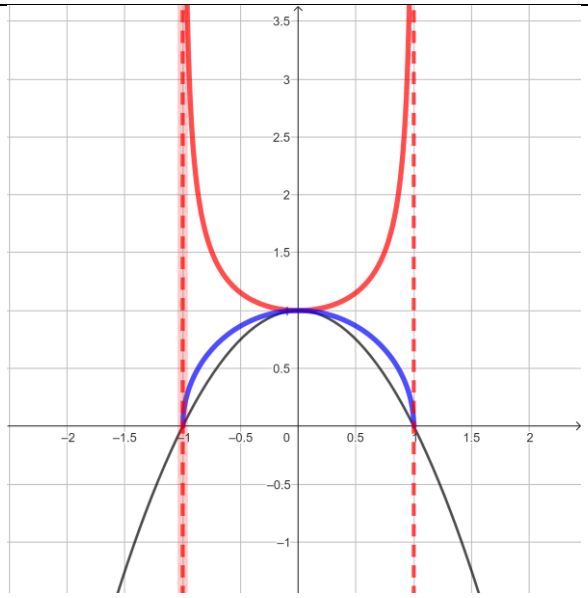
Year 11 Mathematics Extension 1 Task 1 2023

Question No.9 Solutions and Marking Guidelines

Outcomes Addressed in this Question.

ME11-7 communicates making comprehensive use of mathematical language, notation diagrams & graphs

Outcome	Solutions	Marking Guidelines
ME11-7	**(a)** $y^2 = f(x) \leftrightarrow y = \pm\sqrt{f(x)}$. $y = \sqrt{f(x)}$ is the top half of the graph shown, then $y = -\sqrt{f(x)}$ is the reflection of $y = \sqrt{f(x)}$ in the *x* axis.  	**2 marks**: correct solution **1 mark:** $y = \sqrt{f(x)}$ drawn correctly or correct $y = -\sqrt{f(x)}$ from incorrect $y = \sqrt{f(x)}$.
ME11-7	**Note: question said to show any relevant intercepts – this was ignored by many students! Your diagram also needed to indicate it was a smooth curve.** **(b)** To sketch $y = \frac{1}{\sqrt{1-x^2}}$ given $f(x) = 1 - x^2$, first sketch $f(x) = \sqrt{1-x^2}$, which is the top half of a semicircle of centre (0,0) & radius 1 (shown in blue). Then the reciprocal graph $y = \frac{1}{\sqrt{1-x^2}}$ can be drawn. Answer shown in red.	**2 marks**: correct solution **1 mark:** correct solution without detail shown such as asymptotes or incorrect shape.

ME11-7	 **Note: if you have more than one graph drawn on your diagram you must label your graphs, so that you indicate to the marker, which graph is your final answer.** **(c)** Graph of $g(x) = \frac{\lvert x-1\rvert}{x}$ could be drawn by sketching y $= \lvert x-1\rvert$ and $y = \frac{1}{x}$ on using multiplication of ordinates. The final solution is shown in red.  	**2 marks:** correct solution **1 mark:** significant progress towards correct solution with significant detail shown.