

JRAHS Ext1 Term2 2009

QUESTION 1 (9 Marks)

Marks

- (a) The velocity of a particle, v metres per second, moving in a straight line is given as $v = 8t^2 + 8t - 24$, where t is the time in seconds. The particle is initially 7 metres from the origin.
- | | | |
|-------|--|---|
| (i) | Find the displacement as a function of t . | 2 |
| (ii) | Find the acceleration as a function of t . | 2 |
| (iii) | What is the magnitude of the acceleration when $t = 2$. | 1 |
| (iv) | When does the particle change direction? | 1 |
- (b) Twelve people are to be seated around a table.
- | | | |
|-------|--|---|
| (i) | In how many ways can they be seated? | 1 |
| (ii) | In how many ways can they be seated if two particular people are not to be put together? | 1 |
| (iii) | Find the probability that 2 friends will be seated together. | 1 |

QUESTION 2 (9 Marks) START A NEW PAGE

- (a) A particle is moving in a straight line with its acceleration as a function of x given by $\ddot{x} = -e^{-2x}$. It is initially at the origin and is travelling with a velocity of 1 metre per second.
- | | | |
|------|--|---|
| (i) | Show that $\dot{x} = \frac{1}{e^x}$. | 2 |
| (ii) | Hence derive an expression for the displacement of the particle as a function of t . | 2 |
- (b) Consider a particle undergoing SHM, with its displacement, in metres, is given by
- $$x = 2 \cos\left(t + \frac{\pi}{4}\right) \text{ at time } t \text{ seconds.}$$
- | | | |
|-------|--|---|
| (i) | Find the time at which the particle will first be at the origin. | 1 |
| (ii) | Calculate the velocity of the particle when it passes through the origin for a second time. | 2 |
| (iii) | What is the magnitude of the greatest acceleration for this particle and when does it first occur? | 2 |

QUESTION 3 (9 Marks) START A NEW PAGE**Marks**

- (a) The letters A, E, I, O and U are vowels.
- (i) How many arrangements of the letters in the word *MATHEMATICS* are possible? 1
- (ii) How many arrangements of the letters in the word *MATHEMATICS* are possible if the vowels must occupy the 3rd, 5th, 7th and 10th positions? 2
- (b) In SHM, the acceleration of a particle at any time is proportional to its displacement from the origin and is directed towards the origin. Show that a particle with displacement $x = a \tan nt$ is not moving in SMH. 3
- (c) Carbon-14 is a radioactive isotope of carbon that has a half life of 5600 years. It is used extensively in dating organic material that is tens of thousands of years old. What percentage of the original amount of Carbon-14 in a sample would be present after 10,000 years? [Assume $N = N_0 e^{kt}$] 3

QUESTION 4 (9 Marks) START A NEW PAGE

- (a) A freshly caught fish, initially at 18°C , is placed in a freezer that has a constant unknown temperature of $x^\circ \text{C}$. The cooling rate of the fish is proportional to the difference between the temperature of the freezer & the temperature $T^\circ \text{C}$, of the fish.

It is known that T satisfies the equation $\frac{dT}{dt} = -k(T - x)$,

where t is the number of minutes after the fish is placed in the freezer.

- (i) Show that $T = x + Ae^{-kt}$ satisfies this equation. 2
- (ii) If the temperature of the fish is 10°C after $7\frac{1}{2}$ minutes, show that the fish's temperature after t minutes is given by
$$T = x + (18 - x)e^{\frac{2}{15} \log_e \left[\frac{10 - x}{18 - x} \right] t}.$$
 3
- (iii) Find the temperature of the fish after 15 minutes when the initial freezer temperature is 5°C . Answer to the nearest degree. 1
- (b) The velocity of a particle is given by $v = 4\sqrt{x+1}$. If the particle's displacement after 2 seconds is 3 metres, find its displacement after 1 second. 3

QUESTION 5 (9 Marks) START A NEW PAGE**Marks**

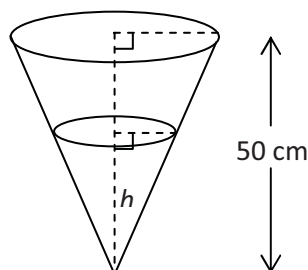
- (a) Four digit numbers are to be formed from the digits 4, 5, 6, 7, 8, 9. Find how many 4-digit numbers can be formed if no digit to appear more than once in the number. 2
- (b) A rocket is fired at 30 metres per second at an angle of 30° to the horizontal at a fireworks display. After 2 seconds, it explodes into two equal particles. One part falls vertically downwards, while the other part gets projected at 60 metres per second at an angle of 60° to the horizontal.
- Assume acceleration due to gravity is 10 metre per second squared and that both parts fall back on the same level ground.
- (i) Through what distance does the vertically falling particle travel when it hits the ground? 2
- (ii) What is the time taken for the second particle to fall back to the ground? (correct to the nearest second). 3
- (iii) What is the distance travelled by the second particle land from its launching site, to one decimal place? 2

QUESTION 6 (9 Marks) START A NEW PAGE

- (a) 5 cards are dealt out from a well-shuffled standard 52 card pack. Find the probability that this hand will contain:
- (i) the 4 queens and another card. 1
- (ii) 2 jacks and 3 kings. 2
- (iii) a 3, 4, 5, 6 and 7. 1
- (b) A particle moves in a straight line and its position, x in metres at time t seconds is given by $x = 4 + \sin 2t + \sqrt{3} \cos 2t$
- (i) Prove that the particle is moving in simple harmonic motion about $x = 4$. 2
- (ii) Find the period and amplitude of the motion. 2
- (iii) What is the speed of the particle as it travels through the equilibrium position? 1

END of PAPER

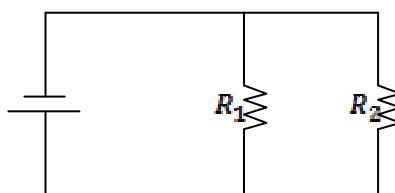
- (c) An inverted conical vessel, as shown below, is 50 centimetres in radius and 50 centimetres in depth. The vessel is being filled with water at a constant rate of $25 \text{ cm}^3/\text{s}$. The depth of the water at any time t seconds is h centimetres.



Not to Scale

- (i) Show that the surface area of the cone can be expressed as $A = \pi r^2(1 + \sqrt{2})$, where r is the radius of the cone. 2
- (ii) Hence, or otherwise determine the rate of increase of the surface area of the water when the depth is 20 centimetres. 3
- (c) A plane flying horizontally at an altitude of one kilometre and at a constant speed of 800 kilometres per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing at the instant when the plane is four kilometres away from the station. 2
- (a) If two resistors with resistances R_1 and R_2 are connected in parallel, as shown in the figure below, then the total resistance R , measured in ohms, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



If R_1 and R_2 are increasing at rates of 0.3 ohm per second and 0.2 ohm per second respectively, how fast is R changing when $R_1 = 80$ ohms and $R_2 = 100$ ohms? Express answer to 3 significant figures. 3

Y.12 T2 24

MATHS EXT 1
MATHEMATICS: Question...

Suggested Solutions	Marks	Marker's Comments
a) $x(t) = \frac{8}{3}t^3 + \frac{8}{2}t^2 - 24t + c$ $x = 7$ when $t = 0 \therefore c = 7$ $\therefore x(t) = \frac{8}{3}t^3 + 4t^2 - 24t + 7$	1 1	
ii) $\ddot{x} = 16t + 8$	1 + 1	
iii) $\ddot{x}(2) = 40 \text{ m/s}^2$	1	forgot $\text{m/s}^2 - \frac{1}{2} \text{ m}$
iv) $8t^2 + 8t - 24 > 0$ $t^2 + t - 3 > 0$ $t = \frac{-1 \pm \sqrt{13}}{2}$		no marks if stops here.
$t > 0 \therefore t = \frac{-1 + \sqrt{13}}{2} \text{ sec}$	1	forgot sec $-\frac{1}{2} \text{ m}$
b) 39916800	1	11! only $\frac{1}{2} \text{ m}$
ii) 10! $\times 32659200$	1	10! $\times 9$ only $\frac{1}{2} \text{ m}$
iii) $\frac{2}{11}$	1	$\frac{10! \times 2}{11!}$ only $\frac{1}{2} \text{ m}$

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
a) (i) $\ddot{x} = -e^{-2x}$ $t=0$ $x=0$ $\dot{x}=1$ $\frac{d}{dx} \left[\frac{1}{2} \dot{x}^2 \right] = -e^{-2x}$ $\frac{1}{2} \dot{x}^2 = \int -e^{-2x} dx$ $\frac{1}{2} \dot{x}^2 = \frac{1}{2} e^{-2x} + C$ $x=0 \quad \dot{x}=1$ $\frac{1}{2} = \frac{1}{2} + C \quad \therefore C=0$ $\dot{x}^2 = e^{-2x}$ $ \dot{x} = e^{-x}$ as $e^{-2x} \neq 0 \quad \therefore \dot{x} \neq 0$ $\therefore$ object never stops and cannot change direction when $t=0$ $\dot{x}=1$ $\therefore \dot{x} > 0$ for all motion $\therefore x = e^{-x} = \frac{1}{e^x}$	(2)	(1/2) using $d(\frac{1}{2}\dot{x}^2)$ (1/2) correct integration (1/2) "C" value (1/2) full explanation that $\dot{x} > 0$
(ii) $\ddot{x} = \frac{1}{e^x} = e^{-x}$ $\frac{d\dot{x}}{dx} = e^{-x}$ $\frac{d\dot{x}}{dx} = e^x$ $t = \int e^x dx$ $= e^x + C$ $t=0 \quad x=0$ $0 = 1 + C \quad \therefore C = -1$ $t = e^x - 1$ $e^x = t + 1$ $x = \ln(t + 1)$	(2)	(1/2) $\frac{d\dot{x}}{dx} = e^x$ (1/2) correct integral including "C" (1) $x = \ln(t + 1)$

MATHEMATICS Extension 1: Question... (2)

Suggested Solutions

Marks

Marker's Comments

$$(b) (i) x = 2 \cos\left(t + \frac{\pi}{4}\right)$$

origin $x = 0$

$$0 = 2 \cos\left(t + \frac{\pi}{4}\right)$$

$$t + \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$t = \frac{\pi}{4} \quad (\text{first time})$$

(time is $\frac{\pi}{4}$ sec)

(ii) at origin second time

$$\frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4} \text{ sec.}$$

$$x_0 = 2 \cos\left(t + \frac{\pi}{4}\right)$$

$$x = -2 \sin\left(t + \frac{\pi}{4}\right)$$

when $t = \frac{5\pi}{4}$

$$x = -2 \sin\left(\frac{3\pi}{2}\right)$$

$$= +2$$

velocity is 2 m/s.

$$(iii) \ddot{x} = -2 \cos\left(t + \frac{\pi}{4}\right)$$

max acceleration is +2 m/s.
occurs when

$$\ddot{x} = 2 = -2 \cos\left(t + \frac{\pi}{4}\right)$$

$$-1 = \cos\left(t + \frac{\pi}{4}\right)$$

$$t + \frac{\pi}{4} = \pi, 3\pi$$

$$\therefore \text{First time is } \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\text{time is } \frac{3\pi}{4} \text{ sec.}$$

(1)

(1) correct answer.

(1) $\frac{5\pi}{4}$ 2nd time at origin

(1) correct answer.

(1) correct accel.
 ± 2 accepted.
 (-2) not accepted.

(1) correct time

Ext 1 MATHEMATICS: Question 3

Suggested Solutions

Marks

Marker's Comments

(a) (i) $\frac{11!}{2!2!2!}$
 $= 4989600$

1 mark

If they forget the 2!2!2! — 0 marks

(ii) $\frac{4!}{2!} \times \frac{7!}{2!2!}$
 $= 15120$

2 marks

1 mark off for each error

(b) $x = a \tan(nt)$
 $\dot{x} = a \sec^2(nt) = a n (\cos(nt))^{-2}$

1 mark

$\ddot{x} = -2an (\cos(nt))^{-3} \cdot -n \sin(nt)$
 $= \frac{2an^2 \sin(nt)}{\cos^3(nt)}$

1 mark

$= +2an^2 \tan(nt) \cdot \sec^2(nt)$

$= 2n^2 \sec^2(nt) \cdot x$ (as $x = a \tan(nt)$)
 $\neq -m^2 x$

1/2 mark

as n must be a positive constant
 and $\sec^2(nt)$ is not a constant
 $\therefore x = a \tan(nt)$ is not in SHM.

1/2 mark

(c) $N = N_0 e^{-\frac{5600}{K} t}$

$\frac{1}{2} N_0 = N_0 e^{-\frac{5600}{K} t}$

$K = \frac{\ln 0.5}{\frac{5600}{N_0}}$

$= 7.24 \times 10^{-4}$

1 mark

$N = N_0 e^{-\ln 0.5 \frac{t}{5600}}$

when $t = 10000$, $N = ?$

$N = N_0 e^{-\frac{\ln 0.5}{5600} \times 10000}$

1 mark

$= N_0 \times 0.2900$

1 mark

$\therefore$ Percentage left is 29%

Suggested Solutions	Marks	Marker's Comments
a); Substitute $T = x + Ae^{-kt}$ into $\frac{dT}{dt} = k(T-x)$ $\text{LHS: } \frac{dT}{dt} = -kAe^{-kt} \quad (\text{x constant}) \quad \text{RHS: } -k(T-x) = -k(Ae^{-kt}) = -kAe^{-kt}$ LHS = RHS $\therefore T = x + Ae^{-kt}$ satisfies $\frac{dT}{dt} = k(T-x)$ 2. ii) When $t=0, T=18$ $\therefore 18 = x + A \quad (\text{by substitution})$ $A = 18 - x$ When $t = 15/2, T = 10$ $\therefore 10 = x + Ae^{-15k/2}$ $10 = x + (18-x)e^{-15k/2}$ $\therefore \frac{10-x}{18-x} = e^{-15k/2} \Rightarrow \frac{18-x}{10-x} = e^{15k/2}$ $\therefore \frac{15k}{2} = \ln\left(\frac{18-x}{10-x}\right) \Rightarrow k = \frac{2}{15} \ln\left(\frac{18-x}{10-x}\right)$ $\therefore T = x + (18-x)e^{-\frac{2}{15}t \ln\left(\frac{18-x}{10-x}\right)}$ $= x + (18-x)e^{\frac{2t}{15} \ln\left(\frac{10-x}{18-x}\right)}$ When $t=15$ and $x=5$ $T = 5 + 13e^{2 \ln(5/13)} = 5 + \frac{25}{13}$ $= 7^\circ\text{C} \quad (\text{to nearest degree C})$	1/2 off for no conclusion 1 1 1 1 1 1 1 1 1	Some leniency here because of question ambiguity.
b) $v = \frac{dx}{dt} = 4(x+1)^{1/2}$ $\int \frac{dx}{(x+1)^{1/2}} = 4 \int dt$ $2(x+1)^{1/2} = 4t + k$ When $t=2, x=3 \therefore k=-4$ $(x+1)^{1/2} = 2t-2$ When $t=1, (x+1)^{1/2}=0$ $\therefore x=-1 \quad (1\text{ m to left of } x=0)$	1 1 1 1 1	

Suggested Solutions

Marks

Marker's Comments

Question 5:

$$(a) \quad 0 \ 0 \ 0 \ 0$$

$$6 \times 5 \times 4 \times 3$$

$$\text{No. of 4 digit no's} = 6 \times 5 \times 4 \times 3$$

$$= 360$$

$$\text{OR } {}^6C_4 \times 4! = 360$$

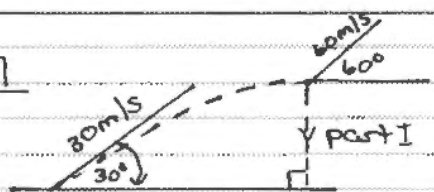
$$\text{OR } {}^6P_4 = 360$$

$$\text{OR } \frac{6!}{2!} = 360$$

②

Question 7

(a) (i)



$$\ddot{y} = -10$$

$$\dot{y} = -10t + C$$

$$\text{but } \dot{y} = 30 \sin 30^\circ \text{ when } t=0$$

$$\therefore \dot{y} = 15 - 10t$$

$$y = -\frac{1}{2}10t^2 + 15t + C$$

$$\text{When } t=0, y=0, C=0$$

$$\therefore y = -5t^2 + 15t$$

$$\text{When } t=2$$

$$y = -20 + 30$$

$$y = 10$$

$\therefore$ Part I falls vertically for 10m when it hits the ground.

①

(ii)

$$\ddot{y} = -10$$

$$\dot{y} = -10t + 60 \sin 60^\circ$$

$$\dot{y} = -10t + 30\sqrt{3}$$

$$y = -5t^2 + 30\sqrt{3}t + 10 \text{ from (i)}$$

$$\text{When } y=0 \quad t^2 - 6\sqrt{3}t - 2 = 0$$

$$t = \frac{6\sqrt{3} \pm \sqrt{36 \times 3 - 4 \times 1 \times 2}}{2}$$

$$= \frac{6\sqrt{3} \pm \sqrt{108+8}}{2}$$

$$= 3\sqrt{3} \pm \sqrt{29}$$

$$\therefore t = -0.189 \text{ or } 10.58$$

$$\text{as } t > 0 \quad t = 10.58$$

$$\text{Nearest second is 11 sec}$$

$$\text{Total time} = 2 + 11$$

$$= 13 \text{ seconds}$$

①

①

$\frac{1}{2}$ mark lost for not eliminating $t = -0.189$

①

11 seconds scored
Full marks

TERM 2

MATHEMATICS Extension 1 : Question 5 and 7 2009

Suggested Solutions

Marks

Marker's Comments

Question 7 (cont.)

$$(iii) \text{ When } t=2, \quad x = 30 \cos 30^\circ \times 2$$

$$= 60 \times \frac{\sqrt{3}}{2}$$

$$x = 30\sqrt{3}$$

$$\text{Now } x_1 = 60 \cos 60^\circ t$$

$$x_1 = 30t$$

$$= 30(3\sqrt{3} + \sqrt{3})$$

$$= 317.4 \text{ m}$$

$$\therefore \text{Total distance} = 317.4 \text{ m} + 30\sqrt{3} \text{ m}$$

$$= 369.4 \text{ m}$$

①

1 mark for each part

①

⑧

		mark	comment
6a(i)	$\text{Prob.} = \frac{{}^4C_4 \times {}^{48}C_1}{{}^{52}C_5}$ $= \frac{1}{54145}$ Or $\text{Prob.} = \left(\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{48}{48} \right) \times \left(\frac{5!}{4!} \right)$	1	No half marks
6a(ii)	$\text{Prob.} = \frac{{}^4C_2 \times {}^4C_3}{{}^{52}C_5}$ $= \frac{1}{108290}$ Or $\text{Prob.} = \left(\frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{3}{49} \times \frac{2}{48} \right) \times \left(\frac{5!}{3! \times 2!} \right)$	2	1 for numerator containing ${}^4C_2 \times {}^4C_3$ with any other term
6a(iii)	$\text{Prob.} = \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_5}$ $= \frac{64}{162435}$ Or $\text{Prob.} = \left(\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times \frac{4}{49} \times \frac{4}{48} \right) \times (5!)$	1	
			Repeated incorrect expressions (i) $\text{Prob.} = \left(\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{48}{48} \right)$ (ii) $\text{Prob.} = \left(\frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{3}{49} \times \frac{2}{48} \right)$ (iii) $\text{Prob.} = \left(\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times \frac{4}{49} \times \frac{4}{48} \right)$ Max 2/4

6b(i)	$x = 4 + \sin 2t + \sqrt{3} \cos 2t$ $\dot{x} = 2 \cos 2t - 2\sqrt{3} \sin 2t$ $\dot{x} = -4 \sin 2t - 4\sqrt{3} \cos 2t$ $= -4(\sin 2t + \sqrt{3} \cos 2t)$ $= -4(x - 4) \text{ since } \sin 2t + \sqrt{3} \cos 2t = x - 4$	2	1 for $\dot{x}$ $\frac{1}{2}$ for $\ddot{x}$ $\frac{1}{2}$ for showing $\ddot{x} = -4(x - 4)$ Could also use $x = 4 + 2 \sin(2t + \frac{\pi}{3})$ Or $x = 4 + 2 \cos(2t - \frac{\pi}{6})$
6b(ii)	Period = π sec Amplitude = 2 m	2	1 mark for each answer
6b(iii)	Speed = 4 m/s	1	1 for correct speed $\frac{1}{2}$ if speed negative