QUESTION 1 (15 Marks) **Marks**

(a) Calculate the exact value of $\cos\left(\frac{5\pi}{12}\right)$. **3**

(b) Evaluate the expression $x^2 + \frac{1}{x^2}$ when $x = \frac{1+2\sqrt{3}}{1-2\sqrt{3}}$. **3**

(c) The second derivative of the function $y = f(x)$ is $\frac{d^2y}{dx^2} = \frac{1}{x^3} - \frac{2}{x^4}$. **2**

Show that the graph of $y = f(x)$ has a point of inflexion.

(d) The point $P(x, y)$ moves so that its distance PA from the point $A(1, 5)$ is always twice its distance PB from the point $B(4, -1)$.

(i) Show that the equation of the locus of P is $x^2 + y^2 - 10x + 6y + 14 = 0$. **2**

(ii) Describe the locus. **2**

(e) Sketch on a number plane the solution set for: **3**

$$\{(x, y) : xy > 4\} \cap \{(x, y) : x - y \geq 0\}.$$

QUESTION 2 (15 Marks) START A NEW PAGE

(a) Differentiate with respect to x:

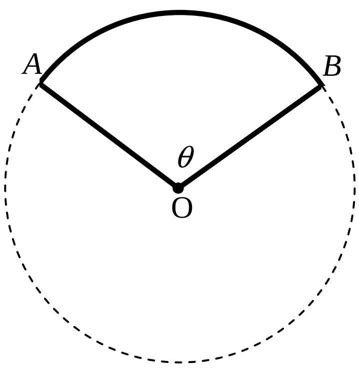
(i) $y = xe^x$ **1**

(ii) $y = \log_x 2$ **2**

(b) Solve the equation for x and verify the solution(s). **2**

$$\frac{|x-3|}{3-x} = x$$

(c) A pond is designed in the shape of a sector of a circle, so that the perimeter of the pond is 200 m and the area is 1600 m^2.



NOT TO SCALE

Calculate the value of the angle, θ radians, subtended at the centre of the circle. **4**

Question 2 continued over page

(d) The point $P(-7, y)$ divides the interval AB in the ratio $a:b$, where A is $(-3,1)$ and B is $(2,-9)$. Both a and b are integers.

(i) Find the ratio $a:b$ expressed in simplest terms. **2**

(ii) Calculate the value of y. **1**

(e) The line m is the tangent to the curve $y = x^4 - 2x^3 - 5x$ at the point $P(2,-10)$. **3**
Find the angle that m makes with the positive direction of the x-axis.
Give your answer to the nearest degree.

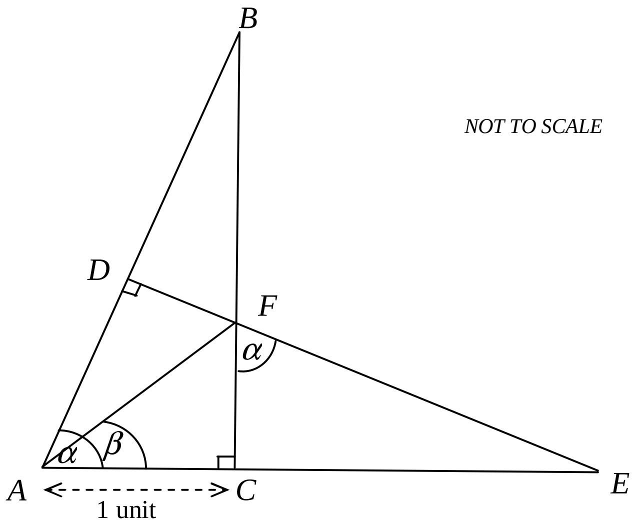
QUESTION 3 (15 Marks) START A NEW PAGE

(a) State whether the function **2**

$$f(x) = \frac{2^x - 1}{2^x + 1}$$

is ***odd***, ***even*** or ***neither odd nor even***, giving reasons.

(b) In the diagram below ΔABC is right-angled at C, with $\angle BAC = \alpha^{\circ}$. ED is drawn perpendicular to AB meeting BC at F such that $\angle EFC$ is also equal to α°.
$\angle FAC = \beta^{\circ}$ and $AC = 1$ unit.



(i) Copy the diagram and show that $BF = \tan\alpha - \tan\beta$ **1**

(ii) Express the length AE in terms of $\tan\alpha$ and $\tan\beta$. **1**

(iii) Prove that ΔBDF is similar to ΔADE. **2**

(iv) Hence prove the formula: **2**

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}.$$

Question 3 continued over page

Question 3 continued **Marks**

(c) Solve the equation for x:

$$f(x) = f[f(x)] \text{ where } f(x) = x^2 - 2x.$$ **3**

(d) If m and n are the roots of the equation $x^2 - x + 3 = 0$,
Find:

(i) the values of $m+n$, mn and $m^2 + n^2$. **2**

(ii) the quadratic equation which has roots **2**

$$\alpha = m^2 - mn \text{ and } \beta = n^2 - mn.$$

QUESTION 4 (15 Marks) START A NEW PAGE

(a) Solve the equation for x: **3**

$$9^x - 3^{x+2} = 10.$$

Give your answer to 2 decimal places.

(b) Use first principles to find the derivative of $y = 8^x$. **3**
You may assume that:

$$\lim_{u \to 0}\left[\frac{2^u - 1}{u}\right] = \ln 2.$$

(c) Consider the function $y = \dfrac{12x^2}{x^3 + 4}$.

(i) Find $\dfrac{dy}{dx}$ in simplest terms. **2**

(ii) Find the coordinates of all the stationary points for the function. **2**

(iii) Determine the nature of the stationary points using the second derivative. **2**

You may assume that $\dfrac{d^2y}{dx^2} = \dfrac{24(x^6 - 28x^3 + 16)}{(x^3 + 4)^3}$. (Do not show this)

(iv) Sketch the graph of $y = \dfrac{12x^2}{x^3 + 4}$, showing all turning points and asymptotes. **3**

END OF EXAMINATION

Question 1:

a) $\cos\left(\frac{5\pi}{12}\right) = \cos(75^\circ)$

$= \cos(30^\circ + 45^\circ) =$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$ | 1 Correct Expansion

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ | 1 + 1 for correct values

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

b) $\left(\frac{1+2\sqrt{3}}{1-2\sqrt{3}}\right)^2 + \left(\frac{1-2\sqrt{3}}{1+2\sqrt{3}}\right)^2$

$= \frac{1 + 4\sqrt{3} + 12}{1 - 4\sqrt{3} + 12} + \frac{1 - 4\sqrt{3} + 12}{1 + 4\sqrt{3} + 12}$ | 1 Expand.

$= \frac{13 + 4\sqrt{3}}{13 - 4\sqrt{3}} + \frac{13 - 4\sqrt{3}}{13 + 4\sqrt{3}}$

$\frac{(13 + 4\sqrt{3})^2 + (13 - 4\sqrt{3})^2}{(13)^2 - (4\sqrt{3})^2}$ | 1 Rationalize

$= \frac{169 + 104\sqrt{3} + 48 + 169 - 104\sqrt{3} + 48}{169 - 48}$

1 Simplify

$= \frac{434}{121}$

c) $\frac{d^2y}{dx^2} = \frac{1}{x^3} - \frac{2}{x^4}$

For possible point of inflexion $\frac{d^2y}{dx^2} = 0$.

$\frac{x-2}{x^4} = 0$

$\therefore x = 2$. | 1 solve $\frac{d^2y}{dx^2} = 0$

TEST concavity

x	1	2	3
$\frac{d^2y}{dx^2}$	-1	0	$+1/81$

1 Concavity Test

$\therefore$ change in concavity and $\frac{d^2y}{dx^2} = 0$

$\therefore$ Point of inflexion when $x = 2$.

Question 1.

d) i) $PA = 2PB$

$PA^2 = 4PB^2$

$(x-1)^2 + (y-5)^2 = 4[(x-4)^2 + (y+1)^2]$ ① distance equation

$x^2 - 2x + 1 + y^2 - 10y + 25 =$
$4(x^2 - 8x + 16 + y^2 + 2y + 1)$

$x^2 - 2x + 26 + y^2 - 10y =$
$4x^2 - 32x + 4y^2 + 8y + 68$

$0 = 3x^2 - 30x + 3y^2 + 18y + 42$ ① Expansion and simplification

$0 = x^2 - 10x + y^2 + 6y + 14$

ii) $x^2 - 10x + 25 +$
$y^2 + 6y + 9 = -14 + 25 + 9$

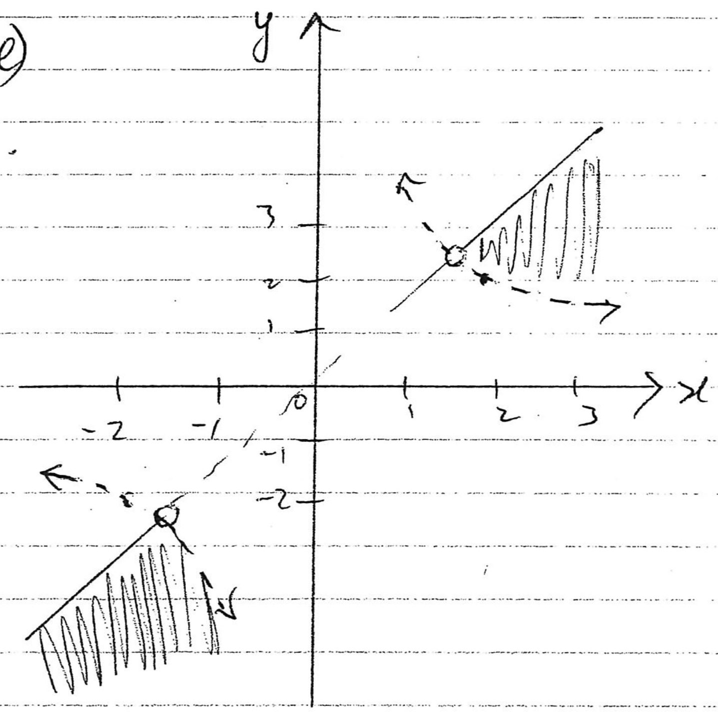
$(x-5)^2 + (y+3)^2 = 20$ ① Complete Squares + Factor

Locus – circle, centre $(5, -3)$, radius $2\sqrt{5}$ units

½ circle

½ centre + radius

e)



① + ① each graph including boundary.

① coloured region

$-\frac{1}{2}$ if circles at point of intersection not shown

JRAHS Ext 1 2010 Q2

Comments

Q2 m a i) $y' = e^x(x+1)$ 1m

2m ii) $y = \frac{\ln 2}{\ln x}$ 1m $y' = \frac{-\ln 2}{x(\ln x)^2}$ 1m

$\ln x^2 \neq (\ln x)^2$

$y = \frac{\log_{10} 2}{\log_{10} x}$ 0m

2m b) Solve for x 1m

Reject $x=-1$, 3 and verify $x=1$

$\frac{|1-3|}{3-1} = \frac{|-2|}{2} = \frac{2}{2} = 1$ 1m

most students got 1m as they forgot to verify

many drew graphs but did not verify $x=1$

4m c) $2r + r\theta = 200$
$\frac{1}{2}r^2\theta = 1600$ 1m

$\left(\frac{200}{2+\theta}\right)^2 \theta = 3200$ 1m

or $3200 = \left(\frac{200-2r}{r}\right) r^2$ 1m

$\theta = 8$ or $\frac{1}{2}$ 1m

Reject $\theta = 8$ as $\theta < 2\pi$ 1m
$\therefore \theta = \frac{1}{2}$ only

many forgot their formulae and can't proceed

most students forgot to reject $\theta = 8$

2m d i) $\frac{2a-3b}{a+b} = -7$ 1m

$a : b = -4 : 9$ 1m

accept $\frac{a}{b} = -\frac{4}{9}$

$9a = -4b$ only $\frac{1}{2}$m

1m ii) $y = 9$ 1m

3m e) $y' = 4x^3 - 6x^2 - 5$ 1m

At $x=2$ $y' = 3$ 1m

$\tan\theta = 3$ $\theta = 72^\circ$ (nearest degree) 1m

many students write $y' = 4x^3 - 6x - 5$

$\theta = 71^\circ 34'$ $\frac{1}{2}$m only

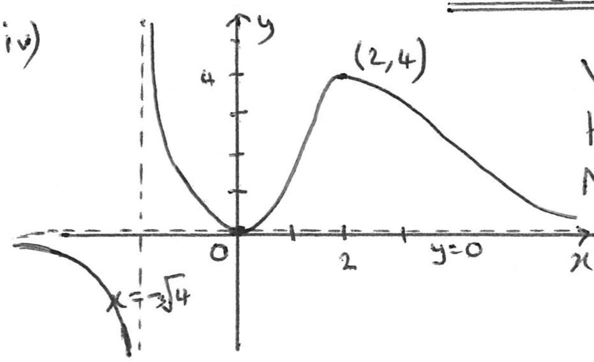
Year 11 Half Yearly 2010 Question 3

	Solution	Marks	Comments
(a)	$f(x)=\dfrac{2^x-1}{2^x+1}$ $f(-x)=\dfrac{2^{-x}-1}{2^{-x}+1}$ $=\dfrac{2^{-x}-1}{2^{-x}+1}\times\dfrac{2^x}{2^x}$ $=\dfrac{1-2^x}{1+2^x}$ $=-\left(\dfrac{2^x-1}{2^x+1}\right)$ $=-f(x)$ $f(-x)=\dfrac{2^{-x}-1}{2^{-x}+1}\times\dfrac{2^x}{2^x}$ $f(-x)=\dfrac{1-2^x}{1+2^x}$ $f(-x)=-\left(\dfrac{2^x-1}{2^x+1}\right)$ $f(-x)=-f(x)$ $\therefore\ f(x)$ is odd	2	$f(-x)=\dfrac{2^{-x}-1}{2^{-x}+1}$ …………½ mark $f(-x)=\dfrac{1-2^x}{1+2^x}$ …………½ mark $f(-x)=-\left(\dfrac{2^x-1}{2^x+1}\right)$ …………½ mark $f(-x)=-f(x)$ …………½ mark $\therefore\ f(x)$ is odd
(b)(i)	$BF=BC-FC$ $\dfrac{BC}{AC}=\tan\alpha$ $BC=AC\tan\alpha$ $=\tan\alpha$ $\dfrac{FC}{AC}=\tan\beta$ $FC=AC\tan\beta$ $=\tan\beta$ $BF=\tan\alpha-\tan\beta$	1	$BC=\tan\alpha$ …………½ mark $FC=\tan\beta$ …………½ mark

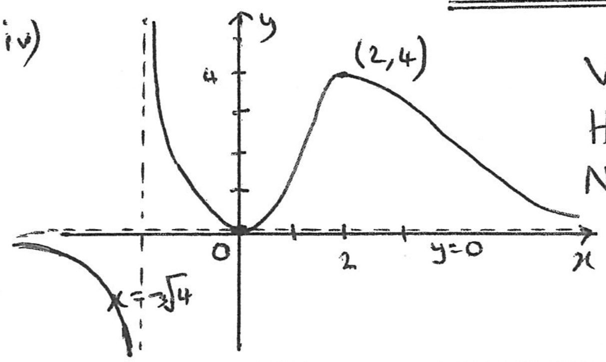
(b)(ii)	$AE = AC + CE$ $\dfrac{CE}{CF} = \tan\alpha$ $CE = CF\tan\alpha$ $= \tan\alpha\tan\beta$ $AE = 1 + \tan\alpha\tan\beta$	1	$CE = \tan\alpha\tan\beta$ …………½ mark $AE = 1 + \tan\alpha\tan\beta$ …………½ mark
(b)(iii)	$\angle BFD = \alpha$ (vertically opposite angles are equal) In ΔBFD and ΔADE $\angle BDE = \angle ADE$ (both $90°$) $\angle BFD = \angle DAE$ (both $= \alpha$) $\Delta BFD \; \vert\vert\vert \; \Delta ADE$ (equiangular) Could also have $\angle BDF = \angle AED \quad (= 90° - \alpha)$ (angle sum of triangles $= 180°$)	2	$\angle BFD = \alpha$ …………½ mark 2 equal angles …………½ mark each angle Test (equiangular)…………½ mark
(b)(iv)	$\dfrac{DF}{DA} = \tan(\alpha - \beta)$ $\dfrac{BF}{AE} = \dfrac{DF}{DA}$ (ratios of corresponding sides in similar triangles are equal) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$	2	$\dfrac{DF}{DA} = \tan(\alpha - \beta)$ ………. 1 mark $\dfrac{BF}{AE} = \dfrac{DF}{DA}$ (ratios of corresponding sides in similar triangles are equal) …….1 mark

(c)	$x^2-2x=(x^2-2x)^2-2(x^2-2x)$ $let\ u=x^2-2x$ $u=u^2-2u$ $u^2-3u=0$ $u(u-3)=0$ $u=0\ or\ 3$ $x^2-2x=0$ or $x^2-2x=3$ $x(x-2)=0$ or $x^2-2x-3=0$ $x=0\ or\ 2$ or $(x-3)(x+1)=0$ $x=3\ or\ -1$ $\therefore\ x=0\ or\ 2\ or\ 3\ or\ -1$ * can also expand, simplify and use remainder theorem to get roots $x^4-4x^3+x^2+6x=0$ $x(x-2)(x-3)(x+1)=0$	3	$x^2-2x=(x^2-2x)^2-2(x^2-2x)$ Correct equation ………………1 mark ½ mark for each solution 0 mark for correct solutions from incorrect working ** $(x^2-2x)^2-2(x^2-2x)=0$ $\Rightarrow x(x-2)=0$ $\Rightarrow x=0\ or\ 2$ ………… max. 1 mark
(d)(i)	$m+n=1$ $mn=3$ $m^2+n^2=(m+n)^2-2mn$ $=1-6$ $=-5$	2	$m+n=1$ …………½ mark $mn=3$ …………½ mark $m^2+n^2=(m+n)^2-2mn$ $=1-6$ …….1 mark $=-5$
(d)(ii)	Quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$ $\alpha+\beta=(m^2-mn)+(n^2-mn)$ $=m^2+n^2-2mn$ $=(m+n)^2-4mn$ $=1-12$ $=-11$ $\alpha\beta=(m^2-mn)(n^2-mn)$ $=m^2n^2-m^3n-mn^3+m^2n^2$ $=2(mn)^2-mn(m^2+n^2)$ $=2(9)-3(-5)$ $=33$ Quadratic equation is $x^2+11x+33=0$		$\alpha+\beta=-11$ …………½ mark $\alpha\beta=33$ …………½ mark ……$+11x$ …… (or correct sign for their coeff. of x) …………½ mark correct position of (their) x-coefficient and constant and $=0$ …………½ mark

YR11 ½YR 2010 EXT 1 **MATHEMATICS: Question** 4

Suggested Solutions	Marks	Marker's Comments
a) $3^{2x} - 9.3^{x} - 10 = 0$ $u^2 - 9u - 10 = 0$ (Let $u = 3^x$) $(u-10)(u+1) = 0$ $u = 10$ or -1 But $3^x > 0$ ∴ $3^x = 10$ only $x = \frac{\log 10}{\log 3}$ $= 2.10$ (2DP)	 1 1 1	Too many people took logs of $3^{2x} - 3^{x+2}$ and got no marks. Four people arrived at 2.10 with no valid working – no marks.
b) $\frac{dy}{dx} = \lim_{h\to 0}\left\{\frac{2^{3(x+h)} - 2^{3x}}{h}\right\} = 2^{3x}\lim_{h\to 0}\left\{\frac{2^{3h}-1}{h}\right\}$ $= 2^{3x}\lim_{h\to 0}\left\{\left(\frac{2^{3h}-1}{3h}\right)3\right\}$ $= 2^{3x}\lim_{h\to 0}\left(\frac{2^{3h}-1}{3h}\right).3$ $= 2^{3x}.3.\ln 2 = 8^x \ln 8$	1 1 1	One mark for 1st principles definition. One mark for knowing answer. Some justification required that $\lim_{h\to 0}\frac{8^h-1}{h} = \ln 8$
c) i) $\frac{dy}{dx} = \frac{(x^3+4)24x - 12x^2(3x^2)}{(x^3+4)^2} = \frac{-12x^4+96x}{(x^3+4)^2}$	2	$\frac{12x(8-x^3)}{(x^3+4)^2}$ better
ii) At S.P., $\frac{dy}{dx} = 0 \Rightarrow x(2-x)(x^2+2x+4) = 0$ ∴ $x = 0$ or $x = 2 \Rightarrow$ S.P. at $(0,0)$ and $(2,4)$	 2	• Too many numerical errors in Quotient Rule. • S.P. are points not x values.
iii) At $(0,0)$, $\frac{d^2y}{dx^2} = 6$ ∴ Concave up ∴ Local Minimum at $(0,0)$ At $(2,4)$, $\frac{d^2y}{dx^2} = -2$ ∴ Concave down Local Maximum at $(2,4)$	1 1	• In part (iii) some mention of concavity was required.
iv)  Vert Asy $x = -4^{1/3}$ Horiz Asy $y = 0$ No intercepts except $(0,0)$	½ per asym ½ per SP 1 for shape and axes.	• Scales were scarcer than expected. At least 0 and 1 other value on each axis.

YR11 ½YR 2010 EXT 1 **MATHEMATICS: Question** 4

Suggested Solutions	Marks	Marker's Comments
a) $3^{2x} - 9.3^{x} - 10 = 0$ $u^2 - 9u - 10 = 0$ (Let $u = 3^x$) $(u-10)(u+1) = 0$ $u = 10$ or -1 But $3^x > 0$ $\therefore 3^x = 10$ only $x = \dfrac{\log 10}{\log 3}$ $= \underline{\underline{2.10}}$ (2DP)	1 1 1	Too many people took logs of $3^{2x} - 3^{x+2}$ and got no marks. Four people arrived at 2.10 with no valid working – no marks.
b) $\dfrac{dy}{dx} = \lim\limits_{h \to 0} \left\{ \dfrac{2^{3(x+h)} - 2^{3x}}{h} \right\} = 2^{3x} \lim\limits_{h \to 0} \left\{ \dfrac{2^{3h} - 1}{h} \right\}$ $= 2^{3x} \lim\limits_{h \to 0} \left\{ \left(\dfrac{2^{3h} - 1}{3h} \right) 3 \right\}$ $= 2^{3x} \lim\limits_{h \to 0} \left(\dfrac{2^{3h} - 1}{3h} \right) . 3$ $= 2^{3x} . 3 . \ln 2 = \underline{\underline{8^x \ln 8}}$	1 1 1	One mark for 1st principles definition. One mark for knowing answer. Some justification required that $\lim\limits_{h \to 0} \dfrac{8^h - 1}{h} = \ln 8$
c) i) $\dfrac{dy}{dx} = \dfrac{(x^3+4)24x - 12x^2(3x^2)}{(x^3+4)^2} = \underline{\underline{\dfrac{-12x^4 + 96x}{(x^3+4)^2}}}$ ii) At S.P., $\dfrac{dy}{dx} = 0 \Rightarrow x(2-x)(x^2+2x+4) = 0$ $\therefore x = 0$ or $x = 2 \Rightarrow$ S.P. at $\underline{\underline{(0,0) \text{ and } (2,4)}}$ iii) At $(0,0)$, $\dfrac{d^2y}{dx^2} = 6$ $\therefore$ Concave up $\therefore$ $\underline{\underline{\text{Local Minimum at } (0,0)}}$ At $(2,4)$, $\dfrac{d^2y}{dx^2} = -2$ $\therefore$ Concave down $\underline{\underline{\text{Local Maximum at } (2,4)}}$	2 2 1 1	$\dfrac{12x(8-x^3)}{(x^3+4)^2}$ better • Too many numerical errors in Quotient Rule. • S.P. are points not x values. • In part (iii) some mention of concavity was required.
iv)  Vert Asy $x = -4^{1/3}$ Horiz Asy $y = 0$ No intercepts except $(0,0)$	½ per asym ½ per SP 1 for shape and axes.	• Scales were scarcer than I expected. At least 0 and 1 other value on each axis.