QUESTION 1 (9 MARKS)

Marks

(a) (i) If $2\tan^2 x - \sec^2 x = 2$, find the exact values of $\tan x$ **2**

(ii) Hence, find the values of x in the domain $0 \le x \le 2\pi$ **2**

(b) The gradient of a function is given by $\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}$ and it passes through the point (4,3). Find the equation of the function. **3**

(c) Solve for x: $\frac{x}{x+3} > 1$ **2**

QUESTION 2 (9 Marks) – START A NEW PAGE

Marks

(a) Using the substitution $u^2 = x$, where $u > 0$, evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ **3**

(b) If $\sin^{-1} t$, $\cos^{-1} t$, and $\sin^{-1}(1-t)$ are acute

(i) Show that $\sin\left(\sin^{-1} t - \cos^{-1} t\right) = 2t^2 - 1$ for $0 < t < 1$. **2**

(ii) Hence, solve the equation: $\sin^{-1} t - \cos^{-1} t = \sin^{-1}(1-t)$ **2**

(c) Find $\lim_{x \to 0} \frac{\sin 3x}{\tan 4x}$ **2**

QUESTION 3 (9 Marks) – START A NEW PAGE

Marks

(a) Use Simpson's rule with five function values to estimate the volume of the solid formed by rotating the curve $y = \frac{2}{1+x^2}$ about the *x*-axis between 2 and –2. **3**

(b) The flow rate of water first into and then out of a horse trough is R, where

$$R = 3\pi t(8 - t)\text{litres/second.}$$

(i) When does water cease to flow into the horse trough **and** how much water has flowed into the trough in this time? Leave your answer in exact form. **3**

(ii) The trough was initially empty. Find how long the trough takes to empty again, **and** the rate at which the trough was then losing water. **3**

QUESTION 4 (9 Marks) – START A NEW PAGE

Marks

(a) If (x_0, y_0) and (x_1, y_1) are two points on the line whose equation is $y = mx + k$, show that the distance between (x_0, y_0) and (x_1, y_1) in terms of x_0 and x_1 is $|x_0 - x_1|\sqrt{1+m^2}$ units. **4**

(b) The area bounded by the curve $y = \frac{x}{\sqrt{1+x}}$, the line $x = 1$ and the *x* axis is rotated about the *x* axis. Find the volume of the generated solid of revolution. **5**

QUESTION 5 (9 Marks) – START A NEW PAGE

Marks

(a) A function is defined by $f(x) = x - \frac{1}{x}$, $x > 0$.

(i) Show that $f(x)$ has an inverse and that this inverse is given by **3**

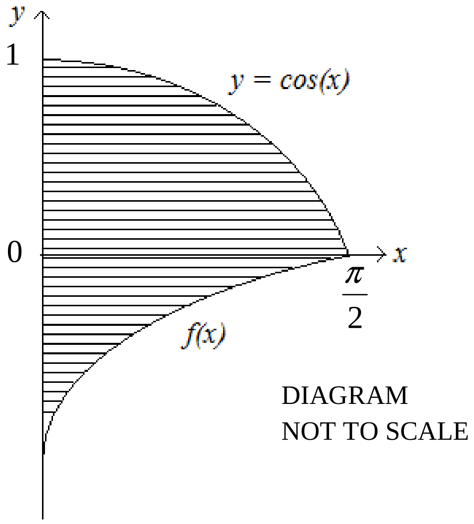
$$g(x) = \frac{x}{2} + \sqrt{1 + \frac{x^2}{4}}.$$

(ii) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same set of axes.

State the domain and range of $g(x)$. **3**

(b) The shaded area enclosed by the curves $y = \cos(x)$ and $y = f(x) = k\left(x - \frac{\pi}{2}\right)^2$ and the y −axis is 4 square units.

Find the exact value of k. **3**



QUESTION 6 (9 Marks) – START A NEW PAGE

Marks

(a) (i) Differentiate $2x\tan^{-1}(2x) - \log_e\sqrt{1+4x^2}$ with respect to x. **2**

(ii) Hence, evaluate $\int_0^{\frac{1}{2}} tan^{-1}(2x)\,dx$ in exact terms. **2**

(b) By finding the intercepts with the x and y-axes and any stationary points and by determining their nature, sketch the curve **5**

$$y = e^x cosx \text{ for } 0 \leq x \leq \frac{\pi}{2}, \text{ neatly.}$$

QUESTION 7 (9 Marks) – START A NEW PAGE **Marks**

(a) Write the general solution in radians for $\cos 2x = -\sin 3x$ **3**

(b) The Agriculture Faculty at James Ruse have a budget of $\$m$ to spend on constructing a rectangular lucerne paddock ABCD as shown in the diagram below. The side AB runs along the edge of the school dam and costs $\$n$ per metre to fence. The remaining three sides of the paddock cost $\$r$ per metre to fence.

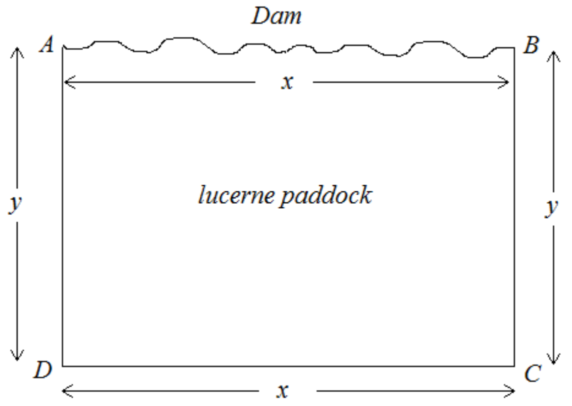


DIAGRAM
NOT TO SCALE

(i) Find in terms of *m*, *n*, *r* and *x*, the width *y* of the paddock. **2**

(ii) Hence, find the area of the paddock in terms of *x*, *m*, *n* and *r*. **1**

(iii) Find the length *(x)* of the paddock in order to maximise the area of lucerne planted. **3**

END OF PAPER

TERM 4 ASSESSMENT 1, 2011

MATHEMATICS EX 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
a) i) $2\tan^2 x - \left(1+\tan^2 x\right) = 2$		
$\tan^2 x = 3$		
$\tan x = \pm\sqrt{3}$	2	
ii) $\tan x = \sqrt{3} \Rightarrow x = \pi/3$ or $4\pi/3$		
$\tan x = -\sqrt{3} \Rightarrow x = 2\pi/3$ or $5\pi/3$		
$\therefore x = \pi/3, 2\pi/3, 4\pi/3$, or $5\pi/3$	2	
b) $\frac{dy}{dx} = x^{1/2} - x^{-1/2}$		
$y = \frac{2x^{3/2}}{3} - 2x^{1/2} + c$	1	Many people did not read the question properly
When $x = 4$, $y = 3$		
$\therefore 3 = \frac{16}{3} - 4 + c$		
$c = 5/3$	1	
$\therefore$ Eqn is $y = \frac{2x^{3/2}}{3} - 2x^{1/2} + \frac{5}{3}$	1	
c) Multiply by $(x+3)^2$ (>0) $(x \neq -3)$		
$x(x+3) > (x+3)^2$	1	
$x^2+3x > x^2+6x+9$		
$3x < -9$		
$x < -3$	1	

MATHEMATICS Extension 1 : Question..2...

Suggested Solutions	Marks	Marker's Comments
(a) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ Let $u^2 = x$. $u>0$		
$\frac{dx}{du} = 2u$		
$dx = 2u\,du$.		
When $x=1$, $u=1$ $x=4$, $u=2$ $u>0$	①	Change Limit values
$\int_1^2 \frac{e^u}{u} . 2u\,du$		
$2\int_1^2 e^u\,du$	①	Substitution
$2\left[e^u\right]_1^2 = 2(e^2 - e)$	①	Integration and answer.
(b) (i) $\sin^{-1}t$, $\cos^{-1}t$ $\sin^{-1}(1-t)$ are acute.		
$-\frac{\pi}{2} < \sin^{-1}t < \pi/2$ ⋮ $-1<t<1$ $0 < \cos^{-1}t < \frac{\pi}{2}$ ⋮ $0<t<1$ $-\frac{\pi}{2} < \sin^{-1}(1-t) < \pi/2$ ⋮ $0<t<2$ $\left.\right\}$ $0<t<1$		* Note. acute angles can be negative
Let $\alpha = \sin^{-1}t$ $\therefore \sin\alpha = t$ $-1<t<1$ $\beta = \cos^{-1}t$ $\therefore \cos\beta = t$ $0<t<1$ $\therefore 0<t<1$		
$\cos^2\alpha + \sin^2\alpha = 1$ $\cos\alpha = +\sqrt{1-t^2}$ $-\pi/2 < \alpha < \pi/2$ $\therefore \cos\alpha > 0$		
$\cos^2\beta + \sin^2\beta = 1$ $\sin\beta = +\sqrt{1-t^2}$ $0 < \beta < \pi/2$ $\therefore \sin\beta > 0$		
$\sin(\sin^{-1}t - \cos^{-1}t)$		
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$	($\frac{1}{2}$)	Expansion
$= t \times t - \sqrt{1-t^2} \times \sqrt{1-t^2}$	①	substitution
$= t^2 - 1 + t^2$		
$= 2t^2 - 1$ $0<t<1$	($\frac{1}{2}$)	answer.
		($-\frac{1}{2}$) if $0<t<1$ not mentioned and part (ii) not completed.

MATHEMATICS Extension 1 : Question 2

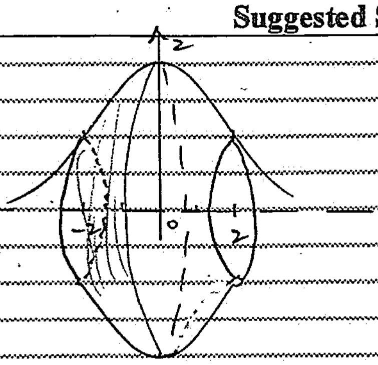
Suggested Solutions	Marks	Marker's Comments
(b)(ii) $\sin^{-1} t - \cos^{-1} t = \sin^{-1}(1-t)$		
consider $\sin(\sin^{-1} t - \cos^{-1} t) = \sin(\sin^{-1}(1-t))$		
$2t^2 - 1 = n\pi + (-1)^n (1-t) \quad n \in Z$		
But $0<t<1$ and $\sin^{-1}(1-t)$ is acute.		
$\therefore 2t^2 - 1 = 1 - t$		
$2t^2 + t - 2 = 0$	½	quadratic equation
$t = \dfrac{-1 \pm \sqrt{1-4(2)(-2)}}{4}$	½	solution
$t = \dfrac{\sqrt{17}-1}{4}$ as $0<t<1$	½	one solution
only	½	for restriction $0<t<1$
(c) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\tan 4x} = \dfrac{3}{4} \lim\limits_{x \to 0} \dfrac{\sin 3x}{3x} \times \dfrac{4x}{\tan 4x}$	½	must show "x's"
$= \dfrac{3}{4} \times 1 \times 1$	½	Must show "1"'s
$= \dfrac{3}{4}$	1	Answer

Y12 ASSESSMENT TEST 1, TERM 4, 2011

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments

3 (a)



* students are NOT reading the Q.

$$\text{VOLUME} = \pi\int_{-2}^{2} y^2\,dx$$

$$= \pi\int_{-2}^{2}\left(\frac{2}{1+x^2}\right)^2 dx \qquad \left(\tfrac{1}{2}\right)$$

$$= 2\pi\int_{0}^{2}\frac{4}{(1+x^2)^2}dx$$

(even function)

$$h = \frac{2-(-2)}{4} = \frac{2-0}{2} = 1$$

x	-2	-1	0	1	2
$y^2 = \frac{4}{(1+x^2)^2}$	$\frac{4}{25}$	1	4	1	$\frac{4}{25}$
	Y_1	Y_2	Y_3	Y_4	Y_5

$\frac{1}{2} + \left(\frac{1}{2}\right)$

$$\therefore V = \pi\int_{-2}^{2} y^2\,dx \doteq \pi \times \frac{h}{3}\left[Y_1 + Y_5 + 4(Y_2 + Y_4) + 2Y_3\right] \qquad 1$$

or equiv. formula

$$= \pi \times \frac{1}{3}\left[\frac{4}{25} + \frac{4}{25} + 4(1+1) + 2\times 4\right]$$

$$= \frac{\pi}{3}\left[\frac{8}{25} + 8 + 8\right]$$

$\boxed{3}$

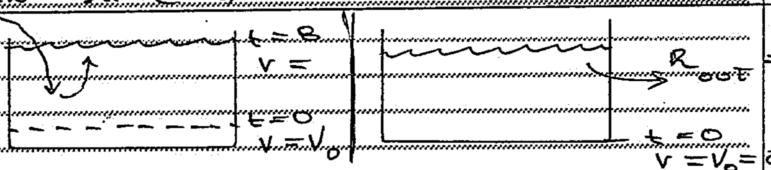
$$\therefore \underline{\text{VOLUME}} = \frac{408\pi}{75} \,/\, \frac{136\pi}{25} \,/\, 5{\cdot}44\pi \text{ units}^3 \qquad 1$$

$-\frac{1}{2}$ if not "volume = ... u^3"

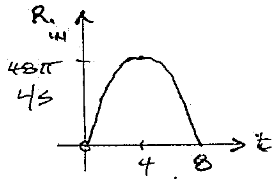
$-\frac{1}{2}$ for 17.09...

Rate in $= 3\pi t(8-t)$

(b) (i)



$R_{out} = -3\pi t\,(8-t)$



cease flow when $R_{IN} = \frac{dV}{dt} = 3\pi t(8-t) = 0 \qquad \frac{1}{2}$

$\therefore t = 0$ or $t = 8$

but $t \neq 0$ $\therefore$ cease flow after 8 seconds $\qquad 1$

$$\text{VOLUME flowed IN} = \int_0^8 R\,dt = \int_0^8 3\pi(8t - t^2)\,dt$$

$$= 3\pi\left[4t^2 - \tfrac{1}{3}t^3\right]_0^8 \qquad \tfrac{1}{2}$$

$$= 3\pi \times \left(256 - \tfrac{512}{3} - 0\right]$$

$\boxed{3}$

$$\therefore \text{VOLUME IN} = 256\pi \text{ L} \qquad 1\tfrac{1}{2}$$

OR $V = \int 3\pi(8t - t^2)\,dt = 3\pi\left(4t^2 - \frac{1}{3}t^3\right) + C$

$= 24\pi t^2 - \pi t^3 + C$

$t = 0 \quad V = V_0 (\geq 0) \quad \therefore C = V_0$

$\therefore V(t) = 24\pi t^2 - \pi t^3 + V_0$

$\therefore$ volume flowed in $V(8) - V(0) = 256\pi$ L.

MATHEMATICS Extension 1 : Question 3

Suggested Solutions | **Marks** | **Marker's Comments**

Q 3(b)(ii) $R_{OUT} = -3\pi t(8-t)$

<u>Data</u>: $t=0 \quad V = V_0 = 0$

$R = \frac{dV}{dt} = -3\pi t(8-t) = 3\pi t^2 - 24\pi t$

$\therefore V = \int 3\pi t^2 - 24\pi t \, dt =$

$V = \pi t^3 - 12\pi t^2 + C$

ie $V = \pi t^3 - 12\pi t^2 + V_0$ — $c = V_0 \equiv 0$ (data)

$\therefore \underline{V = \pi t^3 - 12\pi t^2}$ ✓ — 1

Empty when $V=0$

$\therefore \pi t^2(t-12) = 0$

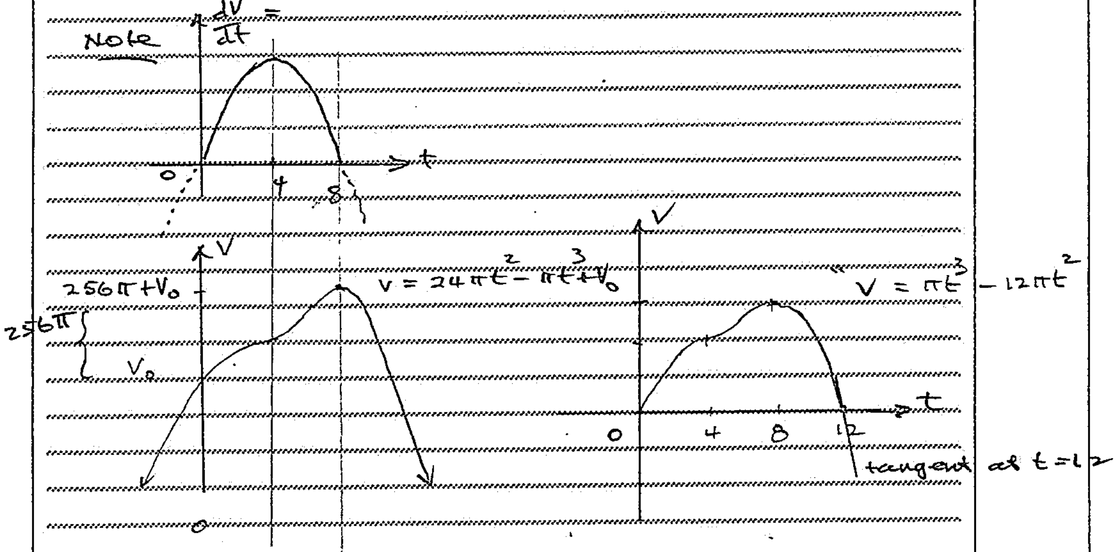
$\therefore t=0$ or $t=12$ but $t>0$

$\therefore$ <u>after 12sec</u> no volume in trough — 1 — $\boxed{3}$

$\therefore$ Rate $= -3\pi \times 12(8-12) = 144\pi$

$\therefore$
- flow rate <u>out</u> $= 144\pi L/s$
- trough <u>lossing</u> <u>at $144\pi L/s$</u> — 1 — $-\frac{1}{2}$ if wording incorrect
- "Rate is $-144\pi L/s$"

Note



b(i) b(ii)

Relative sketch

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks Awarded	Marker's Comments

a) Sub (x_0, y_0) into $y = mx + k$.

$y_0 = mx_0 + k$. —— eq 1

Sub (x_1, y_1) into equation

$y_1 = mx_1 + k$ —— eq 2. } ✓

eq ① − ②

$y_0 - y_1 = m(x_0 - x_1)$. — eq 3 ✓

Now $d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$ — eq 4

sub ③ into ④

$$d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$$

$$= \sqrt{(x_0-x_1)^2 + m^2(x_0-x_1)^2}$$ ✓

$$= \sqrt{(1+m)^2\,(x_0-x_1)^2}$$

as $\sqrt{(x_0-x_1)^2} = |x_0-x_1|$ reason compulsory $(\frac{1}{2}$ mark$)$

~~$= \sqrt{(x_0-x_1)^2} \cdot \sqrt{1+m^2}$~~

$$= |x_0-x_1|\sqrt{1+m^2}$$

Alternatively:

② Some used $m = \dfrac{y_0 - y_1}{x_0 - x_1}$

$\therefore\ y_0 - y_1 = m(x_0 - x_1)$

③ Some divided the distance formula.

$$d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$$

by $\sqrt{(x_0-x_1)^2}$ and times by the same amount

$$d = \sqrt{\frac{(x_0-x_1)^2}{(x_0-x_1)^2} + \frac{y_0-y_1}{(x_0-x_1)^2}} \times \sqrt{(x_0-x_1)^2}$$

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks Awarded	Marker's Comments
b) . $V = \pi \int_0^1 \frac{x^2}{1+x}\,dx$ ✓ $= \pi \int_0^1 \left((x-1) + \frac{1}{x+1}\right) dx$ ✓ $= \pi \left[\frac{x^2}{2} - x + \ln(x+1)\right]_0^1$ ✓ $= \pi \left[\frac{1}{2} - 1 + \ln 2\right) - \left(0 - 0 + \ln 1\right)$ ✓ $= \pi \left[\ln 2 - \frac{1}{2}\right]$ unit^3. ✓		- show multiplied by π and squared y - limits $\int_0^1$ * if no dx or limit lost $\left(-\frac{1}{2}\right)$ Long division $x+1 \overline{)\, x^2+0x+0}$ quotient $x-1$ x^2+x $0 - x$ $-x-1$ 1 $\therefore \frac{x^2}{x+1} = (x-1) + \frac{1}{x+1}$
Alternatively Method 2 . Let $u = x+1$ $x = u-1$. $x^2 = u^2-2u+1$ when $x=0 \Rightarrow u=1$ $x=1 \Rightarrow u=2$. $\therefore V = \pi \int_1^2 \frac{u^2-2u+1}{u}\,du$. Method 3. $V = \pi \int \frac{x^2}{1+x} = \int_0^1 \frac{x^2+2x+1}{x+1} - \frac{2x+1}{x+1}\,dx$ $= \int_0^1 x+1\,dx - \int \frac{2x+2}{x+1}dx + \int \frac{1}{x+1}\,dx$ OR . $V = \pi \int \frac{x^2 dx}{1+x} = \int \frac{x^2-1+1}{1+x} dx = \int \frac{x^2-1}{1+x} dx + \int \frac{1}{1+x}\,dx$		Some gave: -0.19314718π unit^3 or 0.606789763 unit^3 - accepted both. Note. * If no π and not squared. and simplified their ~~answers~~ working to get answer — got maximum of 2 marks * Marks taken off for simplifying their working making question easier.
Some used $\tan^{-1}$ Note instead of getting $\int \frac{x^2}{1+x}\,dx$ you have taken $\int \frac{x^2}{1+x^2}\,dx$ Here the maximum marks you got was 3 marks.		

Q5

(a) (i) $f(x) = x - \frac{1}{x}$ where $x > 0$

let $y = f(x)$

$\therefore y = x - \frac{1}{x}$

inverse function: $x = y - \frac{1}{y}$ ½ $\therefore y > 0$

Range is the ½
domain of the
original functi

$yx = y^2 - 1$

$\therefore 0 = y^2 - yx - 1$ ½

$\therefore y = \frac{x \pm \sqrt{x^2 - 4(1)(-1)}}{2}$ ½

③

$y = \frac{x}{2} \pm \frac{\sqrt{x^2+4}}{2}$ ½

$\therefore y = \frac{x}{2} + \frac{\sqrt{x^2+4}}{2}$ as $y > 0$

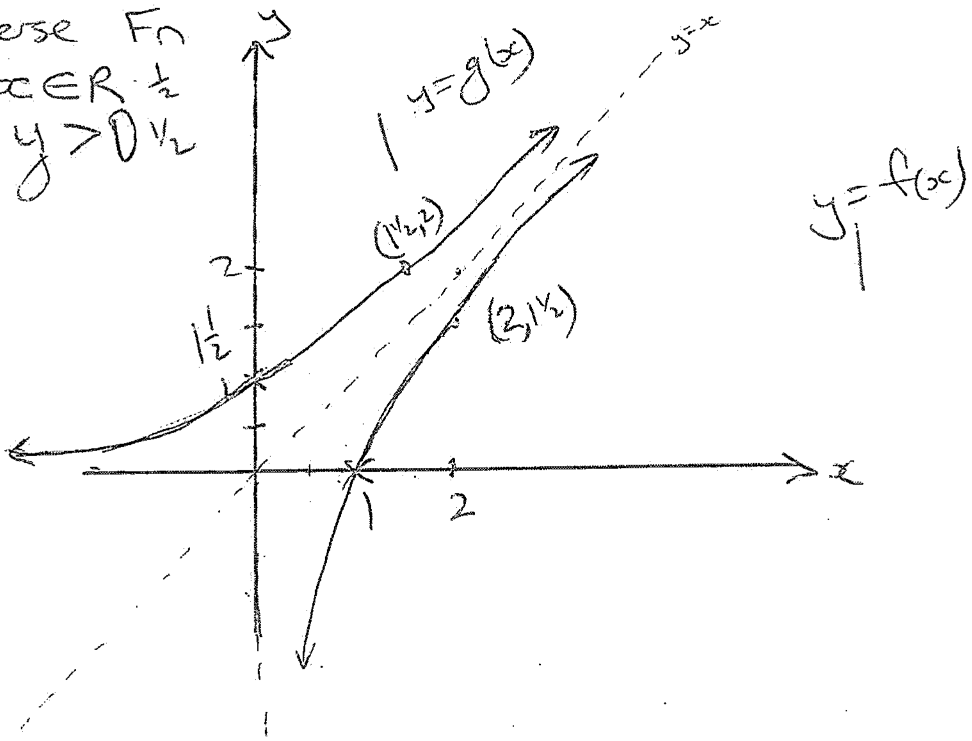
$y = \frac{x}{2} + \frac{1}{2} \cdot 2\sqrt{\frac{x^2}{4}+1}$ ½

$y = \frac{x}{2} + \sqrt{\frac{x^2}{4}+1}$

$\therefore g(x) = \frac{x}{2} + \sqrt{\frac{x^2}{4}+1}$

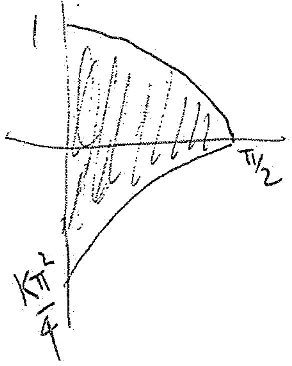
(ii)

Inverse Fn
D: $x \in R$ ½
R: $y > 0$ ½



③

(b)



$$\text{Area} = \overset{1/2}{\int_0^{\pi/2} \cos x\,dx} - K\int_0^{\pi/2} (x - \pi/2)^2\,dx$$

$$\therefore 4 = \Big[\sin x\Big]_0^{\pi/2} - \frac{K}{3}\Big[(x-\pi/2)^3\Big]_0^{\pi/2} \quad \frac{1}{2}$$

$$4 = \sin\pi/2 - \sin 0 - \frac{K}{3}\Big[(\tfrac{\pi}{2}-\pi/2)^3 - (0-\pi/2$$

$$4 = 1 - \frac{K}{3}\Big[\frac{\pi^3}{8}\Big] \quad \frac{1}{2}$$

$$9 = -K\left(\frac{\pi^3}{8}\right)$$

③

$$-\frac{72}{\pi^3} = K \quad 1/2$$

$$\therefore K = \frac{-72}{\pi^3}$$

* 1/2 off for each mistake.

<u>Alternative method</u>

$$\text{Area} = \int_0^{\pi/2} \cos x\,dx + \left|K\int_0^{\pi/2}(x-\pi/2)^2\,dx\right|$$

very similar to above but when they remove the absolute value signs they need to take the negative case which nearly all students did not do!!

Yr 11 yrly 2011 Q6

4m a i) $y' = \underbrace{\frac{2x\cdot 2}{1+4x^2} + 2\tan^{-1}(2x)}_{1m} - \underbrace{\frac{8x}{(1+4x^2)} \times \frac{1}{2}}_{1m}$

If final answer is wrong −1 m

$y' = 2\tan^{-1}(2x)_{\#}$

ii) $\int_0^{\frac{1}{2}} \tan^{-1}(2x)\,dx = \frac{1}{2}\left[2x\tan^{-1}2x - \ln\sqrt{1+4x^2}\right]_0^{\frac{1}{2}}$ 1m

MANY forgot $\frac{1}{2}$

$= \frac{1}{\cancel{2}}\tan^{-1}1 - \frac{1}{4}\ln 2 + \cancel{\frac{\ln 1}{4}}^{\,0}$

$= \frac{\pi}{8} - \frac{1}{4}\ln 2$ 1m

#

5m b) $y = e^x\cos x \quad 0 \le x \le \frac{\pi}{2}$

$y' = e^x(\cos x - \sin x)$ $\frac{1}{2}$m

SP $y' = 0$ when $\cos x = \sin x$ $\frac{1}{2}$m $\frac{1}{2}$m

i.e. $\tan x = 1$ $x = \frac{\pi}{4},\ y = \frac{e^{\frac{\pi}{4}}}{\sqrt{2}}$ $(\doteqdot 1.551)$

$y'' = e^x[-\sin x - \cancel{\cos x}] + e^x(\cancel{\cos x} - \cancel{\sin x}]$

$y'' = -2e^x\sin x$ $\frac{1}{2}$m

when $x = \frac{\pi}{4}$ $y'' = -\frac{2e^{\frac{\pi}{4}}}{\sqrt{2}} < 0$ $\therefore$ concave down

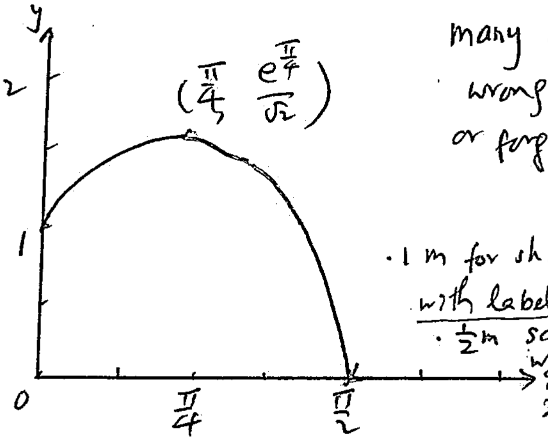
$\frac{1}{2}$m rel max at $\left(\frac{\pi}{4}, \frac{e^{\frac{\pi}{4}}}{\sqrt{2}}\right)$

when $x = 0$, $y = 1$ $\frac{1}{2}$m

$y = 0$, $x = \frac{\pi}{2}$ $\frac{1}{2}$m

MAX 4 out of 5 for part b

~~for~~ $y = e^{\frac{\pi}{4}}\cos\frac{\pi}{4}$ without simplify to $e^{\frac{\pi}{4}}/\sqrt{2}$

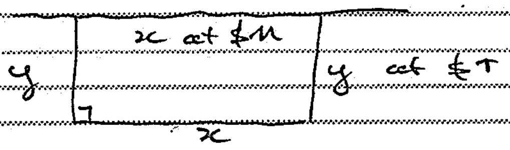


many have wrong scale $-\frac{1}{2}$m

or forgot origin $-\frac{1}{2}$m

- 1 m for shape with labelled TP & intercepts
- $\frac{1}{2}$m scale with correct graph

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
Q7(a) $\cos 2x = -\sin 3x$ $\equiv \sin(-3x)$ ODD function $= \cos\left[\frac{\pi}{2} - (-3x)\right]$ $\cos 2x = \cos\left(\frac{\pi}{2} + 3x\right)$ $\therefore\ 2x = 2m\pi \pm \left(\frac{\pi}{2} + 3x\right)$ $2x = 2m\pi + \frac{\pi}{2} + 3x$ or $2x = 2m\pi - \frac{\pi}{2} - 3x$ $\therefore\ x = \begin{cases} -(4m+1)\frac{\pi}{2} \\ (4m-1)\frac{\pi}{10} \end{cases},\ m \in \mathbb{Z}$ ∘ IF $\sin\left(\frac{\pi}{2} - 2x\right) = \sin(-3x)$ from $\cos 2x = -\sin 3x$ $x = \frac{(2m-1)\pi}{2\left[3(-1)^m - 2\right]}$ or $\frac{\left[2m + (-1)^m\right]\pi}{2\left[2(-1)^m - 3\right]}$ ∘ IF $-\cos 2x = \sin 3x$ $= \cos\left(\frac{\pi}{2} - 3x\right)$ $x = \begin{cases} (4m-1)\frac{\pi}{2} \\ (3-4m)\frac{\pi}{10} \end{cases}\ m \in \mathbb{Z}.$	1 1 each	$\boxed{3}$
(b) (i)  $m = ux + vx + 2vy$ $m = (u+v)x + 2vy$ $\therefore\ y = \frac{m - (u+v)x}{2v}$	1 1	$\boxed{2}$
(ii) $A = xy$ —— (1) $y = \frac{m - (u+v)x}{2v}$ —— (2) $\therefore\ A(x) = x\left[\frac{m - (u+v)x}{2v}\right]$ $A(x) = \frac{m}{2v}x - \frac{(u+v)}{2v}x^2$	1	$\boxed{1}$

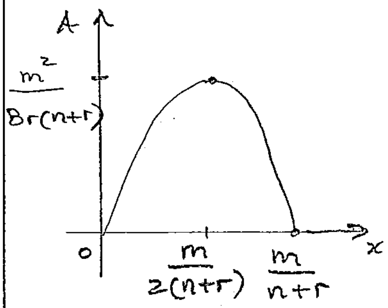
(iii) PTO.

MATHEMATICS Extension 1 : Question 7

Suggested Solutions | **Marks** | **Marker's Comments**

Q 7 (b) (iii) $A(x) = \frac{m}{2r}x - \frac{(n+r)}{2r}x^2$

$0 \le x \le \frac{m}{n+r}$



METHOD 1 From Quadratic Polynomials

As $a = -\frac{(n+r)}{2r} < 0$

∴ concave downwards

Hence a maximum value will exist

at $x = -\frac{b}{2a}$ or $\frac{\alpha+\beta}{2}$

i.e $x = \frac{m}{2(n+r)}$

∴ $y = \frac{m}{2r} - \frac{n+r}{2r} \times \frac{m}{2(n+r)} = \frac{m}{4r}$

$A = \frac{m}{2(n+r)} \times \frac{m}{4r} = \frac{m^2}{8r(n+r)}$

∴ abs max Area when $x = \frac{m}{2(n+r)}$

$a = -\frac{(n+r)}{2r}$; $b = \frac{m}{2r}$

$\alpha = 0$; $\beta = \frac{m}{n+r}$.

METHOD 2 $A = \frac{m}{2r}x - \frac{(n+r)}{2r}x^2$

$\frac{dA}{dx} = \frac{m}{2r} - \frac{(n+r)}{r}x$ 1

$\frac{d^2A}{dx^2} = -\frac{(n+r)}{r}$

For possible max/min values of A to occur $\frac{dA}{dx} = 0$

∴ $\frac{m}{2r} - \frac{(n+r)x}{r} = 0$

∴ $x = \frac{m}{2(n+r)}$ 1

$Y = \frac{m}{4r}$, $A = \frac{m^2}{8r(n+r)}$

$\boxed{3}$

TEST nature

As $\frac{d^2A}{dx^2} = -\frac{(n+r)}{r} < 0$; r and n > 0

∴ concave downwards and since a SP at $x = \frac{m}{2(n+r)}$

∴ a Rel. min TP at $x = \frac{m}{2(n+r)}$ 1

Since continuous and no other SPs for $x \ge 0$

∴ the abs. max. Area occurs when $x = \frac{m}{2(n+r)}$

OR

x	$\frac{m}{4(n+r)}$	$\frac{m}{2(n+r)}$	$\frac{m}{(n+r)}$
← $\frac{dA}{dx}$	$\frac{m}{4r}$	0	$-\frac{m}{2r}$
	⊕		⊖
	/	—	\

∴ a Rel. min TP at $x = \frac{m}{2(n+r)}$

etc: see above