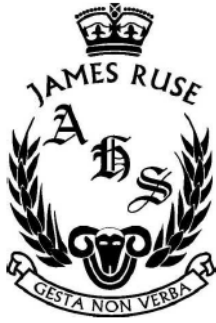


Student Number _____

Class _____



2014

Higher School Certificate

Assessment 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Board approved calculators may be used
- Write using black or blue pen
- Marks may not be awarded for careless or badly arranged work
- A table of standard integrals is provided
- All necessary working should be shown

Write your student number and/or name at the top of every page

Total marks – 63

This examination consists of 7 questions worth 9 marks each.

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc.

This paper MUST NOT be removed from the examination room.

Question 1 (Start a new page)**9 Marks**

- (a) Given that $f(x) = \sin^{-1}x$, find $f'(\frac{1}{2})$ 2
- (b) Given $y = \tan^{-1}\left(\frac{a}{x}\right)$, show that $\frac{dy}{dx} = \frac{-a}{x^2 + a^2}$ 1
- (c) Evaluate $\int_1^4 y \, dx$ if $xy = 1$. Leave answer in exact form. 2
- (d) Find $\int \cos^2(2x) \sin(2x) \, dx$ 2
- (e) Find the coefficient of x^3 in the expansion of $(3 - 2x)(x + 1)^4$ 2

Question 2 (Start a new page)**9 Marks**

- (a) (i) Differentiate $y = x \sin^{-1}x + \sqrt{1 - x^2}$ 2
- (ii) Hence evaluate $\int_0^1 \sin^{-1}x \, dx$ 1
- (b) Use the substitution $u = \sin^2x$ to evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \sin^2 x} dx$ 4
- (c) The curve $y = \frac{1}{\sqrt{1+x^2}}$ is rotated about the x-axis. Find the volume of revolution of the solid enclosed by $x = \frac{1}{\sqrt{3}}$ and $x = \sqrt{3}$ 2

Question 3 (Start a new page)**9 Marks**

- (a) The function $f(x) = \sec x$ for $0 \leq x < \frac{\pi}{2}$, and is not defined for other values of x .
- (i) State the domain and range of the inverse function $f^{-1}(x)$ 2
- (ii) Show that $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$ 1
- (iii) Hence find $\frac{d}{dx} f^{-1}(x)$ 1
- (b) Find the exact values of x and y which satisfy the simultaneous equations
- $$\sin^{-1}x + \frac{1}{2} \cos^{-1}y = \frac{\pi}{3} \quad \text{and}$$
- $$3 \sin^{-1}x - \frac{1}{2} \cos^{-1}y = \frac{2\pi}{3}$$
- (c) Find and simplify the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ 3

Question 4 (Start a new page)**9 Marks**

- (a) Evaluate $\int_1^e \frac{1}{t} dt$
1
- (b) The rate of descent of a submarine into an ocean, starting from the surface, is given in terms of the descent t seconds by $\frac{dh}{dt} = 1 - (1 + t)^{-2}$, where h is the depth of the submarine in metres.
- (i) Find an expression for h in terms of t . 2
- (ii) Hence find the depth of the submarine after 1 minute. 1
- (c) (i) Sketch the curve $y = (x - 1)^2 - 2$ 1
- (ii) Find the largest positive domain such that the graph defines a function $y = f(x)$ which has an inverse. 1
- (iii) Write down the equation of its inverse function $y = f^{-1}(x)$ and state its domain and range. 2
- (iv) On the same graph as (i), draw a neat sketch of $y = f^{-1}(x)$. 1

Question 5 (Start a new page)**9 Marks**

- (a) Use the substitution $u = x + 1$ to evaluate $\int_0^1 \frac{x}{\sqrt{x+1}} dx$ 3
- (b) (i) Write the expansion $(x + 2y)^{10}$ in the form $\sum_{k=0}^n T_k$
1
- (ii) Find the ratio $\frac{T_{k+1}}{T_k}$ 2
- (iii) Find the greatest term if $x = \frac{1}{2}$ and $y = \frac{1}{3}$. 1
- (c) Prove that $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$ 2

Question 6 (Start a new page)**9 Marks**

- (a) Find $\int \frac{dx}{x(\log_e x)^{11}}$ using the substitution $u = \log_e x$ 3
- (b) If $(1 + px)^n = 1 + 15x + 90x^2 + \dots$ find the values of p and n . 3
- (c) Given $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$,
Show that: $x^2 + y^2 + z^2 + 2xyz = 1$ 3

Question 7 (Start a new page)**9 Marks**

- (a) Express $\cos(\tan^{-1} x)$ in terms of x , $0 \leq x \leq \frac{\pi}{2}$ 1
- (b) By using the binomial expansion, show that
- (i) $(q + p)^n - (q - p)^n = 2\binom{n}{1} q^{n-1}p + 2\binom{n}{3} q^{n-3}p^3 + \dots$ 2
- (ii) What is the last term in the expansion when n is odd? 1
- (iii) What is the last term in the expansion when n is even? 1
- (c) (i) Differentiate $y = \tan^{-1} \frac{1}{x}$, $x \neq 0$, and hence show that 2
- $$\frac{d}{dx} \left(\tan^{-1} x + \tan^{-1} \frac{1}{x} \right) = 0$$
- (ii) Sketch the curve $y = \left(\tan^{-1} x + \tan^{-1} \frac{1}{x} \right)$ 2

END OF EXAMINATION

MATHEMATICS Extension 1 : Question...1....

Suggested Solutions

Marks

Marker's Comments

$$a) \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

1

Well done

$$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1-\frac{1}{4}}}$$

$$= \frac{2}{\sqrt{3}}$$

1

Some students introduce a $\pm$ or $||$ which were irrelevant

~~some~~

$$(b) \quad f(x) = \frac{-\frac{a}{x^2}}{1 + \frac{a^2}{x^2}}$$

$$= \frac{-a^2}{x^2 + a^2}$$

1

well done

$$(c) \quad y = \frac{1}{x}$$

$$\int_1^4 \frac{1}{x} dx = [\ln x]_1^4$$

$$= \ln 4 - \ln 1$$

$$= \ln 4 \text{ OR } 2 \ln 2$$

1

1

$$(d) \quad \int \cos^2 2x \sin 2x dx$$

$$= -\frac{\cos^3 2x}{6} + C$$

1 for $\cos^3 2x$

1 for correct $-\frac{1}{6}$

poorly done
students spent a lot of time not going anywhere

students didn't recognize this

and did a number of conversions
please note the relation

$$\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2} \text{ — must be proven.}$$

MATHEMATICS Extension 1 : Question.....

Suggested Solutions

Marks

Marker's Comments

$$(e) (3-2x)(1+4x+6x^2+4x^3+x^4)$$

coefficient of x^3

$$3 \times 4 + -2 \times 6$$

$$= 12 - 12$$

$$= 0$$

1

Students tried to use formulae there who expanded more likely to get correct answer.

1

Students stated $12x^3 - 12x^3 = 0$ but never commented on the coefficient

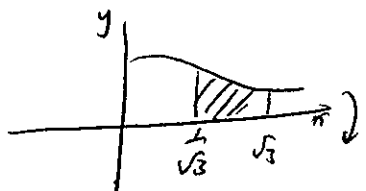
a) $y = x \sin^{-1} x + \sqrt{1-x^2}$

$$y' = \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x + \frac{-2x}{2\sqrt{1-x^2}}$$

$$y' = \sin^{-1} x$$

i) $\int_0^1 \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1-x^2} \Big|_0^1$
 $= (\sin^{-1} 1 + 0) - (0 + 1)$
 $= \frac{\pi}{2} - 1$

b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \sin^2 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2 \sin x \cos x}{1 + \sin^2 x} dx$
 $= \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{du}{1+u} = \ln(1+u) \Big|_{\frac{1}{2}}^{\frac{3}{4}}$ $\left\{ \begin{array}{l} u = \sin^2 x \\ x = \frac{\pi}{4}, u = \frac{1}{2} \\ x = \frac{\pi}{3}, u = \frac{3}{4} \end{array} \right.$
 $= \ln \frac{3}{2} - \ln \frac{1}{2} = \ln \frac{3}{1} = \ln 3$
 $= \ln \left(\frac{3}{1} \right) \#$



c) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{\pi dx}{1+x^2}$
 $= \pi \tan^{-1} x \Big|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$
 $= \pi (\tan^{-1} \sqrt{3} - \tan^{-1} \frac{1}{\sqrt{3}})$
 $= \pi (\frac{\pi}{3} - \frac{\pi}{6})$
 $= \frac{\pi^2}{6} \text{ mark 3} \#$

1m for $\frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$

or
1m for $\frac{-2x}{2\sqrt{1-x^2}}$

2m for getting everything correct

1m

1m for changing the limit $u = \frac{1}{2}$ & $u = \frac{3}{4}$

1m for $\frac{du}{1+u}$

1m for $\ln(1+u)$

1m for simplifying answer

accept 0.154 (3dp)

many students wrote

$\int_{\frac{1}{2}}^{\frac{3}{4}} \frac{du}{1+u} = \tan^{-1} u \Big|_{\frac{1}{2}}^{\frac{3}{4}}$ mark 2

1m for $\pi \cdot \tan^{-1} x$

1m for $\frac{\pi^2}{6}$

many student forgot $\frac{\pi^2}{6}$

wrote $\frac{\pi}{6}$.

MATHEMATICS Extension 1 : Question 3.....

Suggested Solutions	Marks	Marker's Comments
(a) (i) $y = \sec x$ D: $0 \leq x < \pi/2$ R: $y \geq 1$ ∴ Inverse function is D: $x \geq 1$ R: $0 \leq y < \pi/2$ (ii) $y = \sec x$ For the inverse function, interchange x and y. $\therefore x = \sec y$ $x = \frac{1}{\cos y}$ $\cos y = \frac{1}{x}$ $\therefore y = \cos^{-1}\left(\frac{1}{x}\right)$ $\therefore f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$ (iii) $\frac{d}{dx} \left(\cos^{-1} \frac{1}{x} \right) = \frac{-1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}} \times \frac{-1}{x^2}$ $= \frac{1}{x^2 \sqrt{1 - \frac{1}{x^2}}}$ $= \frac{1}{x \sqrt{x^2 - 1}} \quad x > 1$ (b) $\sin^{-1} x + \frac{1}{2} \cos^{-1} y = \frac{\pi}{3} \dots \dots \textcircled{1}$ $3 \sin^{-1} x - \frac{1}{2} \cos^{-1} y = \frac{2\pi}{3} \dots \dots \textcircled{2}$ $\textcircled{1} + \textcircled{2}$ $4 \sin^{-1} x = \pi$ $\sin^{-1} x = \frac{\pi}{4}$ $\therefore x = \frac{1}{\sqrt{2}}$	① ① ① ①	① mark each; totally right to get the mark. * some students did not write what they were doing * some students tried to work backwards from the answer and scored zero! * A lot of students forget to multiply by $\frac{-1}{x^2}$!! ①

MATHEMATICS Extension 1 : Question 3..... continued.

Suggested Solutions	Marks	Marker's Comments
sub $x = \frac{1}{\sqrt{2}}$ into ① $\therefore \frac{\pi}{4} + \frac{1}{2} \cos^{-1} y = \frac{\pi}{3}$ $\frac{1}{2} \cos^{-1} y = \frac{\pi}{6}$ $\therefore \cos^{-1} y = \frac{\pi}{3}$ $y = \frac{\sqrt{3}}{2}$	①	
(c) $T_k = {}^9C_k \left(\frac{3x^2}{2}\right)^{9-k} \left(\frac{-1}{3x}\right)^k$ $= {}^9C_k (-1)^k 3^{9-k} \cdot 2^{k-9} x^{18-3k}$ $= {}^9C_k (-1)^k \cdot 3^{9-2k} \cdot 2^{k-9} x^{18-3k}$	①	
constant term is when x^0 i.e. $x^{18-3k} = x^0$ $18-3k=0$ $k=6$	①	
$\therefore$ coefficient $= {}^9C_6 (-1)^6 3^{-3} \cdot 2^{-3}$ $= \frac{7}{18}$	①	← don't leave it like this, the question asked for the <u>simplified term</u>.

Suggested Solutions

Marks

Marker's Comments

$$\begin{aligned}
 \text{a) } \int_1^e \frac{dt}{t} &= [\ln t]_1^e \\
 &= \ln e - \ln 1 \\
 &= 1 - 0 = \underline{\underline{1}}
 \end{aligned}$$

1

Better not to leave answer in form $\ln e$

$$\begin{aligned}
 \text{b) i) } \frac{dh}{dt} &= 1 - \frac{1}{(1+t)^2} \\
 \therefore h &= t + \frac{1}{(1+t)} + c
 \end{aligned}$$

But, when $t=0, h=0 \therefore c=-1$

$$\therefore h = t + \frac{1}{1+t} - 1 = \underline{\underline{\frac{t^2}{1+t}}}$$

1

Surprising number of people forgot the constant of integration.

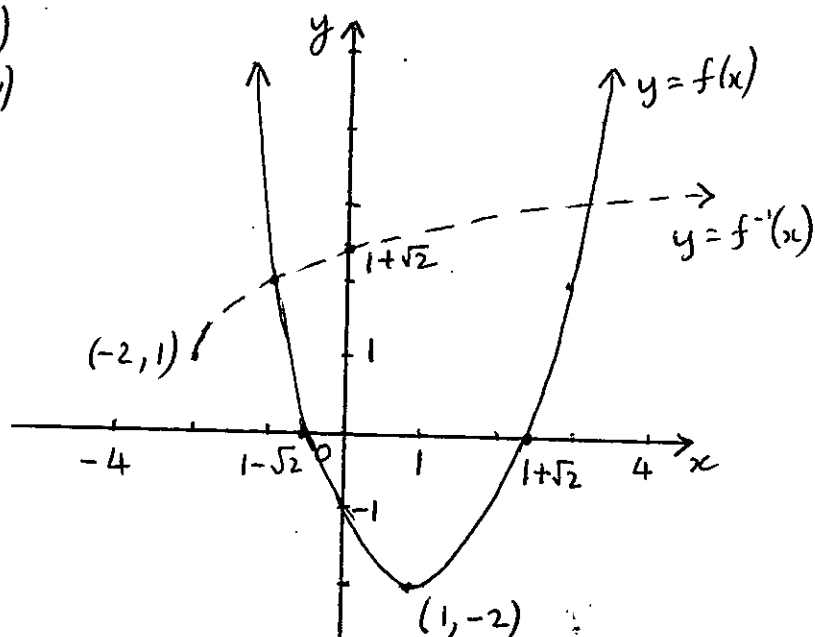
ii) When $t=60$ (seconds!)

$$h = \frac{60^2}{61} = \underline{\underline{59\frac{1}{61} = 59.02 \text{ m}}}$$

1

No marks were awarded if a 'c' left in the answer.

c) i)
iv)



1

If no intercepts shown on curve, a mark was lost.

1

The curve $y = f'(x)$ should be vertical at $(-2, 1)$. Few showed this.

$y = f'(x)$ is increasing for all $x \geq -2$

Suggested Solutions

Marks

Marker's Comments

c) ii) Largest positive domain
to allow inverse is
 $\{x: x \geq 1\}$

iii) Swap x, y in equation
and make y subject.

$$x = (y-1)^2 - 2$$

$$x+2 = (y-1)^2$$

$$(y-1) = \pm \sqrt{x+2}$$

$$y = 1 \pm \sqrt{x+2}$$

But if $x \geq 1$ in $f(x)$
then $y \geq 1$ in $f^{-1}(x)$

$\therefore$ use + sign.

$$\underline{\underline{y = 1 + \sqrt{x+2}}}$$

Domain $x \geq -2$

Range $y \geq 1$

Equality needed
to be included

1

1

1

Question 5

	Solution	Marks	Comments
(a)	$I = \int_0^1 \frac{x}{\sqrt{x+1}} dx$ Let $u = x + 1 \Rightarrow du = dx$ and $x = u - 1$ When $x = 0$, $u = 1$ And $x = 1$, $u = 2$ $\therefore I = \int_1^2 \frac{u-1}{\sqrt{u}} du$ $= \int_1^2 (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du$ $= \left[\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^2$ $= \left(\frac{2}{3} \sqrt{8} - 2\sqrt{2} \right) - \left(\frac{2}{3} - 2 \right)$ $= \frac{4-2\sqrt{3}}{3}$	1 1 1	Correctly changing the limits of integration and correctly replacing x by u. Correctly integrating Correctly substituting the upper and lower limits
(b) (i)	$(x+2y)^{10} = \sum_{k=0}^{10} \binom{10}{k} x^k (2y)^{10-k}$ $= \sum_{k=0}^{10} \binom{10}{k} 2^{10-k} x^k y^{10-k}$ Where $T_k = \binom{10}{k} 2^{10-k} x^k y^{10-k}$.	1	Correct expression for T_k
(ii)	Now $T_{k+1} = \binom{10}{k+1} 2^{9-k} x^{k+1} y^{9-k}$ i.e. $\frac{T_{k+1}}{T_k} = \frac{\binom{10}{k+1} 2^{9-k} x^{k+1} y^{9-k}}{\binom{10}{k} 2^{10-k} x^k y^{10-k}}$ $= \frac{10!}{(k+1)!(9-k)!} \cdot \frac{1}{2} \cdot \frac{x}{y} \div \frac{10!}{k!(10-k)!}$	1	Correctly obtaining an un-simplified expression for the ratio

	$= \frac{k!(10-k)!}{(k+1)!(9-k)!} \cdot \frac{x}{2y}$ $= \frac{k!(10-k)(9-k)!}{(k+1)k!(9-k)!} \cdot \frac{x}{2y}$ $= \frac{(10-k)}{2(k+1)} \cdot \frac{x}{y}$ NOTE: (α) If $T_k = \binom{10}{k} x^{10-k} (2y)^k$, then $\frac{T_{k+1}}{T_k} = \frac{(11-k)}{k} \cdot \frac{2x}{y}$ (β) If $T_k = \binom{10}{k} x^k (2y)^{10-k}$, then $\frac{T_{k+1}}{T_k} = \frac{(10-k)}{k+1} \cdot \frac{x}{2y}$	1	Correct final expression
(iii)	For the greatest term, when $x = \frac{1}{2}, y = \frac{1}{3}$, $\frac{T_{k+1}}{T_k} > 1$ i.e. $\frac{(10-k)}{2(k+1)} \cdot \frac{\frac{1}{2}}{\frac{1}{3}} > 1$ $\frac{10-k}{k+1} > \frac{4}{3}$ $30 - 3k > 4k + 4$ $k < 3\frac{5}{7}$ Hence $k = 3$ The greatest term is $T_4 = \binom{10}{4} 2^6 \left(\frac{1}{2}\right)^4 \left(\frac{1}{3}\right)^6$ $= \frac{\binom{10}{4} 4}{729}$ $= \frac{280}{243}$	1	Correct greatest term

(c)	<u>R.T.P.:</u> $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$ <u>Proof:</u> <u>Method i:</u> LHS $= \frac{n!(n-r+1)}{r!(n-r+1)!} + \frac{n! r}{r!(n-r+1)!}$ $= \frac{n!(n-r+1)+n!r}{r!(n-r+1)!}$ $= \frac{n!(n-r+1+r)}{r!(n-r+1)!}$ $= \frac{n!(n+1)}{r!(n-r+1)!}$ $= \frac{(n+1)!}{r!(n-r+1)!} = \text{RHS}$ <u>Method ii:</u> $\frac{n!}{r!(n-r)!} = {}^nC_{n-r}$ and $\frac{n!}{(r-1)!(n-r+1)!} = {}^nC_{n-r+1}$ also, $\frac{(n+1)!}{r!(n-r+1)!} = {}^{n+1}C_{n-r+1}$ But ${}^nC_{n-r} + {}^nC_{n-r+1} = {}^{n+1}C_{n-r+1}$ (${}^{n+1}C_k = {}^nC_{k-1} + {}^nC_k$ Pascal's triangle result: each coefficient is the sum of the two coefficients in the row above it) Hence $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$ is true.	1 1 2	For correctly rewriting $r!$ and $(n - r + 1)!$ For correctly simplifying and arriving at the desired result
-----	--	----------------------------	--

EXTENSION I
MATHEMATICS: Question... 6...

Suggested Solutions	Marks	Marker's Comments
(a) $\int \frac{dx}{x(\log_e x)^{10}}$ $u = \log_e x$ $\frac{du}{dx} = \frac{1}{x}$ $= \int \frac{1}{(\log_e x)^{10}} \cdot \frac{1}{x} dx$ $= \int \frac{1}{u^{10}} du$ $= \int u^{-10} du$ $= \frac{u^{-9}}{-9} + C$ $= -\frac{1}{9(\ln x)^9} + C$	① ① ①	$\frac{du}{dx}$ correct substitution correct answer No Loss for no +C.
(b) $(1+px)^n = 1 + 15x + 90x^2 \dots$ $= {}^nC_0 + {}^nC_1 px + {}^nC_2 p^2 x^2 \dots$ $\therefore$ Equate terms of equal powers of x ${}^nC_1 p = 15$ and ${}^nC_2 p^2 = 90$ $\frac{n!}{(n-1)!} p = 15$ $\frac{n!}{(n-2)!} p^2 = 90$ $pn = 15$ $p^2 n(n-1) = 180$ $p = \frac{15}{n}$ $(\frac{15}{n})^2 n(n-1) = 180$ $\frac{225n - 225}{n^2} = 180$ $45n = 225$ $n = 5$ $\therefore p = 3$	① ① ①	equate terms Expand combination terms correct solution with working
(c) $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ $\cos^{-1} x + \cos^{-1} y = \pi - \cos^{-1} z$ Let $a = \cos^{-1} x$ $0 < a < \pi$ $\cos a = x$ $b = \cos^{-1} y$ $0 < b < \pi$ $\cos b = y$ $c = \cos^{-1} z$ $0 < c < \pi$ $\cos c = z$ $\cos(a+b) = \cos(\pi - c)$ $\cos a \cos b - \sin a \sin b = -\cos c$ $\sin a = +\sqrt{1-x^2}$ as $0 < a < \pi$ $\sin b = +\sqrt{1-y^2}$ as $0 < b < \pi$ $\therefore xy - (\sqrt{1-x^2})(\sqrt{1-y^2}) = -z$ $xy + z = \sqrt{1-x^2} \sqrt{1-y^2}$ $x^2 y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2 y^2$ $\therefore x^2 + y^2 + z^2 + 2xyz = 1$	① ① ①	Applying "cos" to LHS and expanding correctly correct sub of "sin" expressions and cos of RHS *($\cos \pi = -1$) if used. simplifying to get correct answer No Loss for not including restrictions

MATHEMATICS Extension 1 : Question...7...

Suggested Solutions

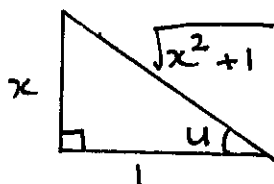
Marks

Marker's Comments

a) $\cos(\tan^{-1}x)$

let $u = \tan^{-1}x$

$\tan u = x$



$\therefore \cos u = \frac{1}{\sqrt{x^2+1}}$

$\therefore \cos(\tan^{-1}x) = \frac{1}{\sqrt{x^2+1}}$

①

b) i) $(q+p)^n - (q-p)^n$

$= {}^nC_0 q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 + \dots + {}^nC_n p^n$

$- ({}^nC_0 q^n - {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 - \dots)$

①

$= 2 {}^nC_1 q^{n-1} p + 2 {}^nC_3 q^{n-3} p^3 + \dots$

$= 2 \binom{n}{1} q^{n-1} p + 2 \binom{n}{3} q^{n-3} p^3 + \dots$

①

ii) $2 \binom{n}{n} p^n = 2p^n$

①

iii) $2 \binom{n}{n-1} q p^{n-1} = 2nq p^{n-1}$

①

c) i) $y = \tan^{-1} \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{1 + (\frac{1}{x})^2} \times \frac{-1}{x^2}$

$= \frac{-1}{x^2+1}$

①

$\frac{d}{dx} (\tan^{-1} \frac{1}{x} + \tan^{-1} x)$

$= \frac{-1}{x^2+1} + \frac{1}{x^2+1}$

$= 0$

①

MATHEMATICS Extension 1 : Question...7...

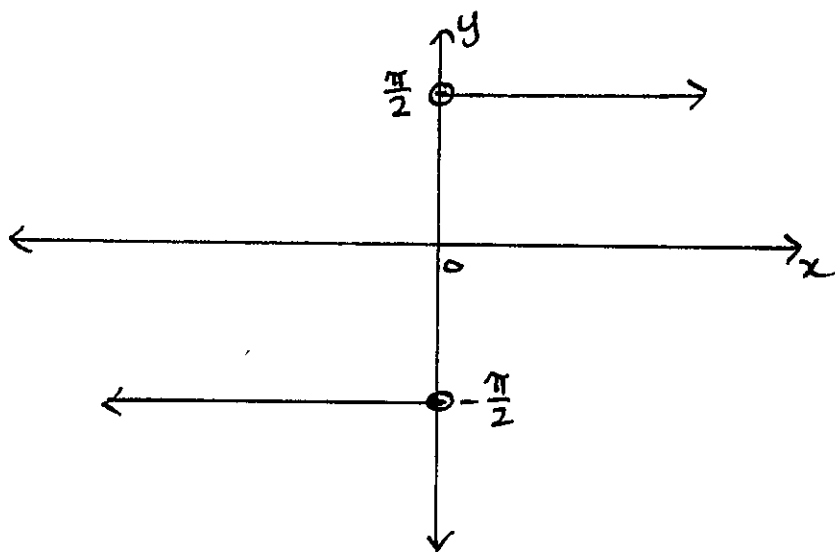
Suggested Solutions

Marks

Marker's Comments

$$\begin{aligned} \text{ii) when } x=1, y &= \tan^{-1} 1 + \tan^{-1} 1 \\ &= \frac{\pi}{4} + \frac{\pi}{4} \\ &= \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} \text{when } x=-1, y &= \tan^{-1}(-1) + \tan^{-1}(-1) \\ &= -\frac{\pi}{4} - \frac{\pi}{4} \\ &= -\frac{\pi}{2} \end{aligned}$$



②

1 for the
two different
lines

1 for open
circles