Name: ____________________

Class: ____________________

2015

Term 4 Assessment (HSC)

Mathematics Extension 1

General Instructions

- Working time – 90 minutes + 5minutes reading
- Board approved calculators may be used
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.

7 questions – 9 marks each

Total marks – 63

Attempt all questions

Start each question on a new sheet of paper.

QUESTION 1 (Start a new page) 9 Marks

a) Given $f(x) = \sqrt{16 - x^2}$.

i) Copy and complete the table below. **1**

x	0	1	2	3	4
$f(x)$	4.000		3.464		0.000

ii) Using the trapezoidal rule and the above function values, approximate the value of

$$\int_0^4 \sqrt{16 - x^2}\, dx$$ **2**

iii) Hence find an approximate value of π. Explain fully how you obtained your answer. **2**

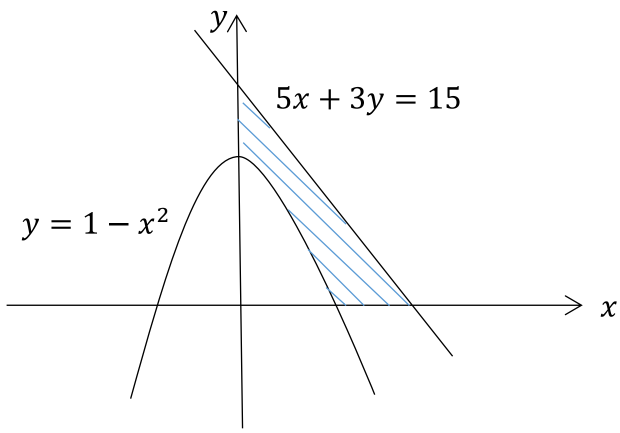
b) Find the volume of the solid of revolution formed when the region bounded by the curve $y = \frac{1}{\sqrt{9+x^2}}$, the x-axis, the y-axis and the line x=3, is rotated about the x-axis. **2**

c) Find the coefficient of x^3 in the expansion of $(x+1)(2x+3)^5$ **2**

QUESTION 2 (Start a new page) 9 Marks

a) Find the tenth term in the expansion $(x-3)^{12}$. Express in the form ${n_C}_r a^{n-r} b^r$ **1**

b) The diagram shows part of the curve $y = 1 - x^2$ and the straight line $5x + 3y = 15$. Find the volume generated when the shaded region is rotated about the y axis.



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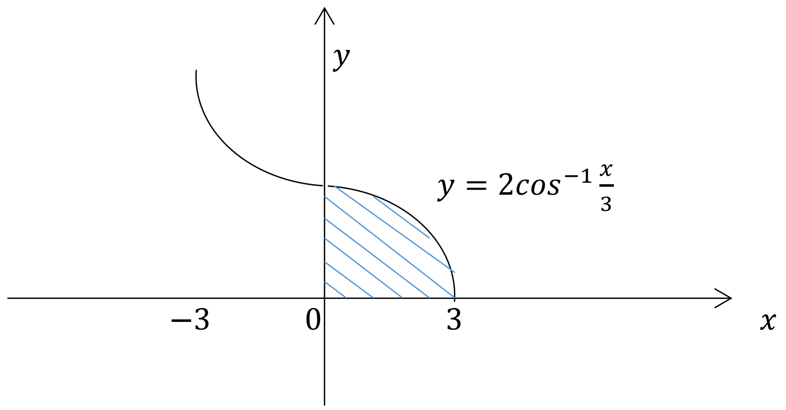
c) Differentiate $x \sin^{-1} \sqrt[3]{x}$ **3**

d) Use Simpson's Rule with 5 function values to estimate the volume of the solid of revolution formed when the curve $y = \frac{1}{\sqrt{1+x^2}}$ is rotated about the x −axis from $x = -1$ to $x = 1$. Express your answer to two decimal places. **3**

QUESTION 3 **(Start a new page)** **9 Marks**

a) Consider the graph of the function $y = 2\ \cos^{-1}\frac{x}{3}$ shown below.



i) Find the y intercept **1**

ii) Determine the inverse function $y = f^{-1}(x)$, and write down the domain D of this inverse function. **2**

iii) Calculate the area of the shaded region. **2**

b) Evaluate the integral $\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$, using the substitution $u = x - 1$ **2**

c) Find $h'(x)$ if $h(x) = tan^{-1}x + tan^{-1}\frac{1}{x}$. Simplify as far as possible **2**

QUESTION 4 **(Start a new page)** **9 Marks**

a) i) Find the exact value $\int_0^{\frac{\pi}{3}} \tan x \, dx$ **1**

ii) Using the trapezoidal rule with ordinates at $x = 0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, estimate

$\int_0^{\frac{\pi}{3}} \tan x \, dx$ to two significant figures. **2**

iii) Explain why the estimation in part (ii) is greater than the exact value obtained in part (i). **1**

b) By using the substitution $x = u^2$, $(u > 0)$, evaluate the definite integral

$$\int_0^1 \frac{\sqrt{x}}{1+x} dx$$ **3**

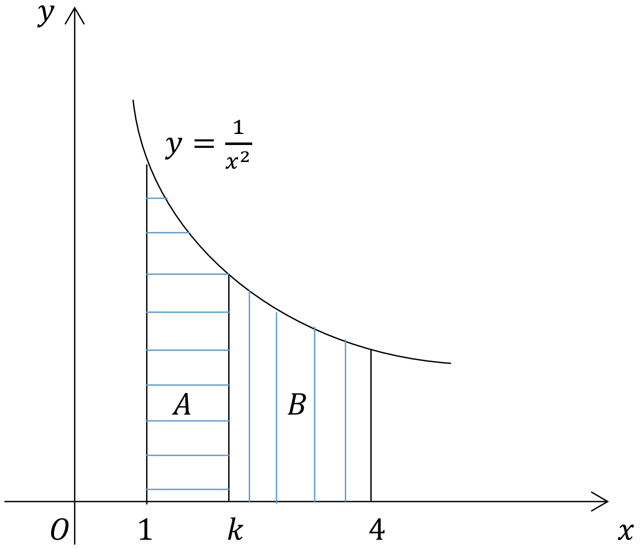
c) Find the exact value of x and y which satisfy the simultaneous equations: **2**

$$sin^{-1}x + \frac{1}{2}cos^{-1}y = \frac{\pi}{3}$$

$$3sin^{-1}x - \frac{1}{2}cos^{-1}y = \frac{2\pi}{3}$$

QUESTION 5 (Start a new page) 9 Marks

a) The diagram shows part of the curve $y = \frac{1}{x^2}$. When the region A is rotated about the x axis, the volume generated is V_A units^3. When the region B is rotated about the x axis, the volume generated is V_B units^3.
If $V_A : V_B = 8 : 1$, find the value of k.



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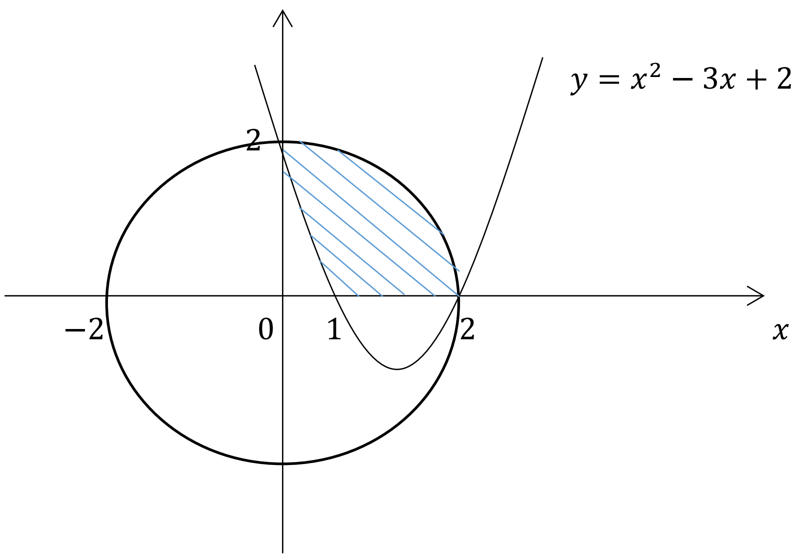
b) The shaded region in the diagram is bounded by the circle of radius 2 centered at the origin, the parabola $y = x^2 - 3x + 2$, and the x axis.
By considering the difference of 2 areas, find the area of the shaded region.



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c) The rate at which snow is melting on a mountain top is modelled by the equation

$$\frac{dm}{dt} = -3 + 5\cos\frac{\pi}{3}t,$$

where m is the mass of ice, in kg, on the mountain top at time t, in hours, at 7 am. It is estimated that there was still 17 000 kg of snow and ice on the mountain at that time.

i) Find m as a function of t. **2**

ii) It is expected that in the next season, in 100 days' time, there will be some snow left. Find how much of snow will be left after this time. (2 significant figure) **1**

QUESTION 6 (Start a new page) 9 Marks

a) Find the greatest coefficient in the expansion $(\frac{1}{3} + 2x)^{18}$ **3**

b) Consider the function $f(x) = \frac{1}{2} tan^{-1} x$.

i) State the range of the function and sketch the graph of $y = f(x)$ **2**

ii) Show that the equation of the tangent at $x = 1$, is $2x - 8y + \pi - 2 = 0$ **2**

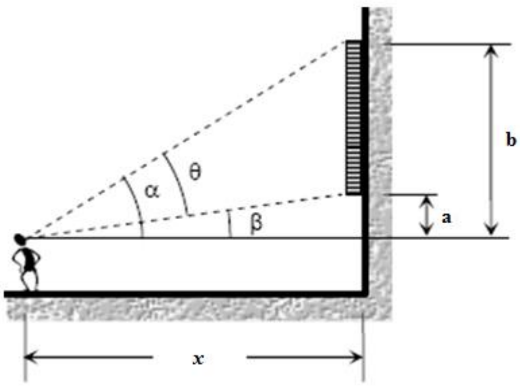
c) Show that $\sum_{k=1}^{n} \binom{n}{k} = 2^n - \binom{n}{0}$ in the expansion of $(1 + x)^n$. **2**

QUESTION 7 (Start a new page) 9 Marks

a) Let p and q be positive integers with $p \leq q$.
Use the binomial theorem to expand $(1+x)^{p+q}$, and hence write down the term of $\frac{(1+x)^{p+q}}{x^q}$ which is independent of x. **2**

b) The lower edge of a picture frame is a metres and the upper edge is b metres, above the eye of an observer standing on a level floor.
The angle of elevation of the bottom edge from the observer is β and the angle of elevation of the top edge from the observer is α.

Let $\theta = \alpha - \beta$ be the angle subtended by the vertical picture and $x =$ the horizontal distance from the observer to the bottom of the wall.



i) Prove that $\tan\theta = \frac{(b-a)x}{x^2+ab}$ **3**

ii) Hence or otherwise, at what horizontal distance , if $a = 1\ m$ and $b = 2.5\ m$, should the observer stand, if the angle subtended by the picture is to be the greatest? **4**

*****END OF EXAMINATION*****

a) $y = \sqrt{16-x^2}$

(i)

x	0	1	2	3	4
f(x)	4	3.873	3.464	2.646	0

1 2 2 2 1

(i) Mark deducted for wrong off error

1 m no half mark

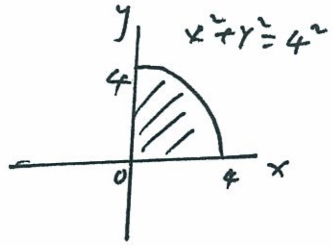
(ii) $\int_0^4 \sqrt{16-x^2}\,dx \doteqdot \frac{1}{2}\left[4 + 2(3.873 + 3.464 + 2.646)\right]$ 1 m

$\doteqdot 11.9828$ 1 m

(iii) Area of quarter of circle $= \frac{\pi}{4} \times 4^2 = 4\pi$

$4\pi \doteqdot 11.9828$ 1 m

$\pi = \underline{2.9957}$ (4dp) 1 m



b) $vol \doteqdot \int_0^3 \pi \frac{dx}{9+x^2} = \frac{\pi}{3} \cdot \tan^{-1}\frac{x}{3}\Big]_0^3$ 1 m

$= \frac{\pi}{3} \times \frac{\pi}{4} = \underline{\frac{\pi^2}{12}}$ (or 0.822467...) 1 m

c) $(x+1)\left[(2x)^5 + \binom{5}{1}(2x)^4 \cdot 3 + \binom{5}{2}(2x)^3 3^2 + \binom{5}{3}(2x)^2 3^3 + \binom{5}{4}(2x)3^4 + 3^5\right]$

terms involving x^3 $\binom{5}{2}(2x)^3 3^2 + \binom{5}{3}(2x)^2 3^3$

coeff of $x^3 = \binom{5}{2} 2^3 \cdot 3^2 + \binom{5}{3} 2^2 \cdot 3^3$

$= 720 + 1080$ 1 m for either 720 or 1080

$= \underline{1800}$ 1 m for 1800

MATHEMATICS Extension 1 Question 2

Suggested Solutions	Marks	Marker's Comments
a) $^{12}C_9 x^3 (-3)^9$ or $-^{12}C_9 x^3 3^9$	1	
b) intercepts for $5x+3y=15$ is $(0,5)$ and $(3,0)$ intercepts for $y=1-x^2$ is $(0,1)$ and $(\pm 1, 0)$ $V = \pi\int_0^5 \left(\frac{15-3y}{5}\right)^2 dy - \pi\int_0^1 (\sqrt{1-y})^2 dy$	1	Alternatively, $\pi\int_0^5 \left(\frac{15-3y}{5}\right)^2 dy$ could be expressed as a cone where $V=\frac{1}{3}\pi r^2 h$ $=15\pi$.
$= \pi\int_0^5 \left(\frac{225-90y+9y^2}{25}\right) dy - \pi\int_0^1 (1-y)\,dy$ $= \frac{\pi}{25}\int_0^5 (225-90y+9y^2)\,dy - \pi\int_0^1 (1-y)\,dy$ $= \frac{\pi}{25}\left[225y - 45y^2 + 3y^3\right]_0^5 - \pi\left[y - \frac{y^2}{2}\right]_0^1$ $= \frac{\pi}{25}(1125 - 1125 + 375) - \pi\left(\frac{1}{2}\right)$ $= 15\pi - \frac{\pi}{2}$ $= 14\frac{1}{2}\pi$.	1	1 mark if: - Work out either one of the definite integrals - rotated about the x-axis instead of y. $\left(\frac{367\pi}{15}\right)$ - Did the question all under 1 integral. $\left(\frac{45\pi}{2}\right)$
c) $f(x) = x\sin^{-1}(\sqrt[3]{x})$ $f'(x) = \boxed{x \cdot \frac{1}{\sqrt{1-x^{2/3}}}} \times \boxed{\frac{1}{3}x^{-2/3}} + \boxed{1 \cdot \sin^{-1}(x^{1/3})}$ $= \frac{x}{3x^{2/3}\sqrt{1-x^{2/3}}} + \sin^{-1}(x^{1/3})$ $= \frac{x^{1/3}}{3\sqrt{1-x^{2/3}}} + \sin^{-1}(x^{1/3})$	1 1 1	

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MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
d) $V = \pi \int_{-1}^{1} \left(\frac{1}{\sqrt{1+x^2}} \right)^2 dx$		
$= \pi \int_{-1}^{1} \frac{1}{1+x^2} dx$ Let $f(x) = \frac{1}{1+x^2}$	1	
(table below)	1	
$\therefore V \doteq \pi \times \frac{1-(-1)}{3 \times 4} \left\{ \frac{1}{2} + 4\left(\frac{4}{5} + \frac{4}{5}\right) + (2 \times 1) + \frac{1}{2} \right\}$		
$= \frac{\pi}{6} \left\{ 3 + \frac{32}{5} \right\}$		
$= \frac{\pi}{6} \times \frac{47}{5}$		
$= \frac{47\pi}{30}$		
$= 4.92 u^3$	1	
- Set question out neatly!		
- Hard to give marks when you have the wrong formula and showed no working leading up to it.		

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	$\frac{1}{2}$	$\frac{4}{5}$	1	$\frac{4}{5}$	$\frac{1}{2}$

ext 1 **MATHEMATICS: Question..3...**

Suggested Solutions	Marks	Marker's Comments
a (i) y intercept $x=0$ $y = 2\cos^{-1} 0$ $= 2 \times \frac{\pi}{2}$ $= \pi$	1	
(ii) $x = 2\cos^{-1}\frac{y}{3}$ $\frac{x}{2} = \cos^{-1}\frac{y}{3}$ $\frac{y}{3} = \cos\frac{x}{2}$ $y = 3\cos\frac{x}{2}$ $f^{-1}(x)$ is $3\cos\frac{x}{2}$ For $f(x)$ range is $0 \le y \le 2\pi$ so domain $f^{-1}(x)$ is $0 \le x \le 2\pi$	1 1	students needed to realise the domain of inverse depended on range of original function
(iii) $A = 2\int_0^3 \cos^{-1}\frac{x}{3}\,dx$ $= 3\int_0^{\pi} \cos\frac{y}{2}\,dy$ $= 3\left[2\sin\frac{y}{2}\right]_0^{\pi}$ $= 6\left[\sin\frac{\pi}{2} - \sin 0\right]$ $= 6 \text{ units}^2$	1 1	a number of students had i-correct limits

MATHEMATICS Extension 1 : Question...3....

Suggested Solutions	Marks	Marker's Comments
$\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$ let $u = x-1$, $du = dx$ when $x=1$ $u=0$ $x=2$ $u=1$ Now $x = u+1$		
$\int_0^1 \frac{1}{\sqrt{3+2(u+1)-(u+1)^2}}\,du.$	1	Many students made basic algebraic errors ~~which~~ led to a function that couldn't be integrated.
$= \int_0^1 \frac{1}{\sqrt{4-u^2}}\,du$		
$= \left[\sin^{-1}\frac{u}{2}\right]_0^1$	1	
$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0$		
$= \frac{\pi}{6}$		
c) $h(x) = \tan^{-1} x + \tan^{-1}\frac{1}{x}$		
$h'x = \frac{1}{1+x^2} + \frac{-\frac{1}{x^2}}{1+\left(\frac{1}{x}\right)^2}$	1	1 for $\frac{1}{1+x^2}$ plus an attempt to differentiate $\tan^{-1}\left(\frac{1}{x}\right)$
$= \frac{1}{1+x^2} - \frac{1}{1+x^2}$	1	1 for answer
$= \underline{0}$		

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

a) i) $\int_0^{\frac{\pi}{3}} \tan x \, dx$

$= \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \, dx$

$= \left[-\ln(\cos x) \right]_0^{\frac{\pi}{3}}$

$= \left(-\ln\left(\cos\frac{\pi}{3}\right) - \left(-\ln(\cos 0)\right)\right)$

$= -\ln\frac{1}{2} + \ln 1$ — 1

$= -\ln\frac{1}{2}$

$= \ln 2$

ii)

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan x$	0	0.27	0.58	1	1.7

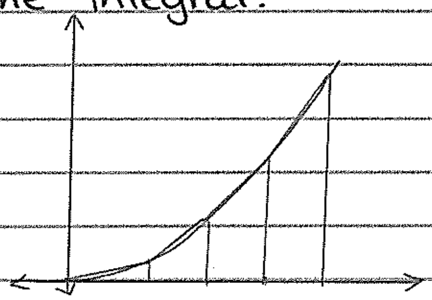
— 1

$\int_0^{\frac{\pi}{3}} \tan x \, dx = \frac{\frac{\pi}{12}}{2}\left[0 + 2(0.27 + 0.58 + 1) + 1.7\right]$

$= \frac{\pi}{24}(5.4)$

$= 0.71$ (2 sig. figs.) — 1

iii) Curve is concave up therefor the trapezoidal rule overestimates the integral. — 1



b) $\int_0^1 \frac{\sqrt{x}}{1+x} \, dx$

$x = u^2$ when $x = 1$, $u = 1$

$\frac{dx}{du} = 2u$ $x = 0$, $u = 0$.

$dx = 2u\,du$

$\int_0^1 \frac{\sqrt{x}}{1+x} \, dx = \int_0^1 \frac{u}{1+u^2} \times 2u \, du$ — 1

Never write u and x together!

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
$= \int_0^1 \frac{2u^2}{1+u^2}\,du$		
$= 2\int_0^1 \frac{1+u^2-1}{1+u^2}\,du$		
$= 2\int_0^1 1 - \frac{1}{1+u^2}\,du$	1	
$= 2\left[u - \tan^{-1}u\right]_0^1$		
$= 2\left[1 - \tan^{-1}(1) - 0\right]$	1	
$= 2\left[1 - \frac{\pi}{4}\right]$		
$= 2 - \frac{\pi}{2}$		
c) $\sin^{-1}x + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$ — (1)		
$3\sin^{-1}x - \frac{1}{2}\cos^{-1}y = \frac{2\pi}{3}$ — (2)		
(1) + (2)		
$4\sin^{-1}x = \pi$		
$\sin^{-1}x = \frac{\pi}{4}$		
$x = \frac{1}{\sqrt{2}}$	1	
when $x = \frac{1}{\sqrt{2}}$,		
$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\cos^{-1}y = \frac{\pi}{3}$		
$\frac{1}{2}\cos^{-1}y = \frac{\pi}{12}$		
$\cos^{-1}y = \frac{\pi}{6}$		
$y = \frac{\sqrt{3}}{2}$	1	
$\therefore\ x = \frac{1}{\sqrt{2}},\ y = \frac{\sqrt{3}}{2}$		

2015 Extension 1 Term 4 Task Q 5

Part (a)

$V_A = \pi \int_1^k \left(\frac{1}{x^2}\right)^2 dx$

$= \pi \left[\frac{x^{-3}}{-3}\right]_1^k$

$= \pi \left[-\frac{1}{3k^3} + \frac{1}{3}\right]$

(1 mark)

$V_B = \pi \int_k^4 \left(\frac{1}{x^2}\right)^2 dx$

$= \pi \left[\frac{x^{-3}}{-3}\right]_k^4$

$= \pi \left[-\frac{1}{192} + \frac{1}{3k^3}\right]$

(1 mark)

$\frac{V_A}{V_B} = \frac{8}{1} \Rightarrow \pi \left[-\frac{1}{3k^3} + \frac{1}{3}\right] = \pi \left[-\frac{8}{192} + \frac{8}{3k^3}\right]$

(1 mark)

$\frac{8}{3k^3} + \frac{1}{3k^3} = \frac{1}{3} + \frac{1}{24}$

$k^3 = 8$

$k \in \mathbb{R} \Rightarrow k = 2$

(1 mark)

Comments:

A mistake in evaluating either integral would carry forward. If errors were made evaluating both integrals, but it was clear the student knew how to compute volumes by rotation, then one mark was awarded for both attempts.

It was essential to construct an equation using the given ratio to earn the final mark.

(b)

The area we want is the area beneath the circle in the first quadrant, less the area beneath the parabola from $x = 0$ to $x = 1$.

Beneath the parabola:

$A_p = \int_0^1 (x^2 - 3x + 2)dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1$

$= \left[\frac{1}{3} - \frac{3}{2} + 2\right]$

$= \frac{5}{6}$ square units

(1 mark)

Area of quarter circle, radius 2:

$A_c = \frac{1}{4} \times \pi \times 4$

$= \pi$ square units

Total $A = A_c - A_p$

$= \pi - \frac{5}{6}$ square units

(1 mark)

Comments:

Some students attempted to integrate the function $y = \sqrt{4 - x^2}$ to find the area under the circle. This earned no marks unless the student correctly completed every step of the required computation.

(c)

(i)

$$m = \int \left(-3 + 5\cos\left(\frac{\pi}{3}t\right)\right) dt$$

$$m = -3t + \frac{15}{\pi}\sin\left(\frac{\pi}{3}t\right) + C$$

(1 mark)

At $t = 0$, $m = 17000 \Rightarrow 17000 = 0 + \frac{15}{\pi}(0) + C$

$C = 17000$

$\therefore m = -3t + \frac{15}{\pi}\sin\left(\frac{\pi}{3}t\right) + 17000$

(1 mark)

Comments:

Many students incorrectly inferred the derivative function was $m' = -3 + \left(5\cos\frac{\pi}{3}\right)t$. In this case, if the rest of the computation was correct, a carry forward error was factored and subsequent marks were awarded.

The second mark was NOT awarded if the student presumed that time began at 7 hours.

(ii)

At $t = 100 \times 24$ hours,

$m = -3(2400) + \frac{15}{\pi}\sin 800\pi + 17000$

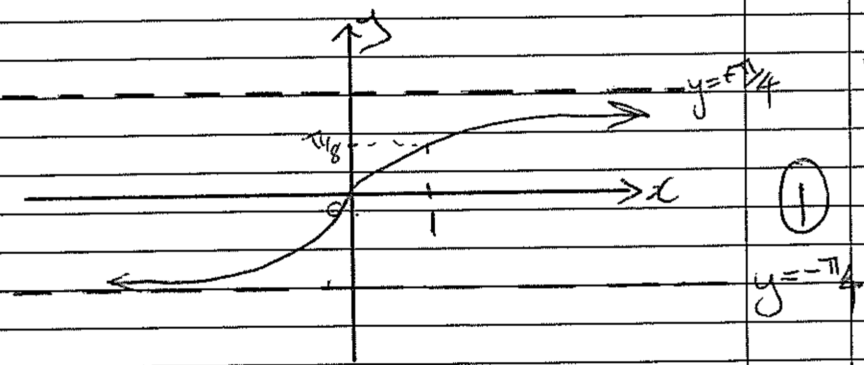
≈ 9814.772

$= 9.8 \times 10^3$ kg (2 s.f.)

(1 mark)

A mark was awarded if the student correctly substituted $t = 2400$ into the function they derived in Part (i)

MATHEMATICS Extension 1: Question 6

Suggested Solutions	Marks	Marker's Comments
(a) $T_{r+1} = {}^{18}C_r (2)^r (1/3)^{18-r}$		
$T_r = {}^{18}C_{r-1} (2)^{r-1} (1/3)^{18-(r-1)}$		
Greatest coefficient when $\frac{T_{r+1}}{T_r} > 1$		
$\frac{{}^{18}C_r\, 2^r\, 3^{r-18}}{{}^{18}C_{r-1}\, 2^{r-1}\, 3^{r-19}} > 1$	1	* or could have used $\frac{{}^{18}C_{r+1}}{{}^{18}C_r}$ etc.
$\frac{\frac{18!}{r!(18-r)!}}{\frac{18!}{(19-r)!(r-1)!}} \times 2 \times 3 > 1$		!
$\frac{19-r}{r} \times 6 > 1$		then you get $r = 15.28$ ..
$114 - 6r > r$		
$-7r > -114$		
$r < 16.285$		
$\therefore r = 16$	1	
$T_{16+1} = {}^{18}C_{16}\, 2^{16} (1/3)^2$	1	A lot of students subbed into T_r :(
(b)(i) $-\pi/4 < f(x) < \pi/4$	1	
	1	very sad to see how BAD this graph was drawn. Use a ruler!!!

2/2

MATHEMATICS Extension 1: Question 6 cont.		
Suggested Solutions	**Marks**	**Marker's Comments**
(ii) $y=\frac{1}{2}\tan^{-1}(x)$		
$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{1+x^2}$		
when $x=1$, $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$		
when $x=1$, $y=\frac{1}{2}\tan^{-1}1 = \frac{1}{2}\times\frac{\pi}{4}$	1	
$y=\frac{\pi}{8}$		
eqn. of tangent is:		
$y-y_1=m(x-x_1)$		
$y-\frac{\pi}{8}=\frac{1}{4}(x-1)$	1	
$8y-\pi = 2x-2$		
$0 = 2x-8y+\pi-2$		

Question 7 - 3 unit

a) ${}^{p+q}C_0(1) + {}^{p+q}C_1(1)(x) + {}^{p+q}C_2(1)(x^2) + \cdots\cdots {}^{p+q}C_r(1)(x^r) + {}^{p+q}C_{p+q}(1)(x^{p+q})$

✓ 1 mark for expansion

* To get one mark one must have the last term

Also accepted

① $\binom{p+q}{0} + \binom{p+q}{1}x + \binom{p+q}{2}x^2 \text{ --------- } \binom{p+q}{q}x^q$

② $\sum_{r=0}^{p+q} \binom{p+q}{r} x^r$ or $\sum_{k=0}^{p+q} {}^{p+q}C_k \, x^k$

* One must have Σ with limits to get one mark.

✓ 2nd mark was allocated for the independent term.

${}^{p+q}C_q$ or $\dfrac{(p+q)!}{q!\,p!}$ or $\binom{p+q}{q}$

$$\frac{T_{k+1}}{x^q} = \frac{(1+x)^{p+q}}{x^q} = \frac{{}^{p+q}C_k x^k}{x^q} = {}^{p+q}C_k \, x^{k-q}$$

Independent of x means $k-q=0$

$\therefore k = q$

$$\therefore {}^{p+q}C_q \, x^{q-q}$$

$$= {}^{p+q}C_q$$

---

b) (i) - 3 marks

$\left.\begin{array}{l}\tan\alpha = \dfrac{b}{x} \\ \tan\beta = \dfrac{a}{x}\end{array}\right\}$ ✓ 1 mark

$$\tan\theta = \tan(\alpha - \beta)$$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{\frac{b}{x} - \frac{a}{x}}{1 + \frac{ba}{x^2}}$$ ✓ 1 mark

$$= \frac{\frac{b-a}{x}}{\frac{x^2+ba}{x^2}} = \frac{(b-a)x}{x^2+ba}$$ ✓ Simplifying

b (ii). We know $\tan\theta = \dfrac{(b-a)x}{x^2+ab}$

when $a=1$ and $b=2.5$

$$\tan\theta = \frac{(2.5-1)x}{x^2+(1)(2.5)}$$

$$\tan\theta = \frac{1.5x}{x^2+2.5} \quad \text{or} \quad \tan\theta = \frac{3x}{2x^2+5}$$

✓ 1 mark for writing in terms of θ

ie $\theta = \tan^{-1}\left(\dfrac{1.5x}{x^2+2.5}\right)$ or $\theta = \tan^{-1}\left(\dfrac{3x}{2x^2+5}\right)$

✓ 1 mark for getting $\dfrac{d\theta}{dx}$

$$\frac{d\theta}{dx} = \frac{(1.5)(x^2+2.5)-(1.5x)(2x)}{(x^2+2.5)^2} \times \frac{1}{1+\left(\frac{1.5x}{x^2+2.5}\right)^2}$$

$$= \frac{3.75-1.5x^2}{(x^2+2.5)^2+2.25x^2}$$

$$= \frac{3.75-1.5x^2}{x^4+7.25x^2+6.25}$$

Alternatively

If used $\tan\theta = \dfrac{3x}{2x^2+5}$

then $$\frac{d\theta}{dx} = \frac{3(2x^2+5)-(4x)(3x)}{(2x^2+5)^2} \times \frac{1}{1+\left(\frac{3x}{2x^2+5}\right)^2}$$

$$= \frac{15-6x^2}{(2x^2+5)^2+(9x^2)}$$

1 mark for getting value of x.

ie $\frac{d\theta}{dx} = 0$

$$\frac{3.75 - 1.5x^2}{x^4 + 7.25x^2 + 6.25} = 0$$

$$3.75 - 1.5x^2 = 0$$

$$1.5x^2 = 3.75$$

$$x^2 = \frac{5}{2}$$

$$x = \pm\sqrt{\frac{5}{2}}$$

$$x = \sqrt{\frac{5}{2}} \text{ or } \sqrt{2.5} \quad \text{as } x > 0$$

accepted $\sqrt{\frac{15}{6}}$ also

✓ 1 mark (4th mark) is for testing the nature

x	1	$\sqrt{5}/2$	2
$\frac{d\theta}{dx}$	$\frac{9}{58}$	0	$\frac{-9}{205}$
or	0.155		-0.04

/ — \

To get full 1 mark one had to write the values of $\frac{d\theta}{dx}$ and not just mention (+) and (−)

$\therefore$ $x = \sqrt{\frac{5}{2}}$ is a relative maximum

$\therefore$ at $x = \sqrt{\frac{5}{2}}$ the angle is the greatest.