JAMES RUSE AHS

GESTA NON VERBA

Name: ____________________

Class: __________

2015

HSC

Term 2 Assessment

# Mathematics Extension 1

*General Instructions*

- Reading time – 5 minutes
- Working time – 90 minutes
- Board approved calculators may be used
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.

**Total marks – 63**

**You may take one A4 sheet of paper into the examination with any notes or formulas written on the sheet. The notes may be hand written or typed and it may be double sided.**

**Attempt all questions**

**Start each question on a new sheet of paper.**

## QUESTION 1 (Start a new page) 9 Marks

(a) In a tennis competition there are eight competitors, three of whom are men and five are women. If there is to be a team of four chosen, what is the probability that at least three of them are women? **2**

(b) In how many ways can the letters of the word **HOLIDAYS** be arranged if

(i) The new arrangement begins with **D** and ends with **Y**? **1**

(ii) The vowels are together at the beginning of the new arrangement? **1**

(c) A particle moving on a straight line has acceleration $a = \frac{-4}{x^2}$.

If $v = 3$ when $x = 1$, then:

(i) Find $v^2$ as a function of $x$ **2**

(ii) Describe the motion **1**

(d) The velocity, $v$ metres per second, of a particle at time $t$ seconds is given by

$$v = t^3 - 3t^2 + 4t - 6.$$

Show that the acceleration is never less than 1 metre per second per second. **2**

## QUESTION 2 (Start a new page) 9 Marks

A steady wind is blowing with speed 36 km/h. From clouds moving horizontally with the wind, heavy raindrops fall to the ground 200m below. (Air resistance may be neglected, and the approximate value $g = 10\ ms^{-2}$ may be assumed)

(a) Show that the position of the raindrop at time t is given by:

$$x = 10t \qquad y = \frac{-gt^2}{2} + 200$$ **2**

(b) Find the time taken for a drop to reach the ground. **2**

(c) Find the speed and angle at which a drop hits the ground (assumed horizontal). **2**

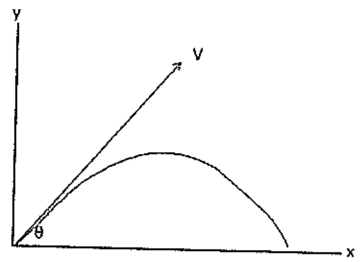
(d) The path of the raindrop is a parabolic arc. Write its equation in Cartesian form. **2**

(e) At what angle does a drop hit the ground when the wind speed is doubled? **1**

## QUESTION 3 (Start a new page) 9 Marks

(a) A particle is moving in a straight line away from the origin. When its displacement is x, its velocity V is given by $V = \frac{k}{x}$, where k is a positive constant and $x > 0$.
A, B, C, and D are four equally spaced points, in order on the line. Show that the times taken by the particle to traverse the segments AB, BC and CD increase in an arithmetic progression. **4**

(b)




The path of an arrow fired from the origin O is given by

$$x = Vt\cos\theta$$

$$y = Vt\sin\theta - 5t^2$$

Where $V$ is the initial speed in metres per second and $\theta$ is the angle of projection as in the diagram and $t$ is the time in seconds.

(i) Find the maximum height reached by the arrow in terms of $V$ and $\theta$. **2**
(ii) Find the range in terms of $V$ and $\theta$. **2**
(iii) Prove that the range is maximum when $\theta$=45°. **1**

## QUESTION 4 (Start a new page) 9 Marks

(a) Kate and Jon are invited to Ashley's dinner party of eight.

Find the number of ways of seating the eight people around a circular table if:

(i) There are no restrictions on where a person sits **1**

**(ii)** Jon wishes to sit between Kate and Ashley **1**

(b) The velocity $v$ m/s of a particle moving in simple harmonic motion along the x-axis is given by

$$v^2 = -5 + 6x - x^2, \text{ where } x \text{ is in metres.}$$

(i) Between which two points is the particle oscillating? **2**

(ii) Find the centre of motion of the particle. **1**

(iii) Find the maximum speed of the particle. **2**

(iv) Find the acceleration of the particle in terms of $x$. **2**

## QUESTION 5 (Start a new page) 9 Marks

(a) One fifth of all jellybeans are black. A random sample of ten jellybeans is chosen.

(i) What is the probability that this sample contains exactly two black jellybeans? Give your answer correct to 3 decimal places. **2**

(ii) What is the probability that the sample contains fewer than two black jellybeans? Give your answer correct to 3 decimal places. **1**

(iii) Which is more likely:

*The sample contains fewer than two black jellybeans,*

*Or*

*The sample contains more than two black jellybeans?*

Give reasons for your answer **2**

(b) High tide at Thomas Harbour occurred at 5 am and low tide at 11.20 am on that same day. The entrance to Thomas Harbour is 12 metres deep at high tide and 4 metres deep at low tide. The tidal motion is assumed to be simple harmonic.

A ship drawing 7.5m wishes to enter the harbour before midday. What is the latest time the ship will be able to enter the harbour safely? 4

## QUESTION 6 (Start a new page) 9 Marks

(a) When Mendel crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf.

Suppose five such offspring were selected at random. Find the probability that:

(i) All of these offspring were tall; **1**

(ii) At least three of these offspring were tall. **2**

(b) A particle moves in such a way that its displacement x cm from the origin 0 at any time t sec is given by the relation $x = \sqrt{3}cos3t - sin3t$.

(i) Show that the particle moves in simple harmonic motion. **2**

(ii) Find the period of the motion. **1**

(c) The carbon isotope $C14$ decays at a rate proportional to its mass. Tree ring experiments suggests that 50% decay takes 5,580 years. A fossil contains 30% of the amount of $C14$ in a similarly sized living organism. Estimate the age of the fossil. **3**

**Question 7 on Next Page**

**QUESTION 7** **(Start a new page)** **9 Marks**

(a) Under ideal conditions, it is estimated that the rate of increase in the population P(t) of the rare spotted tree kangaroo is given by the equation

$\frac{d}{dt}P(t) = kP(t); \ (t \geq 0)$, where k is a positive constant.

(i) Given that the population trebles in two years, write down an expression for $P(t)$ in terms of $t$ and $P(0)$. **3**

(ii) The initial population of a mob of spotted tree kangaroos is 10. Estimate the kangaroo's population after three years. **1**

(b) In practise, population growth is restricted by limitations on food, space and other natural resources. The equation

$$\frac{d}{dt}P(t) = kP(t)(L - P(t)); \ (t \geq 0)$$

where $k$ and $L$ are positive constants and $P(0) < L$, is found to be more appropriate to describe some situations.

(i) Verify that, for any positive constant $C$, the expression

$P(t) = \frac{LC}{C+e^{-kLt}}$ satisfies the equation. **3**

(ii) What can be deduced about P as t increases? Explain what this means. **1**

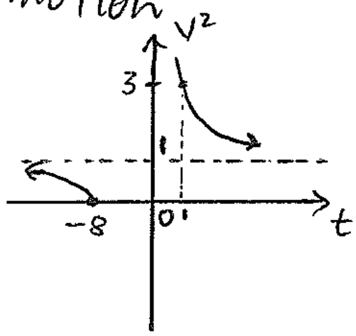
(iii) What can be deduced about $\frac{dP}{dt}$ as t increases? Explain what this means. **1**

END OF EXAMINATION

JRAHS : Y12 3U MATHEMATICS Extension 1 : Question 1 TASK 3 2015

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| a) Sample Space for a team of 4: $^8C_4 = 70$ | 1 | For correct sample space |
| Case 1: 3W, 1M can be arranged in $^5C_3 \times {}^3C_1 = 30$ ways<br>Case 2: 4W, 0M can be arranged in $^3C_0 \times {}^5C_4 = 5$ ways | 1 | For correct number of ways |
| $\therefore P(\text{at least } 3W) = \frac{30+5}{70} = \frac{1}{2}$<br>OR<br>$P(3 \text{ women}) = 1 - [P(1W) + P(2W) + P(0W)]$ → Cannot make a team<br>$= 1 - \left(\frac{^5C_1\,{}^3C_3 + {}^5C_2\,{}^3C_2}{^8C_4}\right)$<br>$= \frac{1}{2}$ | | |
| b) (i) D (H O L I A S) Y — D fixed, Y fixed<br>Number of arrangements $= 6!$<br>$= 720$ ways | 1 | For correct number of ways |
| (ii) (O I A) (H L D Y S) — (O I A) fixed in front<br>Number of way $= 3! \times 5! = 720$ ways | 1 | For correct number of ways |

JRAHS Y12 3U **MATHEMATICS Extension 1 : Question** 1 TASK 3 2015

| Suggested Solutions | Marks | Marker's Comments |
| --- | --- | --- |
| c) (i) $a = -\frac{4}{x^2}$<br>$\boxed{a = v \cdot \frac{dv}{dx}}$<br>$\Rightarrow \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{4}{x^2}$<br>$\therefore \frac{1}{2}v^2 = -4\left(-\frac{1}{x}\right) + C$<br>$= \frac{4}{x} + C$ | 1 | For correct expression for $v^2$ or $\frac{1}{2}v^2$, including integration constant |
| when $x = 1$, $v = 3$<br>$\Rightarrow 4\frac{1}{2} = 4 + C$<br>$\therefore C = \frac{1}{2}$<br>$\therefore \frac{1}{2}v^2 = \frac{4}{x} + \frac{1}{2}$<br>$\therefore v^2 = \frac{8}{x} + 1$ | 1 | For correctly applying boundary conditions and obtaining an expression for $v^2$ |
| (ii) • slows down $(a < 0)$<br>• moving to right (indefinitely)<br>• as $x \to \infty$, $v \to 1$<br>• never comes to a stop (or never changes direction)<br>[All four aspects required for complete description] | 1 | For complete description of motion<br> |

JRAHS Y12 3U **MATHEMATICS Extension 1 : Question** 1 TASK 3 2015

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| d) $v = t^3 - 3t^2 + 4t - 6$ | | |
| $\therefore a = 3t^2 - 6t + 4$ | | |
| $= 3(t^2 - 2t + \frac{4}{3} + 1 - 1)$ | | |
| $= 3[(t-1)^2 + \frac{1}{3}]$ | | |
| $= 3(t-1)^2 + 1$ | 1 | For completing the square |
| But $3(t-1)^2 \geq 0 \ \forall \ t > 0$ | | |
| $\therefore a \geq 1$ | | |
| Hence $a$ can never be less than 1 | 1 | For correct conclusion, based on evidence presented. |

| Method (n+1) | Method (n+2) | Method (n+3) | Method (n+4) |
|---|---|---|---|
| $a = 3t^2 - 6t + 4$ | $a = 3t^2 - 6t + 4$ | $a = 3t^2 - 6t + 4$ | Assume $a < 1$ |
| $\dot{a} = 6t - 6$ | $t = 0, \ a = 4$ | $\Delta = -12 < 0$ | i.e $3t^2 - 6t + 4 < 1$ |
| $\dot{a} = 0 \Rightarrow t = 1$ | $t = \frac{1}{2}, \ a = 1\frac{3}{4}$ | $a = 3 > 0$ | $3t^2 - 6t + 3 < 0$ |
| Test: | $t = 1, \ a = 1$ | $\therefore$ concave up | $t^2 - 2t + 1 < 0$ |
| $\ddot{a} = 6 > 0$ | $t = 2, \ a = 4$ | Vertex: | $(t-1)^2 < 0$ |
| $\therefore a$ is concave up | $t = 3, \ a = 13$ | $t = -b/2a = 1$ | which is impossible $\forall \ t > 0$ |
| Hence $t = 1$ a minimum exists | Clearly minimum 1 | $\therefore a(1) = 1$ | Hence contradiction |
| and $a(1) = 1$ | $\therefore a \not< 1$ | $\therefore V(1, 1)$ | $\therefore$ Assumption not true |
| $\therefore a \not< 1$ | | $\therefore a \geq 1$ | $\therefore a \not< 1$ |
| | | $\therefore a \not< 1$ | |

Ext 1 2015 T2 Yr12

Q2

$36\text{km/hr} = 3600\!/_{3600}\ \text{m/s} = 10\text{m/s}$

a) $\ddot{x} = 0$

$\dot{x} = c_1$

$t=0,\ \dot{x}=10 \quad \therefore c_1 = 10$ (horizontal wind)

must show initial conditions in each case

$x = 10t + c_2$

$t=0,\quad x=0 \quad \therefore c_2 = 0$

1m

$\therefore x = 10t$

$\ddot{y} = -g$

must start from $\ddot{y} = -g$ not $\ddot{y} = -10$

$\dot{y} = -gt + k_1$

$t=0 \quad \dot{y} = 0$ (horizontal wind)

$\therefore k_1 = 0$

$\dot{y} = -gt$

$y = -\dfrac{gt^2}{2} + k_2$

$t=0,\ y=200 \quad \therefore k_2 = 200$

1m

$\therefore y = -\dfrac{gt^2}{2} + 200$

b) $y=0 \quad \therefore -\dfrac{gt^2}{2} + 200 = 0$

1m

$g=10 \qquad 5t^2 = 200$

$t^2 = 40$

$t = 2\sqrt{10} \quad (t \geq 0)$

1m

c) $t = 2\sqrt{10} \quad \dot{y} = -20\sqrt{10} \qquad \dot{x} = 10$

$|V| = \sqrt{(-20\sqrt{10})^2 + (10)^2}$

$|V| = \sqrt{4100} = 10\sqrt{41}$ m/s

1m

$\tan\theta = \dfrac{-20\sqrt{10}}{10} = -2\sqrt{10}$

$\theta = -81° \quad \therefore 81°$ below horizon

1m

some students do

$\tan\theta = \left| \dfrac{-20\sqrt{10}}{10} \right| = 2\sqrt{10}$

$\theta = 81°$ $\left(\text{or } \tan\theta = \dfrac{20\sqrt{10}}{10}\right)$

Need to explain the absolute value or draw diagram.

d) $x = 10t$

$t = \frac{x}{10}$

$y = -\frac{g}{2}\left(\frac{x}{10}\right)^2 + 200$ from (a) 1 m

$g = 10$ $\therefore y = -5\left(\frac{x^2}{100}\right) + 200$

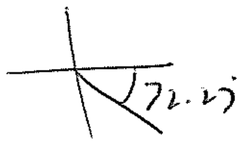
$y = -\frac{x^2}{20} + 200$ # 1 m

Some forgot to answer in terms of $x$ not $g$.

e) $\dot{x} = 20$

$\tan\theta = \frac{-20\sqrt{10}}{20} = -\sqrt{10}$

$\theta = -72.27^\circ$




$\therefore$ $72.27^\circ$ below horizon 1 m

**Question 3 Solutions**

| | |
|---|---|
| (a) Note the restriction $x > 0$<br><br>$v = \frac{k}{x} \;\Rightarrow\; \frac{dx}{dt} = \frac{k}{x}$<br><br>$\int x dx = \int k dt$<br><br>$\frac{x^2}{2k} + C = t$ | 1 mark for integrating correctly |
| We are not given the initial conditions of motion, so at $t = 0$, let $x = x_0$ where $x_0$ is the point A<br><br>$\Rightarrow \; t = \frac{x^2}{2k} - \frac{x_0^2}{2k}$ | 1 mark for understanding the initial conditions and expressing $t$ as a function of $x$<br><br>Note that many students incorrectly presumed that the motion began at the origin, despite the question's stipulation that $x > 0$ |
| The points are equally spaced. Let the distance between them be $d$.<br><br>Then at A: $t = \frac{1}{2k}(x_0^2 - x_0^2) = 0$<br><br>B: $t = \frac{1}{2k}[(x_0 + d)^2 - x_0^2]$<br>$= \frac{1}{2k}(d^2 + 2dx_0)$<br><br>C: $t = \frac{1}{2k}[(x_0 + 2d)^2 - x_0^2]$<br>$= \frac{1}{2k}(4d^2 + 4dx_0)$<br><br>D: $t = \frac{1}{2k}[(x_0 + 3d)^2 - x_0^2]$<br>$= \frac{1}{2k}(9d^2 + 6dx_0)$<br><br>The time from A to B was<br><br>$T_{AB} = \frac{1}{2k}(d^2 + 2dx_0) - 0$<br><br>The time from B to C was<br><br>$T_{BC} = \frac{1}{2k}[(4d^2 + 4dx_0) - (d^2 + 2dx_0)]$<br>$= \frac{1}{2k}[3d^2 + 2dx_0]$<br><br>The time from C to D was<br><br>$T_{CD} = \frac{1}{2k}[(9d^2 + 6dx_0) - (4d^2 + 4dx_0)]$<br>$= \frac{1}{2k}[5d^2 + 2dx_0]$ | 1 mark was awarded for computing the times between each point. |
| To show that the time taken between each | |

| | |
|---|---|
| stage is an AP, we must show that each stage's time differs by the same amount.<br><br>So:<br><br>$T_{BC} - T_{AB} = \frac{1}{2k}[3d^2 + 2dx_0] - \frac{1}{2k}(d^2 + 2dx_0)$<br>$= \frac{2d^2}{2k}$<br>$= \frac{d^2}{k}$<br><br>$T_{CD} - T_{BC} = \frac{1}{2k}[5d^2 + 2dx_0] - \frac{1}{2k}[3d^2 + 2dx_0]$<br>$= \frac{d^2}{k}$<br><br>Therefore, the times taken between each point increase in arithmetic progression, with common difference $\frac{d^2}{k}$ | 1 mark was awarded for showing that the difference in times increased in AP. |
| | Notes:<br><br>This question was attempted by some students without using calculus and simply using the definition of velocity as distance over time.<br><br>If this was done *without* assuming, explicitly or implicitly, that the motion began at the origin, then four marks were awarded. |
| (b)<br><br>(i) $\dot{y} = v\sin\theta - 10t$<br><br>Maximum height is reached when $\dot{y} = 0$<br><br>ie: $10t = v\sin\theta$<br>$t = \frac{v\sin\theta}{10}$ | 1 mark for computing the time the maximum height is reached with respect to $\theta$ |
| Substitute this time into the original function for the vertical component of motion:<br><br>$y = v\left(\frac{v\sin\theta}{10}\right)\sin\theta - 5\left(\frac{v^2\sin^2\theta}{100}\right)$<br>$= \frac{v^2\sin^2\theta}{10} - \frac{v^2\sin^2\theta}{20}$<br>$= \frac{v^2\sin^2\theta}{20}$ | 1 mark for substituting and simplifying |
| (ii) The horizontal range can be found by finding the times at which the vertical component is zero. That is, set $y = 0$ and solve for a non-zero value of $t$.<br><br>$vt\sin\theta - 5t^2 = 0$<br>$t(v\sin\theta - 5t) = 0$<br><br>$t = 0$ or $v\sin\theta - 5t = 0$<br>$t = \frac{v\sin\theta}{5}$ | 1 mark for finding the time at which the arrow |

| | |
|---|---|
| | lands in terms of $\theta$ |
| To find the horizontal range, substitute this value of $t$ back into the $x$ function<br><br>$x = v\left(\frac{v\sin\theta}{5}\right)\cos\theta$<br>$= \frac{v^2\sin\theta\cos\theta}{5}$<br>$= \frac{v^2}{5} \times \frac{\sin 2\theta}{2}$<br>$= \frac{v^2\sin 2\theta}{10}$ | 1 mark for substituting and simplifying to this point |
| (iii) $-1 \leq \sin 2\theta \leq 1 \Rightarrow$ the horizontal range is maximum when $\sin 2\theta = 1$<br><br>$\sin 2\theta = 1$<br>$2\theta = 90°, 450° \ldots$<br>$\theta = 45°, 225° \ldots$<br><br>But $\theta$ is in the first quadrant.<br><br>$\therefore \theta = 45°$ and the range is maximum at this value of $\theta$ | 1 mark for explaining that $\sin 2\theta = 1$ will be where maximum is reached **and** correctly computing the value of $\theta$ that satisfies the constraints |

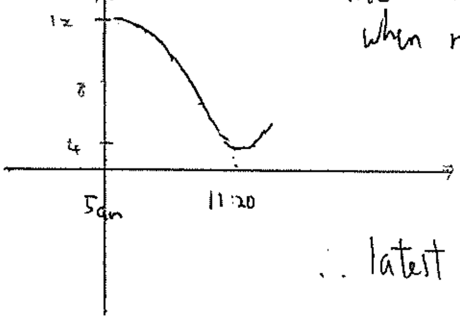
MATHEMATICS Ext1 : Question 4

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| c(i)(i) $1 \times 7! = 5040$ | 1 | Needed to calculate |
| (ii) Jun can sit in between Kate and Ashley in 2 ways<br>Number of ways $= 2 \times 5!$<br>$= 240$ | 1 | |
| (b) (i) $v^2 = -5 + 6x - x^2$<br>when $v = 0$<br>$x^2 - 6x + 5 = 0$ | 1 | |
| $(x-5)(x-1) = 0$<br>$x = 1$ or $5$<br>particle oscillates between 1 and 5 | 1 | |
| (ii) centre of motion<br>$x = \frac{1+5}{2}$<br>$= \underline{3}$ | 1 | |
| (~~iii~~) ~~Ans~~ | | |

| MATHEMATICS Extension 1 : Question 4 | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |
| (iii) Max speed occurs at centre of motion. $x = 3$ | | Do not use other physics formula. |
| $v^2 = -5 + 6 \times 3 - 3^2$ | 1 | |
| $v^2 = 4$ | | |
| $v = \pm 2$ | 1 | |
| so speed is 2 m/s | | |
| (iv) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ | 1 | |
| $= \frac{1}{2}\frac{d}{dx}\left(-5 + 6x - x^2\right)$ | | |
| $= \frac{1}{2}(6 - 2x)$ | | |
| $= \underline{3 - x}$ | 1 | |

| MATHEMATICS Extension 1: Question 5. | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |
| a) i $P(2\text{ black}) = {}^{10}C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8$ | 1 | |
| $= 0.302$ (3dp) | 1 | |
| ii) $P(0\text{ black}) + P(1\text{ black})$ | | |
| $= {}^{10}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} + {}^{10}C_1\left(\frac{1}{5}\right)\left(\frac{4}{9}\right)^9$ | | |
| $= 0.376$ (3 dp) | 1 | |
| iii) $P(> 2\text{black}) = 1 - 0.376 - 0.302$ | | |
| $= 0.322$ | 1 | |
| Since $P(< 2\text{black}) > P(> 2\text{black})$ | | |
| It is more likely that the sample contains fewer than 2 black jelly beans. | 1 | |

**MATHEMATICS Extension 1 : Question 5**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| b) Period $= 12$ hrs $40$ mins $= 12\frac{2}{3}$ (in hours). | | |
| $\therefore \frac{2\pi}{n} = \frac{38}{3}, \quad n = \frac{3\pi}{19}$ | 1 | |
| $A = 4$, centre of motion is at $8$. | | |
| $\therefore x = 8 + 4\cos\left(\frac{3\pi}{19}t + \alpha\right)$ | | |
| when $t = 0$ (5:00am), $x = 12$. $\therefore 12 = 8 + 4\cos\alpha$ $\therefore \alpha = 0$ | | |
| $\therefore x = 8 + 4\cos\left(\frac{3\pi}{19}t\right)$ * | 1 | ✓ with correct setting. (You had to show working for $\alpha$ or make explicit that $t=0$ was at 5am) |
| $x = 7.5 \Rightarrow 7.5 = 8 + 4\cos\left(\frac{3\pi}{19}t\right)$ | | |
| $\therefore 4\cos\left(\frac{3\pi}{19}t\right) = -\frac{1}{2}$ | | |
| $\cos\frac{3\pi}{19}t = \frac{-1}{8}$ | | |
| $\frac{3\pi}{19}t = \cos^{-1}\left(\frac{-1}{8}\right)$ | 1 | |
| $\frac{3\pi}{19}t \approx 2n\pi \pm 1.696$ | | |
| $3\pi t = 38n\pi \pm 32.226$ | | |
| $t = \frac{38n}{3} \pm 3.42$ | | |
|   $\therefore$ the last possible time occurs when $n = 0$., and since $t \geq 0$ $\therefore t = 3.42 = 3$ hrs $25$ mins. | | |
| $\therefore$ latest time is 5:00am + 3hrs 25mins $= 8$ hrs $25$ mins | 1 | <u>Note</u>: If you had made a mistake earlier (especially finding $n$, then the first answer from your general solution may not be the answer required) |

**MATHEMATICS Extension 1 : Question...6...**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| a) i) $P(\text{all tall}) = \left(\frac{3}{4}\right)^5$ | | |
| $= \frac{243}{1024}$ | 1 | |
| ii) $P(\geq 3 \text{ tall}) = {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 +$ | | |
| ${}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1 + {}^5C_5\left(\frac{3}{4}\right)^5$ | 1 | |
| $= \frac{459}{512}$ | 1 | |
| b) i) $x = \sqrt{3}\cos 3t - \sin 3t$ | | |
| $\dot{x} = -3\sqrt{3}\sin 3t - 3\cos 3t$ | 1 | |
| $\ddot{x} = -9\sqrt{3}\cos 3t + 9\sin 3t$ | | |
| $= -9(\sqrt{3}\cos 3t + \sin 3t)$ | | |
| $= -9x$ | | |
| $\therefore \ddot{x} = -3^2 x$ which is in simple haromic motion in the form $\ddot{x} = -n^2 x$ | 1 | |
| ii) $P = \frac{2\pi}{n}$, from (i) $n = 3$ | | |
| $\therefore P = \frac{2\pi}{3}$ | 1 | |
| c) Let the mass at time $t$ be $M$ units and the initial mass be $M_0$ units | | |
| $M = M_0 e^{-kt}$ | 1 | |
| when $t = 5580$, $M = \frac{1}{2}M_0$ | | |
| $\frac{1}{2}M_0 = M_0 e^{-5580k}$ | | |
| $\frac{1}{2} = e^{-5580k}$ | | |
| $-5580k = \ln\left(\frac{1}{2}\right)$ | | |
| $k = \frac{\ln\left(\frac{1}{2}\right)}{-5580}$ | 1 | |

MATHEMATICS Extension 1 : Question........

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| $k = \frac{\ln 2}{5580}$ | | |
| if $M = 0.3M_0$ then | | |
| $0.3M_0 = M_0 e^{-\frac{\ln 2}{5580}t}$ | | |
| $-\frac{\ln 2}{5580}t = \ln 0.3$ | | |
| $t = \dfrac{\ln 0.3}{-\frac{\ln 2}{5580}}$ | | |
| $= 9692.2680\ldots$ | | |
| $= 9692$ (nearest year) | 1 | |
| $\therefore$ the fossil is around 9692 years old. | | |

**MATHEMATICS Extension 1: Question 7**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (i) $\frac{d}{dt}P(t) = kP(t)$ | | Students wasted time trying to prove this question said write down |
| $\therefore P(t) = P(0)e^{kt}$ is a solution | | |
| $3P(0) = P(0)e^{2k}$ | 1 | |
| $e^{2k} = 3$ | | |
| $2k = \ln 3$ | | |
| $k = \frac{\ln 3}{2}$ | 1 | |
| $\therefore P(t) = P(0)e^{\frac{\ln 3}{2}t}$ | 1 | |
| (ii) $t=0$ $P(0)=10$ | | |
| $P(t) = P(0)e^{10\times\frac{\ln 3}{2}}$ | | |
| $= 51.96$ | 1 | |
| estimated number is 52 or 51 whole kangaroos | | |

**MATHEMATICS Extension 1 : Question** 7

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (b) $\frac{d}{dt}P(t) = k\,P(t)(L - P(t))$ | | |
| (i) $P(t) = \dfrac{Lc}{c+e^{-kLt}}$ | | |
| $P'(t) = \dfrac{0 \times c+e^{-kLT} - Lc \times -kLe^{-kLt}}{c+e^{-kLt}}$ | 1 | Many ways of doing this students who got a -ve sign and then ignored it needed to realise they were fudging a step. |
| $= \dfrac{L^2ck\,e^{-kLt}}{c+e^{-kLt}}$ | | |
| $= \dfrac{kLc}{c+e^{-kLt}} \cdot \dfrac{L\,e^{-kLt}}{c+e^{-kLt}}$ | 1 | |
| $= k\,P(t)\left[\dfrac{L(c+e^{-kt})}{c+e^{-kt}} - \dfrac{Lc}{c+e^{-kt}}\right]$ | 4 | |
| | 1 | |
| $= k\,P(t)\left[L - P(t)\right]$ | | |
| (ii) $P(t) = \dfrac{Lc}{c+e^{-kLt}}$ As $t \to \infty$ | | |
| $e^{-kLt} \to 0$ $\quad P(t) \to \dfrac{Lc}{c} \to L$ | | |
| ∴ As time increases the population reaches the limit L, the limitations of population wrt food etc | | |

| MATHEMATICS Extension 1: Question 7 | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |
| (iii) $P'(t) = k\,P(t)(L - P(t))$ | | To gain |
| as $t \to \infty$ $P(t) \to L$ | 1 | This mark |
| $P'(t) \to kL(L-L) = 0$ | | you had |
| The population growth | | to show |
| decreases and ~~approaches~~ | | why it was |
| the population gets close | | 0 not state |
| to constant. | | it and |
| ie # born = # die. | | make a sensible |
| | | comment. |