Question One (9 marks) **Marks**

a) Use five function values of Simpson's Rule to approximate the value of $\int_1^3 \log_{10} x \, dx$, correct to 2 decimal places. 3

bi) Water is leaking out through a hole of the bath-tub at a rate $\frac{dv}{dt} = -8(25 - t)$, where v cubic centimetres is the volume of the liquid in the bath-tub at time t seconds. 2

Initially there was 2500 cubic centimetres of water in the bath-tub.

Find v in terms of t.

ii) Hence find the time taken for the bath-tub to be emptied. 1

c) Find the exact value of $\int_0^1 \frac{t \, dt}{\sqrt{1+t}}$, use the substitution $u = 1 + t$. 3

Question Two (9 marks) Start a New Page

a) Find the derivative of $\sin^{-1}(\frac{2x-1}{3})$ and simplify. 2

b) If ${}^{2n}C_{n+4} = {}^{2n}C_{2n-9,}$ find the value(s) of n. 2

c) Prove the identity: ${}^{n}C_{k} = {}^{n-1}C_{k} + {}^{n-1}C_{k-1}$, $1 \le k \le n-1$ 2

d) Find the values of a and n if $(1-18x+\frac{297}{2}x^2)$ are the first three terms in the expansion of $(1+ax)^n$. 3

Question Three (9 marks) Start a New page

a) Find $\int \frac{3dx}{\sqrt{1-4x^2}}$. 2

bi) On the same set of axes, sketch the graphs of $y = \sqrt{4-x^2}$ and $y = 8 - 2x^2$. 1

ii) Find the area bounded by these two graphs. 3

iii) Find the volume of the solid when this region is rotated about the x-axis. 3

Question Four (9 marks) Start a New Page

a) Find the coefficient of x^6 in the expansion of $y=(1-4x)^2(2+3x^2)^6$. 3

b) Consider the function $f(x)=(x+2)^2-9, \quad -2\le x\le 2$

i) Find the equation of the inverse function $f^{-1}(x)$. 2

ii) On the same diagram, sketch the graphs of $y=f(x)$ and $y=f^{-1}(x)$. 2

Showing clearly the co-ordinates of the end-points and the intercepts on the co-ordinate axes.

iii) Find the *x*-coordinate of the point of intersection of the curves $y=f(x)$ and $y=f^{-1}(x)$, giving the answer in exact form. 2

Question Five (9 marks) Start a New page

a) Evaluate $\int_0^2 4x\sqrt{1-\frac{x}{2}}\,dx$ using the substitution $u=1-\frac{x}{2}$. 3

bi) Find the term independent of x in the expansion of $\left(2x+\frac{3}{x^2}\right)^9$. 2

ii) T_k is the k^{th} term in the expansion of $\left(2x+\frac{3}{x^2}\right)^9$ with decreasing powers of *x*, k =1, 2, 3, etc. 2

Find $\frac{T_{k+1}}{T_k}$ and simplify.

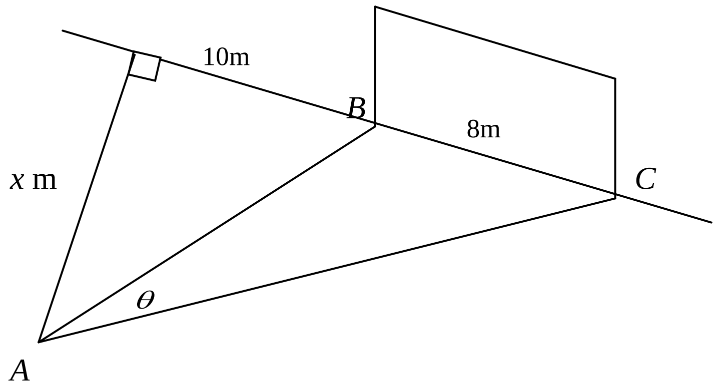
iii) Find the numerically greatest term in the expansion when $x=1$. 2

Question Six (9 marks) Start a New page

a) If $\cos^{-1}x - \sin^{-1}x = k$, where $-\frac{\pi}{2} \le k \le \frac{3\pi}{2}$, show that $x = \frac{1}{\sqrt{2}}\left[\cos\frac{k}{2} - \sin\frac{k}{2}\right]$. 3

b) As shown in the diagram below, a soccer player *A* is *x* metres from a goal line of a soccer field.

He takes a shot at the goal *BC*, with the ball not leaving the ground.



Not to scale

i) Show that the angle θ he must shoot within is given by $\theta = \tan^{-1}\left[\frac{8x}{180 + x^2}\right]$ where he is 10 metres to one side of the near goal post and 18 metres to the same side of the far post. 3

ii) Find the value of *x* which makes the angle θ a maximum. 3

(leave your answer in exact form).

Question Seven (9 marks) Start a New Page

a) Find the exact value of $\sin\left[\cos^{-1}\frac{2}{3} + \tan^{-1}\left(-\frac{3}{4}\right)\right]$. 3

bi) Sketch the graph of $f(x) = 2\cos^{-1}(x-1)$. 2

ii) Find the area enclosed by the graph of $y = f(x)$ and the axes. 4

End OF PAPER

MATHEMATICS Extension 1 : Question....1....		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) $\int_1^3 \log_{10} x \, dx$ $\doteq \frac{1}{6}\left[\log_{10}1 + 4\log_{10}\frac{3}{2} + \log_{10}2\right] + \frac{1}{6}\left[\log_{10}2 + 4\log_{10}\frac{5}{2} + \log_{10}3\right]$ (1) $= \frac{1}{6}(1.005395032) + \frac{1}{6}(2.369911285)$ $= \frac{1}{6} \times 3.375306317$ (1) $= 0.562551052$ $= 0.56$ (2dp.) (1)		* If students got 0.57 because they rounded too early, they lost a mark.
(b)(i) $\frac{dV}{dt} = -200 + 8t$ $V = -200t + 4t^2 + C$ when $t=0$, $V = 2500$ $\therefore C = 2500$ $\therefore V = -200t + 4t^2 + 2500$	(1) (1)	
(ii) when $V=0$, $t = ??$ $0 = 4t^2 - 200t + 2500$ $0 = t^2 - 50t + 625$ $0 = (t-25)(t-25)$ $\therefore t = 25$ seconds only	 (1)	
(c) $\int_0^1 \frac{t\,dt}{\sqrt{1+t}}$ when $u = 1+t$, $t = u-1$; when $t=0, u=1$; when $t=1, u=2$ $\therefore I = \int_1^2 \frac{(u-1)\,du}{\sqrt{u}} = \int_1^2 (\sqrt{u} - u^{-1/2})\,du$ $= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^2$ $= \frac{2}{3}.2^{3/2} - 2.2^{1/2} - \frac{2}{3} + 2$ $= \frac{4}{3} - \frac{2\sqrt{2}}{3}$	 (1) (1) (1)	* If they got $(1-\ln 2) = 1$ mark * If you forget to change the limits you lost 2 marks

MATHEMATICS Extension 1 : Question 2		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
a) Derivative of $\sin^{-1}\left(\frac{2x-1}{3}\right)$	2 marks	
$y' = \frac{1}{\sqrt{1-\left(\frac{2x-1}{3}\right)^2}} \times \frac{d}{dx}\left(\frac{2x-1}{3}\right)$		1 mark for finding the derivative
$y' = \frac{1}{\sqrt{1-\left(\frac{2x-1}{3}\right)^2}} \times \frac{2}{3}$ ✓	✓	1 mark for simplifying to either $\frac{1}{\sqrt{2-x^2+x}}$ or $\frac{1}{\sqrt{(x+1)(2-x)}}$
$y' = \frac{2}{3\sqrt{1-\frac{(4x^2-4x+1)}{9}}}$		
$y' = \frac{2}{\cancel{3}\sqrt{\frac{9-4x^2+4x-1}{\cancel{9}}}}$		
$y' = \frac{2}{\sqrt{8-4x^2+4x}}$		
$y' = \frac{2}{\sqrt{4(2-x^2+x)}}$		
$y' = \frac{\cancel{2}}{\cancel{2}\sqrt{2-x^2+x}}$		
$y' = \frac{1}{\sqrt{2-x^2+x}}$.		

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks Awarded	Marker's Comments
b) If ${}^{2n}C_{n+4} = {}^{2n}C_{2n-9}$ find values of n. Method 1 $n+4 = 2n-9$ $n = 13$ ✓ Since ${}^{n}C_{k} = {}^{n}C_{n-k}$ ${}^{2n}C_{n+4} = {}^{2n}C_{2n-(2n-9)}$ ${}^{2n}C_{n+4} = {}^{2n}C_{9}$ $\therefore n+4 = 9$ $\therefore n = 5$ ✓	2 marks	1 mark for getting $n=13$ 1 mark for getting $n=5$.
Method 2 $\dfrac{2n!}{(n+4)!(2n-n-4)} = \dfrac{2n!}{(2n-9)!(2n-2n+9)!}$ $\dfrac{1}{(n+4)!(n-4)!} = \dfrac{1}{(2n-9)!\,9!}$ $9!(2n-9)! = (n+4)!(n-4)!$		1 mark for reaching $\dfrac{1}{9!(2n-9)!} = \dfrac{1}{(n+4)!(n-4)!}$ Not correct $9(2n-9) = n^2-16$ $18n-81 = n^2-16$ $n^2-18n+65=0$ $\therefore n=13$ or $n=5$

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MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
(c) Prove $^{n}C_{k} = {^{n-1}C_{k}} + {^{n-1}C_{k-1}}$ for $1 \le k \le n-1$	2 marks	Prove LHS = RHS
LHS $^{n}C_{k} = \dfrac{n!}{(n-k)!\,k!}$		
RHS $^{n-1}C_{k} + {^{n-1}C_{k-1}}$		
$\dfrac{(n-1)!}{((n-1)-k)!\,k!} + \dfrac{(n-1)!}{(n-1-(k-1))!\,(k-1)!}$ $\dfrac{(n-1)!}{(n-k-1)!\,k!} + \dfrac{(n-1)!}{(n-k)!\,(k-1)!}$	✓	1 mark.
$= \dfrac{(n-1)!\,(n-k) + k\,(n-1)!}{k!\,(n-k)!}$		
$= \dfrac{(n-1)!\,[n-k+k]}{k!\,(n-k)!}$		
$= \dfrac{n\,(n-1)!}{k!\,(n-k)!}$	✓	1 mark for subsequent working
$= \dfrac{n!}{k!\,(n-k)!} = {^{n}C_{k}}$.		
= LHS		

MATHEMATICS Extension 1 : Question 2		
Suggested Solutions	Marks	Marker's Comments

d) Expand

$(1+ax)^n = \sum_{k=0}^{n} {}^nC_k (1)^{n-k} a^k x^k$.

$(1+ax)^n = 1 + {}^nC_2\, ax + {}^nC_3 (ax)^2 + \ldots$

when $k=0$ $\quad {}^nC_0 (1)^{n-1} = 1$

when $k=1$ $\quad {}^nC_1 (1)^{n-k} a^1 x^1 = {}^nC_1\, ax$

From series

$${}^nC_1\, ax = -18x$$

$$nax = -18x$$

$$\therefore na = -18 \longrightarrow eq\ 1$$

✓

For $k=2$ $\quad {}^nC_2\, a^2 x^2 = \frac{297}{2} x^2$

$$\therefore {}^nC_2\, a^2 = \frac{297}{2}$$

$$\frac{n!\, a^2}{(n-2)!\, 2!} = \frac{297}{2}$$

$$\frac{n!\, a^2}{(n-2)!} = 297$$

$$a^2(n-1)n = 297$$

$$a^2(n^2-n) = 297$$

Sub in a from eq 1

$$\left(\frac{-18}{n}\right)^2 (n^2-n) = 297$$

$$\frac{324}{n^2}(n^2-n) = 297$$

$$= 324 - \frac{324}{n} = 297$$

$$= \frac{324}{n} = 27$$

$$\therefore n = 12$$ ✓

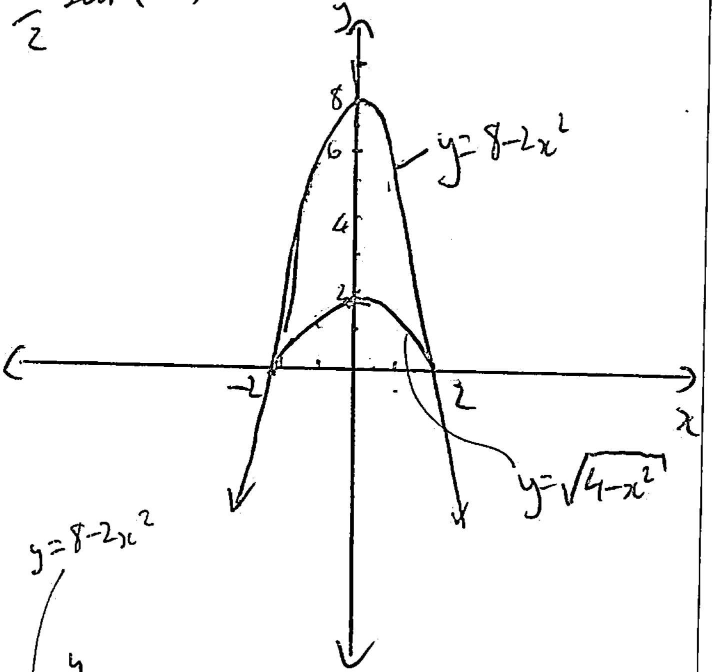
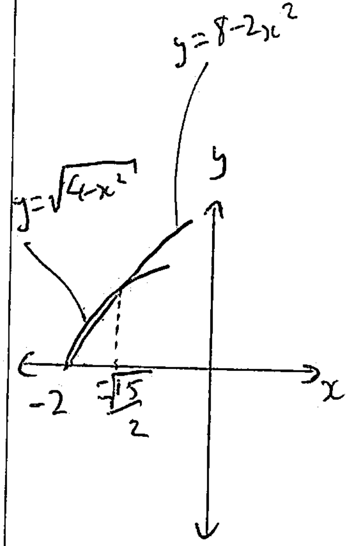
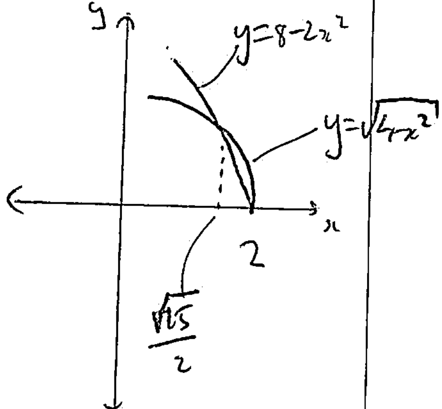
from eq 1

$$a = \frac{-18}{n}$$

$$a = \frac{-18}{12}$$

$$a = \frac{-3}{2}$$ ✓

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $\int \frac{3\,dx}{\sqrt{1-4x^2}}$		
$= \int \frac{3}{2} \frac{dx}{\sqrt{\frac{1}{4}-x^2}}$	1	Students were required to show one intermediate step to earn both marks.
$= \frac{3}{2} \sin^{-1}(2x) + c$	1	
(b) (i) 	1	the mark was awarded for the top, less detailed graph.
 		It is worth noting that the graphs intersect at $x = \frac{\pm\sqrt{15}}{2}$
Detail		

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks Awarded	Marker's Comments
(ii) Find points of intersection:		
$8-2x^2=\sqrt{4-x^2}$		
$64-32x^2+4x^4=4-x^2$		
$4x^4-31x^2+60=0$		
Let $m=x^2$		
$4m^2-31m+60=0$		
$m=4,\ \frac{15}{4}$		
$\Rightarrow x=\pm 2,\ \pm\frac{\sqrt{15}}{2}$	1	
Case 1: $-2\leq x<-\frac{\sqrt{15}}{2}$		
$y=\sqrt{4-x^2}$ is upper function		
Case 2: $-\frac{\sqrt{15}}{2}\leq x\leq\frac{\sqrt{15}}{2}$		
$y=8-2x^2$ is upper function		
Case 3: $\frac{\sqrt{15}}{2}<x\leq 2$		
$y=\sqrt{4-x^2}$ is upper function	1	

Suggested Solutions	Marks	Marker's Comments
xx: $\displaystyle\int_{-2}^{-\frac{\sqrt{15}}{2}} \sqrt{4-x^2}-(8-2x^2)\,dx$ $+\displaystyle\int_{-\frac{\sqrt{15}}{2}}^{\frac{\sqrt{15}}{2}} 8-2x^2-\sqrt{4-x^2}\,dx$ $+\displaystyle\int_{\frac{\sqrt{15}}{2}}^{2} \sqrt{4-x^2}-(8-2x^2)\,dx$ $=2\pi-8\sin^{-1}\left(\frac{\sqrt{15}}{4}\right)+\frac{63\sqrt{15}-128}{6}$ units^2. Alternative: We disregard the points of intersection at $x=\pm\frac{\sqrt{15}}{2}$. Note that this question is assessing the procedure $A=\displaystyle\int_a^b f_1(x)-f_2(x)\,dx$ Since the integrals above require techniques beyond the scope of this course, this simplification is sufficient to earn full marks. Assume $8-2x^2\geq\sqrt{4-x^2}$ for $-2\leq x\leq 2$	1	Students were only required to set up integrals for 3 marks, or make a substantive and clear effort in setting them up.

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks	Marker's Comments
So		
Area $2\int_{-2}^{2} 8-2x^2-\sqrt{4-x^2}\,dx$		
$= 2\int_{0}^{2} 8-2x^2-\sqrt{4-x^2}\,dx$ (even function)	1	
Note $\sqrt{4-x^2}$ is a semicircle.		
So Area $= 2\left[8x-\frac{2x^3}{3}\right]_0^2 - \frac{2\pi(2)^2}{4}$	1	
$= \left(16-\frac{16}{3}\right) - 2\pi$		
$= \frac{64}{3} - 2\pi$ units^2	1	
(iii) Using above simplification:		
$V = \int_{-2}^{2} \pi(8-2x^2)^2 - \pi(4-x^2)\,dx$	1	
$= 2\pi\int_0^2 64-32x^2+4x^4-4+x^2\,dx$ (even function)		
$= 2\pi\left[64x-\frac{32x^3}{3}+\frac{4x^5}{5}-4x+\frac{x^3}{3}\right]_0^2$	1	

MATHEMATICS Extension 1: Question 3

Suggested Solutions	Marks	Marker's Comments
$= 2\pi\left(128 - \frac{256}{3} + \frac{4(32)}{5} - 8 + \frac{8}{3}\right)$		
$= \frac{1888\pi}{15}$ units^2	1	
Alternative: If the $x = \pm\frac{\sqrt{15}}{2}$ intercepts are used then we have 3 cases so: $V = \pi\int_{-2}^{-\frac{\sqrt{15}}{2}} \left(\sqrt{4-x^2} - 8 + 2x^2\right)^2 dx + \pi\int_{\frac{\sqrt{15}}{2}}^{+\frac{\sqrt{15}}{2}} \left(8 - 2x^2 - \sqrt{4-x^2}\right)^2 dx + \pi\int_{\frac{\sqrt{15}}{2}}^{2} \left(\sqrt{4-x^2} - 8 + 2x^2\right)^2 dx$	3	Three marks were awarded if the student set up the three integrals.

Assessment Test 1 Term 4 2016 Extension 1

Question 4

(a) $y=(1-4x)^2(2+3x^2)^6$

$y=(1-8x+16x^2)\left({}^6C_0 2^6+{}^6C_1 2^5(3x^2)+{}^6C_2 2^4(3x^2)^2+{}^6C_3 2^3(3x^2)^3+{}^6C_4 2^2(3x^2)^4+{}^6C_5 2(3x^2)^5+{}^6C_6(3x^2)^6\right)$

Term in $x^6 = {}^6C_3 2^3(3x^2)^3+16x^2({}^6C_2 2^4(3x^2)^2)$

$= 4320x^6+34560x^6$ 1 mark for each term

$= 38\,880x^6$

$\therefore$ Coefficient of x^6 is $38\,880$ 1 mark for answer

Total of 3 marks

Mistakes – lose a mark for not using ${}^6C_3$ (i.e. 20) and ${}^6C_2$ (i.e. 15) to multiply by $2^3\times 3^3$ and $16\times 2^4\times 3^2$ respectively

(ii) If the students left off the squared on the x for the $(2+3x^2)^6$ and made it $(2+3x)^6$ the answer was 55161

i.e. Term in $x^6 = 1\times {}^6C_6(3x)^6-8x\times {}^6C_5 2(3x)^5+16x^2\times {}^6C_4 2^2(3x)^4$

$=729x^6-23328x^6+77760x^6$

$=55161x^6$

$\therefore$ Coeff. of x^6 is 55161. Max. of 2 marks

(iii) If the students put a squared on the x in the first bracket i.e. instead of $(1-4x)^2$ they wrote $(1-4x^2)^2$ then the term in x^6 is as follows:

$(1-8x^2+16x^4)({}^6C_0 2^6+{}^6C_1 2^5(3x^2)+{}^6C_2 2^4(3x^2)^2+\dots)$

Term in x^6 is $-8x^2({}^6C_2 2^4(3x^2)^2)+-16x^4\times {}^6C_1 2^5(3x^2)$

$=-17280x^6-9216x^6$

$=-26496x^6$

$\therefore$ Coeff. of x^6 is -26496 Scored 2 marks if no subsequent error.

(iv) Students who did not have the power of 2 in their answer i.e.

${}^6C_3(3x^2)^3+16x^2\times {}^6C_2(3x^2)^2$

$540 + 2160 = 2700$ received 2 marks.

Question 4

(b)(i) $f(x) = (x+2)^2 - 9$ for $-2 \le x \le 2$

$\therefore x = (y+2)^2 - 9$

$x + 9 = (y+2)^2$

$y + 2 = \pm\sqrt{x+9}$

$y = \pm\sqrt{x+9} - 2$

but $-2 \le y \le 2$ for inverse function

$\therefore y = \sqrt{x+9} - 2$ for $-9 \le x \le 7$

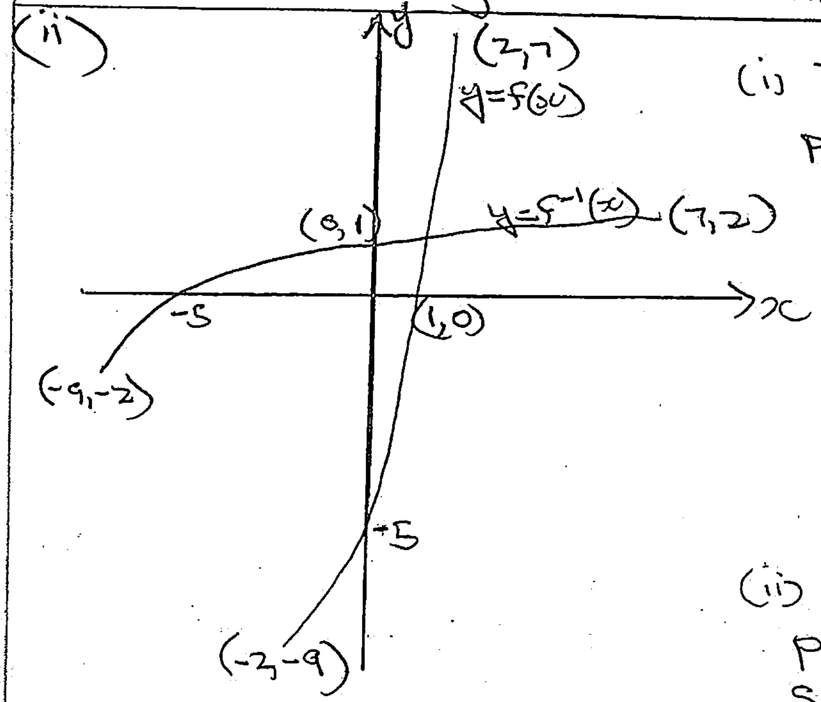
No marks were paid for just interchanging the x and y. $y = -\sqrt{x+9} - 2$ needed to be elimated and a new domain for the inverse function.

i.e. $y = \sqrt{x+9} - 2$ for $-9 \le x \le 7$ or $-2 \le y \le 2$

1 mark for restricted domain

1 mark for inverse function after excluding the -ve i.e. $y = -\sqrt{x+9} - 2$

(ii)



(i) There are 8 essential points on the graph. 4 correct points scored 1 mark if there was no subsequent error. If the whole parabolas were drawn then students lost the mark for 4 correct points.

(ii) Students needed all 8 points and a good shape to score 2 marks

(iii) Graphs intersect on $y = x$

$y = x$ ..... (1)

$y = (x+2)^2 - 9$ ..... (2)

(1) = (2)

$x = (x+2)^2 - 9$

$x = x^2 + 4x + 4 - 9$

$0 = x^2 + 3x - 5$

1 mark for the quadratic eqn.

3

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4\times 1\times -5}}{2\times 1}$$

$$= \frac{-3 \pm \sqrt{9+20}}{2}$$

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

but $x > 0$

$$\therefore x = \frac{-3+\sqrt{29}}{2}$$

1 mark for answer

Some students went the very long way and equated both the function and the inverse function. They then had a very nasty polynomial for which they found a factor and used polynomial division to finally get down to the above quadratic. No marks were awarded for this method if they did not end up with the correct quadratic equation.

JRAHS Y12 T4 MATHEMATICS Extension 1 : Question 5 Task 1 : 2016

Suggested Solutions	Marks Awarded	Marker's Comments
QUESTION 5		
a) $I = \int_0^2 4x\sqrt{1 - x/2}\, dx$		
Let $u = 1 - x/2$ $\Rightarrow du = -\frac{1}{2}dx$ or $dx = -2du$ and $x = 2 - 2u$; when $x = 0$, $u = 1$; when $x = 2$, $u = 0$		
$\therefore I = \int_1^0 (8 - 8u)\sqrt{u}\,(-2du)$	1	For correctly converting integrand from x to u and changing limits.
$= -16\int_1^0 (\sqrt{u} - u^{3/2})\, du$		
$= 16\int_0^1 (u^{\frac{1}{2}} - u^{3/2})\, du$		
$= 16\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$	1	For correctly integrating
$= 16\left(\frac{2}{3} - \frac{2}{5} - 0\right)$		
$= \frac{64}{15}$		
$= 4\frac{4}{15}$	1	For correct final answer

JRAHS Y12 Task 1 MATHEMATICS Extension 1 : Question 5 T4 : 2016		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
b) (i) $\left(2x+\frac{3}{x^2}\right)^9$: Term independent of x		
$\left(2x+\frac{3}{x^2}\right)^9 = \sum_{r=0}^{9} {}^9C_r\,(2x)^r\left(\frac{3}{x^2}\right)^{9-r}$		
$\therefore T_r = {}^9C_r\, 2^r \cdot 3^{9-r} \cdot x^{r-18+2r}$		
$= {}^9C_r\, 2^r \cdot 3^{9-r} \cdot x^{3r-18}$		
Hence Term independent of x occurs when $3r-18=0$ $\therefore r=6$	1	Correct evaluation of r
Hence the term is $T_6 = {}^9C_6 \cdot 2^6 \cdot 3^3 \cdot x^0$		
$= 145\,152$	1	Correct Term
OR If $T_r = {}^9C_r\, 2^{9-r} \cdot 3^r \cdot x^{9-r-2r}$, then $r=3$, in which case $T_3 = {}^9C_3 \cdot 2^6 \cdot 3^3 \cdot x^0$		
$= 145\,152$		

JRAHS Task 1 Y12 MATHEMATICS Extension 1 : Question 5 Term 4 2016

Suggested Solutions	Marks Awarded	Marker's Comments
b ii) Note: $T_k = {}^9C_k (2x)^{9-k}\left(\frac{3}{x^2}\right)^k$ yields decreasing powers of x while $T_k = {}^9C_k (2x)^k \cdot \left(\frac{3}{x^2}\right)^{9-k}$ yield increasing powers of x		
Now $\frac{T_{k+1}}{T_k} = \frac{{}^9C_k\,(2x)^{9-k} \cdot (3x^{-2})^k}{{}^9C_{k-1}\,(2x)^{10-k} \cdot (3x^{-2})^{k-1}}$ $= \frac{{}^9C_k\, 2^{9-k} \cdot 3^k \cdot x^{9-3k}}{{}^9C_{k-1} \cdot 2^{10-k} \cdot 3^{k-1} \cdot x^{12-3k}}$ $= \frac{{}^9C_k \cdot 3 \cdot}{{}^9C_{k-1} \cdot 2 \cdot x^3}$	1	For recognising the correct k^{th} term which gives decreasing powers of x and for obtaining a ratio for T_{k+1} and T_k

JRAHS Task 1 Y12 MATHEMATICS Extension 1 : Question 5 Term 4 : 2016

Suggested Solutions	Marks Awarded	Marker's Comments
$= \dfrac{9!}{k!(9-k)!} \times \dfrac{(k-1)!\,(10-k)!}{9!} \cdot \dfrac{3}{2x^3}$		Note: $\dfrac{k!}{(k-1)!} = k$
$= \dfrac{10-k}{k} \cdot \dfrac{3}{2x^3} = \dfrac{T_{k+1}}{T_k}$	1	for correct expression
(Note $k = 1, 2, 3 \ldots$) $k \neq 0$		
If $\dfrac{T_{k+1}}{T_k} = \dfrac{{}^9C_{k+1} \cdot (2x)^{8-k} \cdot \left(\frac{3}{x^2}\right)^{k+1}}{{}^9C_k \cdot (2x)^{9-k} \cdot \left(\frac{3}{x^2}\right)^k}$		
$= \dfrac{(9-k)}{(k+1)} \cdot \dfrac{3}{2x^3}$		

MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks Awarded	Marker's Comments
b iii) When $x = 1$ $\frac{T_{k+1}}{T_k} = \frac{3(10-k)}{2k}$ If T_{k+1} is the greatest coefficient, then $T_{k+1} \geq T_k$ i.e $30 - 3k \geq 2k$ $k \leq 6$	1	For correct k^{th}-term
$\therefore$ Numerically greatest term is $T_{k+1} = {}^9C_k\, 2^{9-k} \cdot 3^k$ for $x = 1$ i.e $T_7 = {}^9C_6\, 2^3 \cdot 3^6$ $= 489\,888$	1	For correct numerically greatest term
Note $T_{5+1} = {}^9C_5\, 2^4 \cdot 3^5$ $= 489\,888$		

Mathematics Ext I Question 6

Suggested Solutions	Marks	Marker's Comments
6. a)		
Method 1		
(1) $\cos^{-1}x - \sin^{-1}x = k$		
(2) $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$	1	
(1) + (2) $\longrightarrow$ $2\cos^{-1}x = k + \frac{\pi}{2}$		
$\cos^{-1}x = \frac{k}{2} + \frac{\pi}{4}$	1	
$x = \cos\left(\frac{k}{2} + \frac{\pi}{4}\right)$		
$= \cos\frac{k}{2}\cos\frac{\pi}{4} - \sin\frac{k}{2}\sin\frac{\pi}{4}$		
$= \frac{1}{\sqrt{2}}\cos\frac{k}{2} - \frac{1}{\sqrt{2}}\sin\frac{k}{2}$		
$= \frac{1}{\sqrt{2}}\left[\cos\frac{k}{2} - \sin\frac{k}{2}\right]$	1	
$\downarrow$		
Same marking criteria if you did (1) − (2) to solve a sin equation.		

Mathematics ____ Question 6

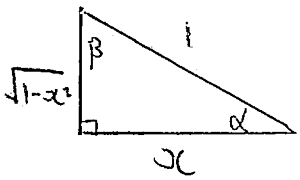
Suggested Solutions	Marks	Marker's Comments

Method 2 — Maximum 2/3 !!!

Let $\alpha = \cos^{-1}x$, $\beta = \sin^{-1}x$

$\sin(\alpha - \beta) = \sin k$

$\therefore \sin k = \sin\alpha\cos\beta - \cos\alpha\sin\beta$



$= \sqrt{1-x^2}\sqrt{1-x^2} - x^2$

$= 1 - 2x^2$ —— (1)

$\therefore 2x^2 = 1 - \sin k$

$= 1 - 2\sin\frac{k}{2}\cos\frac{k}{2}$ (double angle formula) —— (1)

$= \left(\sin^2\frac{k}{2} + \cos^2\frac{k}{2}\right) - 2\sin\frac{k}{2}\cos\frac{k}{2}$

$= \left(\cos\frac{k}{2} - \sin\frac{k}{2}\right)^2$

$\therefore x = \pm\frac{1}{\sqrt{2}}\left(\cos\frac{k}{2} - \sin\frac{k}{2}\right)$

<u>or</u>

$\cos(\alpha - \beta) = \cos k$

$\therefore \cos k = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= x\sqrt{1-x^2} + x\sqrt{1-x^2}$

$= 2x\sqrt{1-x^2}$ —— (1)

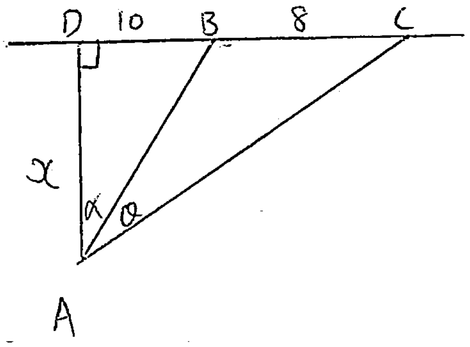
Marker's Comments: As students cannot explain why we take the positive, this method will only yield a maximum of 2 marks out of 3.

Mathematics ____ Question 6

Suggested Solutions	Marks	Marker's Comments
$\cos^2 k = 4x^2(1-x^2)$		
$= 4x^2 - 4x^4$		
$\therefore 4x^4 - 4x^2 + \cos^2 k = 0$		
$x^2 = \dfrac{4 \pm \sqrt{16-4(4)\cos^2 k}}{8}$		
$= \dfrac{4 \pm 4\sin k}{8}$		
$= \dfrac{1 \pm \sin k}{2}$	1	
$\therefore 1 \pm \sin k = 2x^2$		
$\boxed{1 - \sin k = 2x^2}$ $\underline{\underline{\text{or}}}$ $1 + \sin k = 2x^2$		
$\left(\cos^2\frac{k}{2} + \sin^2\frac{k}{2}\right) - 2\sin\frac{k}{2}\cos\frac{k}{2} = 2x^2$		
$2x^2 = \left[\cos\frac{k}{2} - \sin\frac{k}{2}\right]^2$		
$x = \pm\frac{1}{\sqrt{2}}\left(\cos\frac{k}{2} - \sin\frac{k}{2}\right)$		

Mathematics ____ Question 6		
Suggested Solutions	Marks	Marker's Comments

b) i)



Let D be as on the diagram

Let α be $\angle DAB$.

$$\left.\begin{aligned}\tan(\theta+\alpha) &= \frac{18}{x} \\ \tan\alpha &= \frac{10}{x}\end{aligned}\right\} \qquad (1)$$

$$\begin{aligned}\tan\theta &= \tan[(\theta+\alpha)-\alpha] \\ &= \frac{\frac{18}{x} - \frac{10}{x}}{1 + \frac{18}{x}\times\frac{10}{x}} \qquad (1) \\ &= \frac{18-10}{x + \frac{180}{x}} \\ &= \frac{8x}{180+x^2}\end{aligned}$$

$$\therefore \theta = \tan^{-1}\left(\frac{8x}{180+x^2}\right) \qquad (1)$$

or

Mathematics 1 Question 6

Suggested Solutions	Marks	Marker's Comments
$\tan(\theta+\alpha) = \frac{18}{x}$, $\tan\alpha = \frac{10}{x}$	1	
$\tan(\theta+\alpha) = \frac{18}{x}$		
$\frac{\tan\theta + \tan\alpha}{1-\tan\theta\tan\alpha} = \frac{18}{x}$	1	
$x\tan\theta + x\tan\alpha = 18 - 18\tan\theta\tan\alpha$		
$x\tan\theta + 10 = 18 - \frac{180}{x}\tan\theta$		
$x^2\tan\theta + 10x = 18x - 180\tan\theta$		
$\tan\theta(180+x^2) = 8x$		
$\tan\theta = \frac{8x}{180+x^2}$	1	
or		

Mathematics ____ Question 6

Suggested Solutions	Marks	Marker's Comments
$\cos\theta = \dfrac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}$	1	
$= \dfrac{(x^2+100) + (x^2+324) - 64}{2 \times \sqrt{(x^2+100)(x^2+324)}}$		
$= \dfrac{2x^2 + 360}{2\sqrt{x^4 + 424x^2 + 32400}}$		
$= \dfrac{x^2 + 180}{\sqrt{x^4 + 424x^2 + 32400}}$		
$\dfrac{\sin\theta}{8} = \dfrac{\sin\angle ACB}{AB}$ (Sine rule)	1	
$\therefore \dfrac{\sin\theta}{8} = \dfrac{\dfrac{x}{\sqrt{x^2+324}}}{\sqrt{x^2+100}}$		
$\therefore \sin\theta = \dfrac{8x}{\sqrt{x^4+424x^2+32400}}$		
$\therefore \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{8x}{180+x^2}$		
$\therefore \theta = \tan^{-1}\left(\dfrac{8x}{180+x^2}\right)$	1	

Mathematics ____ Question 6

Suggested Solutions	Marks	Marker's Comments

ii) $\theta = \tan^{-1}\left(\frac{8x}{180+x^2}\right)$

$$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{8x}{180+x^2}\right)^2} \times \frac{(180+x^2).8 - 8x(2x)}{(180+x^2)^2}$$

$$= \frac{1440+8x^2-16x^2}{(180+x^2)^2+64x^2}$$

$$= \frac{1440-8x^2}{(180+x^2)^2+64x^2} \qquad (1)$$

$$\frac{d\theta}{dx} = 0 \longrightarrow 1440 - 8x^2 = 0$$

$$\therefore x^2 = 180$$

$$x = \sqrt{180} \quad (x > 0)$$

$$= 6\sqrt{5} \qquad (1)$$

Test:

x	13	$6\sqrt{5}$	14
$\frac{d\theta}{dx}$	$\frac{88}{132617}$	0	$\frac{-2}{2405}$
	/	—	\

$\therefore x = 6\sqrt{5}$ is a local maximum,

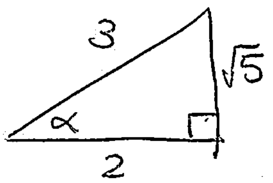
Since θ is continuous for all x, then $6\sqrt{5}$ is also an absolute maximum. (1)

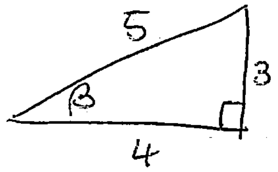
JRAHS Ext 1 Term 4 2016

Question 7

a) Let $\alpha = \cos^{-1}\frac{2}{3}$, $\beta = \tan^{-1}\frac{3}{4}$

$-\beta = \tan^{-1}\left(-\frac{3}{4}\right)$





$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \frac{\sqrt{5}}{3} \times \frac{4}{5} - \frac{2}{3} \times \frac{3}{5}$$
$$= \frac{4\sqrt{5} - 6}{15}$$

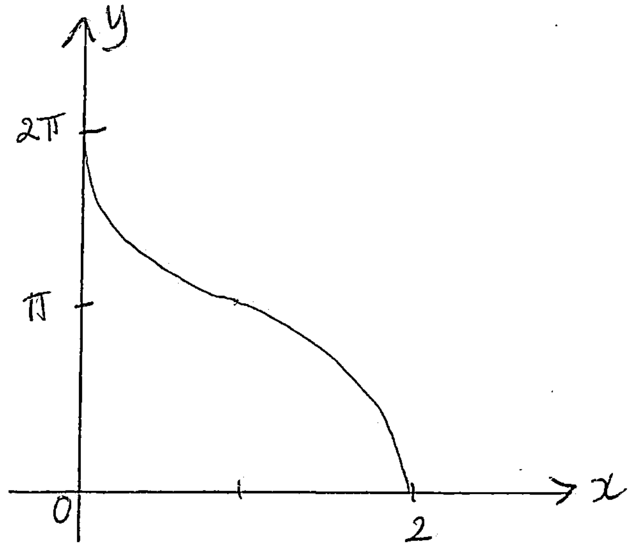
Note:
Many students let $\beta = \tan^{-1}\left(-\frac{3}{4}\right)$ then did not consider that β must then be in 4th quadrant.

① diagrams

① correct substitution into sum/diff. expansion

① simplification

b) i)



① correct domain

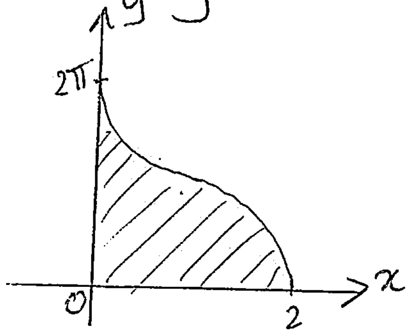
① correct range

b)ii) Method 1 – Area bounded by curve WRT y-axis: (2)

$y = 2\cos^{-1}(x-1)$

$\frac{y}{2} = \cos^{-1}(x-1)$

$x = \cos\left(\frac{y}{2}\right) + 1$



$A = \int_0^{2\pi} \left[\cos\left(\frac{y}{2}\right) + 1\right] dy$

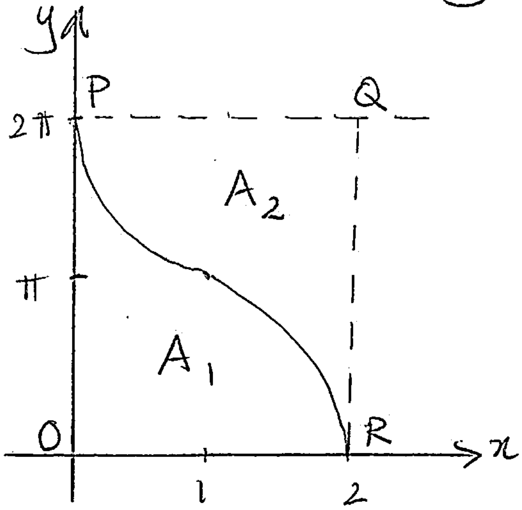
$= \left[2\sin\frac{y}{2} + y\right]_0^{2\pi}$

$= 2\sin\pi + 2\pi - 2\sin 0 - 0$

$= 2\pi$ units^2

① expression for x
① expression for area
① integration
① evaluation of definite integral.

Method 2 :– Using symmetry



Graph of $y = 2\cos^{-1}(x-1)$ is symmetrical about point $(1, \pi)$.

$\therefore A_1 = A_2$

$\therefore A_1 = \frac{1}{2}$ of Area of PQRO

$= \frac{1}{2} \times 2 \times 2\pi$

$= 2\pi$ units^2

Note: Many students said $A_1 = PQRO - A_1$ without any explanation of symmetry. This received 3 marks.

Their solution started with:

$$A = 4\pi - \int_0^{2\pi} \left[\left(\cos\frac{y}{2}\right) + 1\right] dy$$