Name: ____________________

Class: ____________________

2016

Term 2 Assessment (HSC)

Mathematics Extension 1

General Instructions

- Working time – 90 minutes + 5minutes reading
- Board approved calculators may be used
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.

7 questions – 9 marks each

Total marks – 63

Attempt all questions

Start each question on a new sheet of paper.

QUESTION 1 **(Start a new page)** **9 Marks**

a) The acceleration of a particle is given by $a = e^{-2t}$ (in m/s^2) and the particle is stationary at the origin.

i) Find the velocity function. **1**

ii) Sketch the velocity-time graph and describe briefly what happens to the velocity of the particle as the time goes on. **2**

b) Between 5 am and 5 pm on 3rd March 2009, the height, h, of the tide in a harbour was given by $h = 1 + 0.7\sin\frac{\pi}{6}t$ for $0 \leq t \leq 12$, where h is in metres and t is in hours, with $t = 0$ at 5am.

i) What is the period of the function h? **1**

ii) What was the value of h at low tide, and at what time did low tide occur? **2**

iii) A ship is able to enter the harbour only if the height of the tide is at least 1.35m. Find all times between 5am and 5pm on 3rd March 2009 during which the ship was able to enter the harbour. **3**

QUESTION 2 (Start a new page) 9 Marks

a) A particle is moving in a straight line with velocity $v\ m/s$. The acceleration of the particle when it is x meters from a fixed point 0 is given by $\ddot{x} = 6x^2$. Initially $x = 1$ and $v = -2$.

a) Find v^2 as a function of displacement, x. **2**

b) Explain why velocity can never be positive. **2**

b) i) Ten points in a plane are such that no three points lie in the same straight line. Find the number of triangles that can be drawn with these points as vertices. **1**

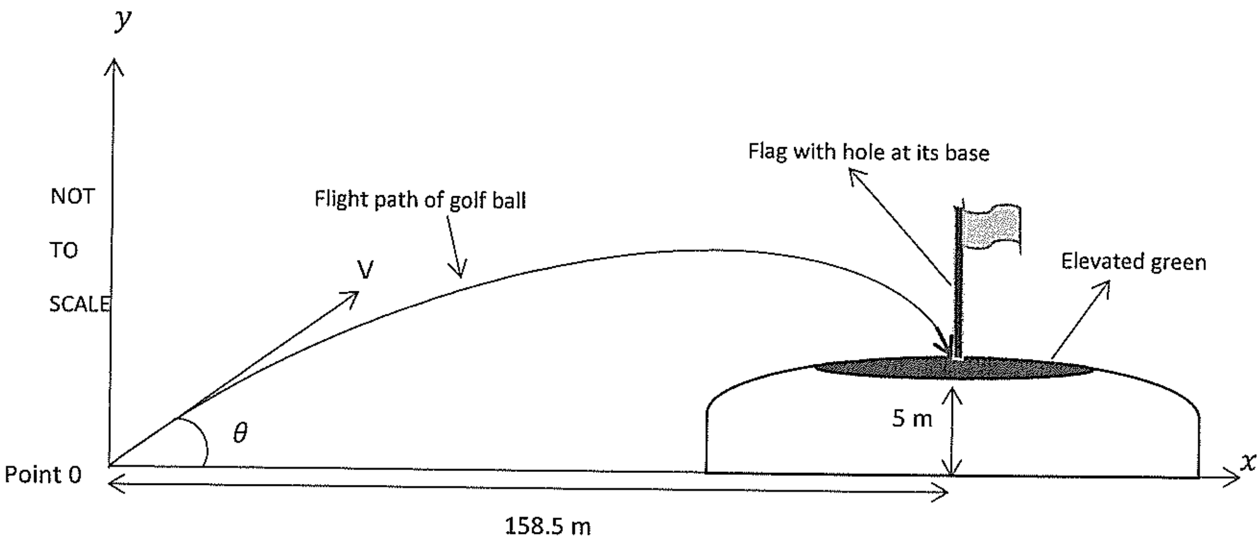
ii) If, in addition, there are 4 points on a straight line that does not pass through any of the given 10 points, find the number of triangles that can be drawn **2**

c) In how many ways can 11 people sit at a round table for dinner if a particular couple insist on sitting next to one another and three individuals insist on being separated ? **2**

QUESTION 3 **(Start a new page)** **9 Marks**

a) A delegation of 4 pupils is to be chosen from 5 boys and 6 girls. Find the number of ways of forming the delegation if the boys must be in the majority. **2**

b) Tiger Woods hits a golf ball from a point 0 towards a flat, elevated green as shown in the diagram below. The hole at the base of the flag is situated in the centre of the green.



The golf ball is projected from the point 0 with an initial velocity of $V\ m/s$ and at an angle of θ to the horizontal. You may assume the only force acting on the golf ball in flight is gravity, and take the acceleration due to gravity to be $10m/s^2$.

i) Taking the point 0 as the origin, show that the parametric equations of the flight path of the golf ball are given by $x = Vtcos\theta$ and $y = -5t^2 + Vtsin\theta$. **2**

ii) If the angle of projection is given by $\theta = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, find the value of V which will enable Woods to hit the golf ball directly into the hole on the green. The hole is situated 158.5 meters horizontally and 5 meters vertically from the point 0. Give your answer correct to the nearest integer. **3**

iii) Using your value of V from part ii), find the alternative angle of projection from the point 0 that would enable Woods to still hit the golf ball directly into the hole. **2**

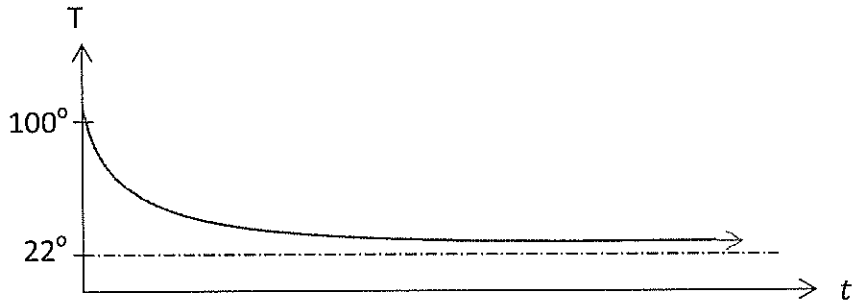
QUESTION 4 (Start a new page) 9 Marks

a) i) Prove $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$ **1**

ii) The speed, $v\ cm/s$, of a particle moving along the x axis is given by
$v^2 = 72 - 12x - 4x^2$.
Show that the motion is simple harmonic. **2**

iii) Find the period and amplitude of the motion. **2**

b) The graph shown below represents the relationship between T, the temperature in °C of a cooling cup of milk, and t, the time in minutes.



The rate of cooling of this milk is given by $\frac{dT}{dt} = -k(T - A)$.
$T = A + Be^{-kt}$ is a solution to the differential equation of $\frac{dT}{dt} = -k(T - A)$.
A, B and k are constants and $k > 0$

i) By examining the graph when $t = 0$ and $t \to \infty$, find the values of A and B. **2**

ii) If the temperature of the milk is 50°C after 90 minutes, show that
$k = -\frac{1}{90} ln\left(\frac{14}{39}\right)$ **1**

iii) Find the rate at which the milk is cooling after 90 minutes. **1**

QUESTION 5 (Start a new page) 9 Marks

a) A company has installed a new computer system at Ruse and some staff are having difficulty logging in to the system. The staff members have been given training and the problems which arose during training were recorded and their probabilities calculated as follows:

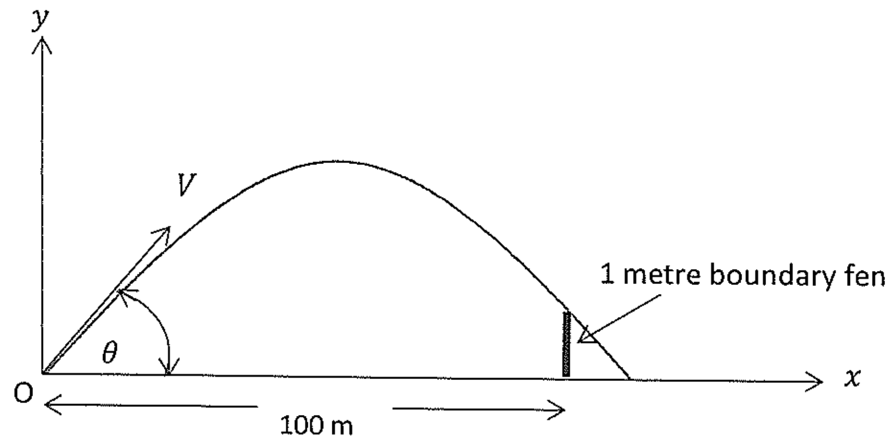
- A staff member has a probability of 0.9 of logging in successfully on their first attempt.
- If at any time the staff logs in successfully then the staff will also be successful on their next attempt with probability 0.9.
- If at any time the staff tries to log in and is NOT successful, then the staff loses confidence and the probability of a successful log-in on their next attempt drops to 0.5.

Use a tree diagram or other methods to find the following probabilities.

i) A staff logs on successfully only once in three attempts. **1**

ii) A staff does not manage to log on successfully in three attempts. **1**

b) A "six" is scored in a cricket game when the ball is hit over the boundary fence on the full as in the diagram. A ball is hit from O with velocity V of 32 m/s at an angle θ to the horizontal and towards the 1 metre high boundary fence 100 metres away.



i) Derive the Cartesian equation of motion for the ball in flight using axes as in the diagram. (Air resistance is to be neglected and the acceleration due to gravity is taken as $10\ m/s^2$). **3**

ii) Show that the ball just clears the boundary fence when
$50000tan^2\theta - 102400tan\theta + 51024 = 0.$ **1**

iii) In what range must θ lie for a "six" to be scored? **2**

iv) Without further calculation, discuss qualitatively the effect of air resistance on your answer in (iii). **1**

QUESTION 6 (Start a new page) 9 Marks

a) In the 1950's, electronic computers were constructed with valves (rather than transistors). Assume that each particular valve has a probability, $p = 0.00002$ of failing during the next hour. (Assume that valve failures are independent events, and are independent of the length of past service of the valve).

i) Find the probability q of a particular valve operating correctly during the next hour. **1**

ii) If the computer contains 10000 valves, find the probability P_0 of no valve failing during the next hour, to two significant figures. **3**

b) In the time just before the Democratic Party general election, the level of confidence in Hillary Clinton is C, where C is expressed as a percentage. The rate of daily change in that confidence level is given by $\frac{dC}{dt} = k(C + 0.1)$ where k is a constant.

i) If C is initially 90%, show that $C = -0.1 + e^{kt}$ **2**

ii) If $C = 60\%$ after 10 days, find the exact value of k **1**

iii) How many days will it take for the level of confidence in Hillary Clinton to reduce to 10%. (to the nearest day) **2**

QUESTION 7 (Start a new page) 9 Marks

a) Jess was first to board to her flight to Perth. She forgot her seat number and picks a random seat for herself. After this, every single person who gets to the flight sits on his/her seat if its available, else chooses any available seat at random. Jack is last to enter the flight and at that moment 99/100 seats were occupied. What's the probability that Jack gets to sit in his own seat ? **1**

b) A pendulum is constructed by attaching a weight to a light rigid rod of length L. The weight can swing in a vertical plane around a fixed point P to which the other end of the rod is attached. Let θ be the angle between the pendulum rod and the (downward) vertical direction, and let g be the acceleration of gravity. Then the equation of motion of the pendulum can be shown to be

$$d^2\theta/dt^2 = -\,(g/L)\sin\theta.$$

(You may assume $\frac{du^2}{dt} = 2u\frac{du}{dt}$ and $\omega = \frac{d\theta}{dt}$)

i) Show that the quantity $E = \frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 - \left(\frac{g}{L}\right)cos\theta$ does not change with time. **2**

ii) The pendulum is started from its equilibrium position (from $\theta = 0$) with an initial angular velocity equal to ω. What is the value of the quantity E of part (i)? **2**

iii) Assume that the pendulum started moving rapidly enough so that it swings all the way around the fixed point P in a full circle. By considering the instant when it is at the very top of the circle, or otherwise, show that the value of the quantity E for such a motion must be greater than g/L. **2**

iv) By considering the results of parts (ii) and (iii), or otherwise, show that the pendulum will swing all the way around in full circle if and only if the absolute value of its initial angular velocity ω exceeds $2\sqrt{(g/L)}$. **2**

****END OF EXAMINATION****

MATHEMATICS Extension 1: Question 1

Suggested Solutions	Marks	Marker's Comments

(a) (i) $a = e^{-2t}$

$V = -\frac{1}{2}e^{-2t} + c$

$V=0,\ x=0,\ t=0$

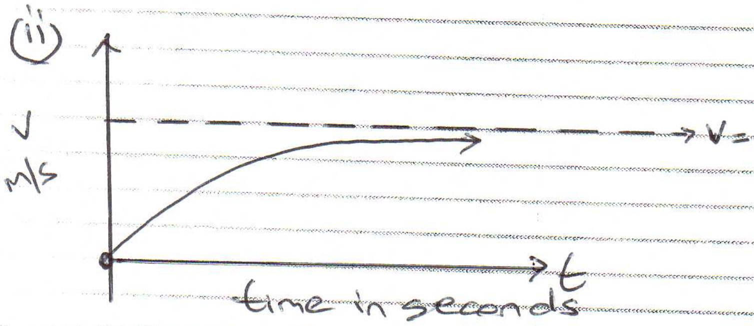
$0 = -\frac{1}{2}e^{0} + c$

$c = \frac{1}{2}$

$\therefore V = -\frac{1}{2}e^{-2t} + \frac{1}{2}$ — (1)

*students forgot to find the constant of integration.

(ii)



(1)

as $t \to \infty$ $v \to \frac{1}{2}$

so as time goes on, the velocity approaches $\frac{1}{2}$ m/s. — (1)

*students forgot to write in words what happened to the velocity?

*Some students thought time could be negative

(b) (i) period $= \frac{2\pi}{\pi/6}$

$= 12$ hours — (1)

a lot of students forgot to cancel out the π's

(ii) Low tide is 2pm — (1)

Height is 0.3m — (1)

Some left it as t = 9hours, but that's not the actual time, so they lost a mark!!

(iii) $h = 1 + 0.7\sin\frac{\pi}{6}t$

$1.35 = 1 + 0.7\sin\frac{\pi}{6}t$

$\frac{1}{2} = \sin\frac{\pi}{6}t$

$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6},$ — (1)

$t = 1$ or 5 — (1)

ship can enter from 6am until 10am — (1)

MATHEMATICS Extension 1: Question 2

Suggested Solutions	Marks	Marker's Comments

Q2. a) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 6x^2$

$\frac{1}{2}v^2 = 6\int x^2\,dx$ 1

$v^2 = 12\left[\frac{x^3}{3}\right]$

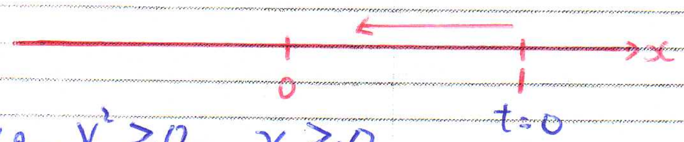
$v^2 = 4x^3 + c$

when $x = 1$, $v = -2$.

$\therefore c = 0$.

$\therefore v^2 = 4x^3$ 1

b) when $t = 0$, $v = -2$.
$\therefore$ The particle starts with a negative velocity. 1



Since $v^2 \geq 0$, $x \geq 0$.
The particle travels towards 0 from 1 while slowing down. When x reaches 0, $v = 0$ and $\ddot{x} = 0$. 1
$\therefore$ It is stationary and experiences no acceleration.
$\therefore$ It stays at $x = 0$ and $\therefore$ v is never positive.

Note: Anyone who wrote "when $x = 1$, $v = -2$, $\therefore v = -\sqrt{4x^3}$" will get a maximum 1 mark for the whole question regardless of what gets written afterwards.

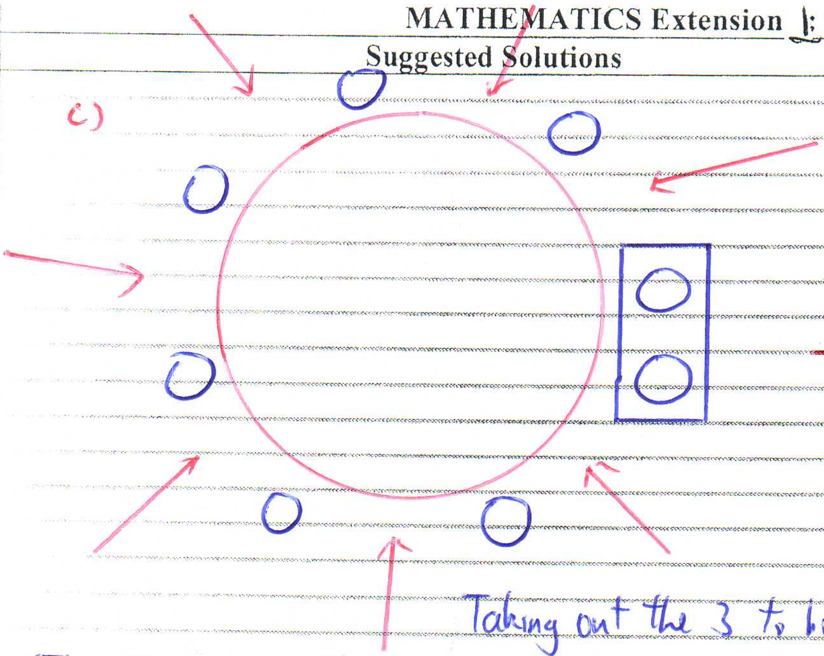
MATHEMATICS Extension 1: Question 2

Suggested Solutions	Marks	Marker's Comments
b) i) ${}^{10}C_3 = 120$		1
ii) Method 1:		
Number of ways to choose 3 points from 14 − Number of ways to choose 3 points from the 4 collinear lines		1
$= {}^{14}C_3 - {}^{4}C_3 = 360$		1
Method 2:		
Number of ways to choose 3 points from original 10 + Number of ways to choose 2 points from 10 and 1 from 4 + Number of ways to choose 1 point from 10 and 2 from 4		1
$= {}^{10}C_3 + {}^{10}C_2 \times {}^{4}C_1 + {}^{10}C_1 \times {}^{4}C_2$		
$= 360$		1

MATHEMATICS Extension 1: Question 2

Suggested Solutions	Marks	Marker's Comments

c)



1

Taking out the 3 to be separated.

There are $6!(2!)$ ways to organise the rest in a circle.

Then there are ${}^7P_3$ ways to slip the 3 individuals in 7 gaps.

$\therefore 6!\,2! \times {}^7P_3 = 302400$ 1

2016 T2 TASK **MATHEMATICS Extension 1: Question 3**

Suggested Solutions	Marks	Marker's Comments

(a) 4 students from 5B, 6G, Boys majority

∴ Boys must be 3 or 4

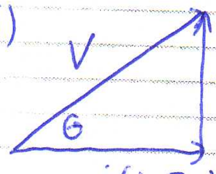
If 3B and 1G chosen: ${}^5C_3 \times {}^6C_1 = 60$ — 1

4B and 0G chosen: ${}^5C_4 \times {}^6C_0 = 5$

Total $= 60 + 5$

$= 65$ ways — 1

(b) (i)



Horizontal

$\ddot{x} = 0$

$\dot{x} = \int dt$

$= C_1$

$t = 0,\ \dot{x} = V\cos\theta \Rightarrow C_1 = V\cos\theta$

$x = \int V\cos\theta\, dt$

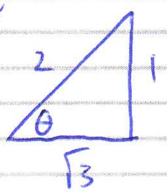
$= Vt\cos\theta + C_2$

$t = 0,\ x = 0 \Rightarrow C_2 = 0$

$\therefore x = Vt\cos\theta$ — 1

Marker's comments: Students were required to account for the constants of integration to earn the mark.

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
<u>Vertical</u>		
$\ddot{y} = -10$		Unless $y=-10$ was used the mark was not awarded.
$\dot{y} = \int -10 \, dt$		
$= -10t + c_1$		
$t=0, \ \dot{y} = V\sin\theta \Rightarrow c_1 = V\sin\theta$		
$y = \int -10t + V\sin\theta \, dt$		
$= -5t^2 + Vt\sin\theta + c_2$		
$t=0 \quad y=0 \Rightarrow c_2 = 0$		
$\therefore y_m = -5t^2 + Vt\sin\theta$	1	
(ii) $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$		
NB: Angle of inclination is in the first quadrant.		
 $\cos\theta = \frac{\sqrt{3}}{2}$		
$\sin\theta = \frac{1}{2}$		
$\begin{cases} y = -5t^2 + Vt\sin\theta \\ x = Vt\cos\theta \end{cases}$ (shown in (i))		
$x = \frac{Vt\sqrt{3}}{2}$		
$t = \frac{2x}{V\sqrt{3}} \rightarrow y$	1	
$y = \frac{-20x^2}{3V^2} + V \times \frac{2x}{V\sqrt{3}} \times \frac{1}{2}$		
$= \frac{-20x^2}{3V^2} + \frac{x}{\sqrt{3}}$		

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
Given condition $x = 158.5,\ y = 5$		
$\therefore 5 = \dfrac{-20(158.5)^2}{3V^2} + \dfrac{158.5}{\sqrt{3}}$	1	
$V^2 = \dfrac{-20\sqrt{3}(158.5)^2}{15\sqrt{3} - 3\times 158.5}$		
$\doteqdot 1395.98$		
$V \doteqdot 43.999\ ms^{-1}$		
$\doteqdot 44\ ms^{-1}$ to nearest integer	1	
(iii) $y = -5t^2 + Vt\sin\theta$ (from (i))		
$x = Vt\cos\theta \Rightarrow t = \dfrac{x}{V\cos\theta} \rightarrow y$		
$y = -5\left(\dfrac{x}{V\cos\theta}\right)^2 + V\left(\dfrac{x}{V\cos\theta}\right)\sin\theta$		
$= \dfrac{-5x^2}{V^2}\sec^2\theta + x\tan\theta$		
$= \dfrac{-5x^2}{V^2}(1+\tan^2\theta) + x\tan\theta$ by pythagorean identity $1+\tan^2\theta = \sec^2\theta$		
Given condition $x = 158.5,\ y = 5,\ V = 44$ (from (ii))		
$5\times 44^2 = -5(158.5)^2(1+\tan\theta) + 158.5\times 44^2\times\tan\theta$		
$0 = 5\times 158.5^2\tan^2\theta - 158.5\times 44^2\tan\theta + (5\times 44^2 + 5\times 158.5^2)$		

Suggested Solutions | Marks | Marker's Comments

Method 1

$$\tan\theta = \frac{158.5\times44^2 \pm\sqrt{(158.5\times44^2)^2-4(5\times158.5^2\times(5\times44^2+5\times158.5^2)}}{2(5\times158.5^2)}$$ (1)

$\tan\theta \doteqdot 1.9655$ or 0.5713

$\theta \doteqdot 61.8^\circ$ or 30°

$\therefore$ alternate angle is 61.8° — 1

Method 2

Let two solution for $\tan\theta$ be α and β.

By sum of roots:

$$\alpha+\beta = \frac{158.5\times44^2}{5\times158.5^2}$$ — 1

$$= \frac{1936}{792.5}$$

We know that one solution is $\tan\theta = \tan^{-1}(1/\sqrt{3})$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$= \alpha$

$$\therefore \frac{1}{\sqrt{3}}+\beta = \frac{1936}{792.5}$$

$$\beta = \frac{1936}{792.5}-\frac{1}{\sqrt{3}} = \tan\theta$$

θ lies in 1st quadrant

$$\therefore \theta = \tan^{-1}\left(\frac{1936}{792.5}-\frac{1}{\sqrt{3}}\right)$$

$$\doteqdot 61.8^\circ$$ — 1

3 unit Question 4

a) (i) Prove $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$

LHS

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \frac{dv}{dx}$$

$$= v \times \frac{dv}{dx}$$

$$= \frac{dx}{dt} \times \frac{dv}{dx} \quad (*)$$

$$= \frac{dv}{dt}$$

$$= \frac{d^2x}{dt^2}$$

$$= RHS$$

Some proved from RHS $\frac{d^2x}{dt^2}$

$$= \ddot{x}$$

$$= \frac{dv}{dt}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{dv}{dx} \cdot v$$

$$(*) \quad = \frac{dv}{dx} \cdot \frac{d\left(\frac{1}{2}v^2\right)}{dv}$$

$$= \frac{d\left(\frac{1}{2}v^2\right)}{dx}$$

(* need these steps)

a (ii) $v^2 = 72 - 12x - 4x$

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$= \frac{d}{dx}\left(36 - 6x - 2x^2\right)$$

$$= -6 - 4x$$

$$= -4\left(x + \frac{3}{2}\right)$$

$$= -2^2\left(x + \frac{3}{2}\right)$$

It is in simple harmonic motion of form $-n^2(x - x_0)$

Those who used this to say $v^2 = n^2\left(a^2 - (x - x_0)^2\right)$ lost a mark. This is a property not a definition

(iii) $P = \frac{2\pi}{2} = \pi$ sec

where $n = 2$

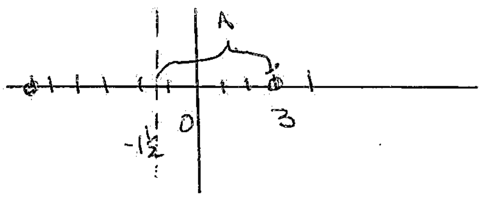
It stops when $v^2 = 0$

$$4x^2 + 12x - 72 = 0$$

$$4(x - 3)(x + 6) = 0$$

$$\therefore x = 3, -6$$

Centre of motion $= \frac{-3}{2}$



amplitude $= 4\frac{1}{2}$ units

well done

4b

rate of cooling $\frac{dT}{dt} = -k(T - A)$

$$T = A + Be^{-kt}$$

$$\therefore Be^{-kt} = T - A$$

$$\frac{dT}{dt} = -k(Be^{-kt})$$

$$\therefore \frac{dT}{dt} = -k(T - A)$$

Alternate methods were accepted

(i) Values of A and B

From graph $T = 100$ when $t = 0$

$$\therefore 100 = A + Be^{-k(0)}$$

$$100 = A + B \quad \text{eq } ①$$

when $t \to \infty \Rightarrow T = 22$ ✓

Sub in $T = A + Be^{-k(\infty)}$

$$22 = A + Be^{-k(\infty)}$$

$$A = 22$$

Sub in $100 = 22 + B$ in eq ①

$$B = 78 \quad ✓$$

(ii) $T = 50$ when $t = 90$

$$T = A + Be^{-kt}$$

$$50 = 22 + 78e^{-k(90)}$$

$$28 = 78e^{-90k}$$

$$-90k = \ln \frac{28}{78}$$

$$\therefore k = -\frac{1}{90} \ln \frac{14}{39} \quad ✓$$

(iii) $\frac{dT}{dt} = -k(T - A)$

$$= -\left(-\frac{1}{90} \ln \frac{14}{49}\right)(50 - 22)$$

$$= \frac{28}{90} \ln \frac{14}{49} \text{ or } \frac{14}{45} \ln \frac{14}{49}$$

$$= -0.31873^\circ/\text{min}$$

Rate is decreasing by $0.32^\circ/\text{min}$

or

cooling at $0.32^\circ/\text{min}$

<u>Note</u> $\frac{dT}{dt}$ has to be negative

✓

① If 0.32 (positive) This means the temperature is increasing (no marks awarded)

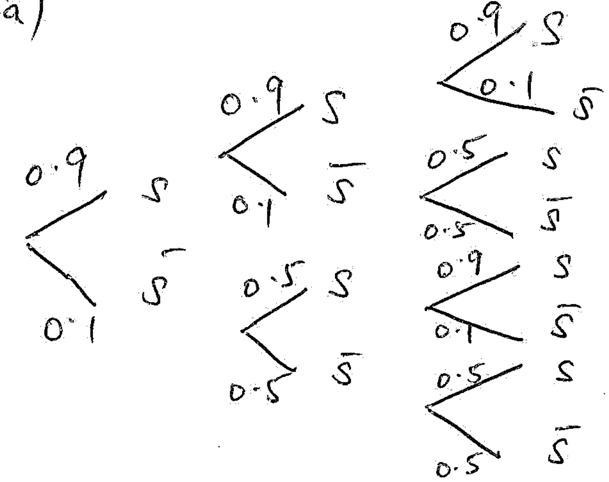
② The answer $\frac{14}{45} \ln \frac{14}{49}$ or $\frac{28}{90} \ln \frac{14}{49}$ Not accepted

Only 1 mark awarded for -0.32.

MATHEMATICS Extension 1 : Question……5…..

Suggested Solutions	Marks Awarded	Marker's Comments

5a)



SSS
$SS\bar{S}$
$S\bar{S}S$
$S\bar{S}\bar{S}$
$\bar{S}SS$
$\bar{S}S\bar{S}$
$\bar{S}\bar{S}S$
$\bar{S}\bar{S}\bar{S}$

i) p (success only once)

$S\bar{S}\bar{S} + \bar{S}S\bar{S} + \bar{S}\bar{S}S$ — 1

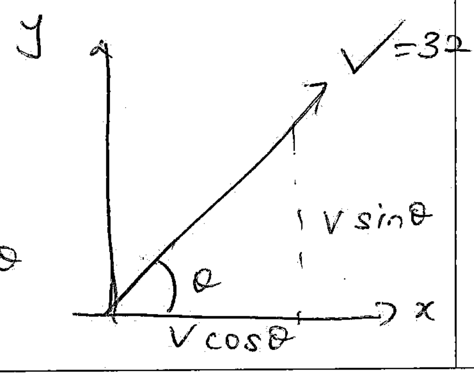
$0.9 \times 0.1 \times 0.5 + 0.1 \times 0.5 \times 0.1 + 0.1 \times 0.5 \times 0.5$

$= 0.075$ or $3/40$

ii) $p(\bar{S}\bar{S}\bar{S}) = 0.1 \times 0.5 \times 0.5$
$= 0.025$ or $1/40$ — 1

5b) Derive Cartesian eqn.

Marker's comments: many left in parametric eqn. Thus didn't answer the question.



Vertical

$\ddot{y} = -10$

$\dot{y} = -10t + C_1$

$t = 0 \quad \dot{y} = V\sin\theta$

$\therefore C_1 = V\sin\theta$

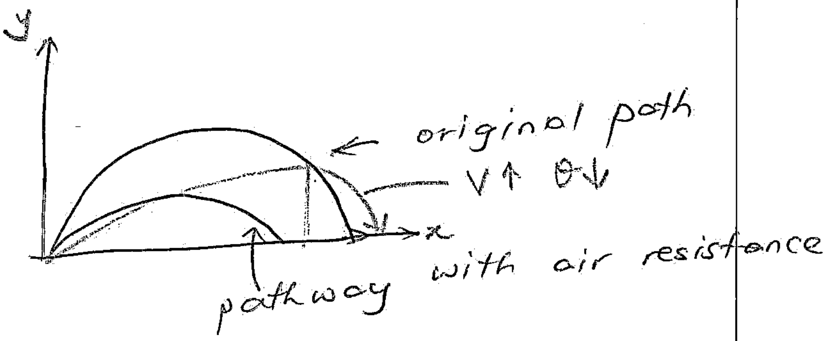
MATHEMATICS Extension 1 : Question 5		Pg 2
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
$\therefore \dot{y} = -10t + v\sin\theta$ $y = \frac{-10t^2}{2} + v\sin\theta\, t + C_2$ $t=0,\ y=0,\ \therefore C_2 = 0$ $\therefore y = -5t^2 + vt\sin\theta$ $v=32 \Rightarrow y = -5t^2 + 32t\sin\theta$ — i Horizontal $\ddot{x} = 0$ $\dot{x} = C_1$ $t=0,\ \therefore C_1 = v\cos\theta$ $\therefore \dot{x} = v\cos\theta$ $x = vt\cos\theta + C_2$ $t=0,\ x=0,\ C_2 = 0$ $v=32 \Rightarrow \therefore x = 32t\cos\theta$ — ii. To find Cartesian eqn $x = 32t\cos\theta$ $t = \frac{x}{32\cos\theta}$ sub into i $y = -5\left(\frac{x}{32\cos\theta}\right)^2 + 32\left(\frac{x}{32\cos\theta}\right)\sin\theta$ $= \frac{-5x^2}{32^2\cos^2\theta} + \frac{\cancel{32}x\sin\theta}{\cancel{32}\cos\theta}$ $\therefore$ $y = \frac{-5x^2\sec^2\theta}{1024} + x\tan\theta$ — iii	1 mark to get i & ii correct. 1 $\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\frac{1}{\cos^2\theta} = \sec^2\theta$	many students did this for part ii.

 OR

MATHEMATICS Extension ~~1~~: Question 5 Pg 3

Suggested Solutions	Marks Awarded	Marker's Comments
$y = \dfrac{-5x^2(1+\tan^2\theta)}{1024} + x\tan\theta$ — iii	$\sec^2\theta = 1+\tan^2\theta$	
5bii) $x = 100$, $y = 1$ sub into iii $\left[1 = \dfrac{-5(100)^2(1+\tan^2\theta)}{1024} + 100\tan\theta\right] \times 1024$ $1024 = -50000(1+\tan^2\theta) + 102400\tan\theta$ $0 = -50000 - 50000\tan^2\theta + 102400\tan\theta - 1024$ $0 = 50000\tan^2\theta - 102400\tan\theta - 51024$		1 mark no mark awarded if didn't show or explain if student showed $\dfrac{3125\tan^2\theta}{64} - 100\tan\theta + \dfrac{3189}{64}$ then $0 = 50000\tan^2\theta - 102400\tan\theta - 5102$
5biii) for scoring 6 $x = 100$, $y > 1$ $\tan\theta = \dfrac{102400 \pm \sqrt{102400^2 - 4 \times 50000 \times 51024}}{2(50000)}$ $\tan\theta = 1.1916$ and 0.8563 $\theta = (40°35')$ & $(50°, 49.996°)$ in degrees. or (0.708^c) & (0.873^c) in radians $\therefore 40°35' < \theta < 50°$		1 mark for $\tan\theta$ 1 mark for θ

MATHEMATICS Extension 1 : Question 5		Pg 4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
5 iv) With the effect of air resistance, both the horizontal and vertical velocities are reduced. Therefore the ball will not rise so far nor cover such great distance with same hit. It will be necessary to increase the initial velocity and reduce the range of θ to still score a six	2 marks 1 mark	must explain effect of air resistance and the angle/range of hit if θ is not mentioned.



MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
a) i) $q = 1 - 0.00002$		
$= 0.99998$	1	
ii) $P_0 = (0.99998)^{10000}$	1	
$= 0.818729\ldots$	1	
$= 0.82$ (2 sig. fig.)	1	
b) i) $C = -0.1 + e^{kt} \Rightarrow e^{kt} = C + 0.1$		
$\frac{dC}{dt} = ke^{kt}$	1	
$= k(C + 0.1)$	1	
$\therefore C$ is a solution of $\frac{dC}{dt}$.		
OR		
C will be in the general form of:		
$C = -0.1 + Ae^{kt}$	1	
when $t=0$, $C=0.9$		
$0.9 = -0.1 + A$		
$\therefore A = 1$	1	
$\therefore C = -0.1 + e^{kt}$ is a solution of $\frac{dC}{dt}$.		
OR		
$\frac{dC}{dt} = k(C+0.1)$		
$\int \frac{dC}{k(C+0.1)} = \int dt$		
$t = \frac{1}{k}(\ln(C+0.1)) + C_1$	1	
when $t=0$, $C=0.9$		
$0 = \frac{1}{k}(\ln(0.9+0.1)) + C_1$		
$\therefore C_1 = 0$		
$\therefore kt = \ln(C+0.1)$		
$e^{kt} = C + 0.1$		
$C = e^{kt} - 0.1$	1	
$C = -0.1 + e^{kt}$		
$\therefore C$ is a solution of $\frac{dC}{dt}$		

MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
ii) when $t=10$, $C=0.6$		
$0.6 = -0.1 + e^{10k}$		
$0.7 = e^{10k}$		
$10k = \ln 0.7$		
$k = \dfrac{\ln 0.7}{10}$	1	
iii) when $C = 0.1$		
$0.1 = -0.1 + e^{kt}$		
$0.2 = e^{kt}$		
$kt = \ln 0.2$		
$t = \dfrac{\ln 0.2}{k}$		
$= \dfrac{\ln 0.2}{\frac{\ln 0.7}{10}}$	1	
$= 45.12\ldots$		To be technically correct, it should be 46 days.
$= 45$ days (nearest day)	1	

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a) $\frac{1}{2}$

If Jack's seat was taken, he would not get his own seat, otherwise he would. | 1 m

b) i) $\frac{dE}{dt} = \frac{1}{2} \times 2\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right) + \frac{g}{L}\sin\theta \frac{d\theta}{dt}$ | 1 m

$= \frac{d\theta}{dt}\left[\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta\right]$

$= \frac{d\theta}{dt}\left[-\frac{g}{L}\sin\theta + \frac{g}{L}\sin\theta\right]$ | 1 m

$\frac{dE}{dt} = \frac{d\theta}{dt} \times 0 = 0$ ∴ E not change with time

Alternatively (bi)

$\frac{d^2\theta}{dt^2} = \frac{dw}{dt} = \frac{dw}{d\theta}\cdot\frac{d\theta}{dt} = w\frac{dw}{d\theta}$

$\int w\,dw = \int -\frac{g}{L}\sin\theta\,d\theta$

$\therefore \frac{w^2}{2} = \frac{g}{L}\cos\theta + \frac{k}{2}$

$E = \frac{1}{2}w^2 - \frac{g}{L}\cos\theta$

$E = \frac{g}{L}\cos\theta + \frac{k}{2} - \frac{g}{L}\cos\theta$

$E = \frac{k}{2}$ ∴ constant

not change with time

b ii) when $\theta = 0$, $\frac{d\theta}{dt} = w$, $\cos 0 = 1$

$E = \frac{1}{2}w^2 - \frac{g}{L}\cdot\cos 0 = \frac{1}{2}w^2 - \frac{g}{L}$ #

bii 1 m @ w or cos 0 = 1 but must apply to equation E

iii) At top of circle $\theta = \pi$

$E = \frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 - \frac{g}{L}\cos\pi = \frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 - \frac{g}{L}(-1)$

$E = \frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 + \frac{g}{L}$ | 1 m

For complete circle, $\frac{d\theta}{dt} > 0$ $\left(\therefore \left(\frac{d\theta}{dt}\right)^2 > 0\right)$

must give reason why $E > g/L$

$E > \frac{g}{L}$ # | 1 m

iv) From ii, iii

$\frac{1}{2}w^2 - \frac{g}{L} > \frac{g}{L}$ | 1 m

using this method results from ii & iii this line must be right before student get the second mark

$\frac{1}{2}w^2 > \frac{2g}{L}$

$w^2 > \frac{4g}{L}$

$|w| > 2\sqrt{\frac{g}{L}}$ # | 1 m