Student Name ____________________

Class ____________________

2017

Higher School Certificate

Term 4 T1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- NESA approved calculators may be used
- Write using black pen
- Marks may not be awarded for careless or badly arranged work
- An HSC Reference Sheet is provided
- All necessary working should be shown

Write your student number at the top of every page

Total marks – 60

This examination consists of 6 questions worth 10 marks each.

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc.

This paper MUST NOT be removed from the examination room.

Question 1 (10 Marks)

(a) Find the coefficient of the x^4 term in the expansion of $\sum_{k=0}^{6}\binom{6}{k}(x^2)^{6-k}(3)^k$ **2**

(b) Evaluate x and y if

$$(\sqrt{3}-1)^5 = x + y\sqrt{3}$$ **2**

(c) Use the substitution $u = t^2{+}2$ to find $\int t^3\sqrt{t^2+2}\,dt$ **3**

(d) If $\alpha = \cos^{-1}x$ and $\beta = \cos^{-1}3y$ show that

$$\cos\left(\cos^{-1}x - \cos^{-1}3y\right) = 3xy + \sqrt{(1-x^2)(1-9y^2)}$$ **3**

Question 2 (10 Marks) Start a new sheet of paper

(a) Find the constant term in the expansion

$$(5+x)^4\left(1-\frac{2}{x}\right)^3$$ **2**

(b) Differentiate $x^2\cos^{-1}(3x)$ **2**

(c) Find the exact value of $\sin^{-1}(\sin\pi)$ **1**

(d) Find the gradient to the curve $y = \tan^{-1}(\sin x)$ at $x = \pi$ **2**

(e) Find the general solution of the equation

$$2\cos 3x\sin 4x + 2\cos 3x - \sin 4x - 1 = 0$$ **3**

Question 3 (10 Marks) Start a new sheet of paper

(a) The coefficients of x and x^2 in the expansion of $(1 + ax)^n$ are -20 and 180 respectively. Find the values of a and n **3**

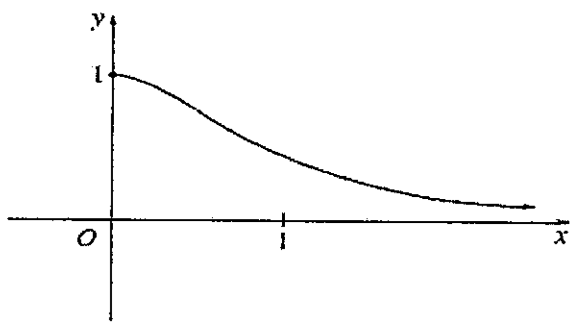
(b) Find the exact value of $\cos(2\cos^{-1} x + \sin^{-1} x)$ when $x = \frac{1}{5}$ **3**

(c) (i) Find the domain and range of the function $f(x) = 2\cos^{-1}(1 - x)$ **2**

(ii) Sketch the graph of the function $f(x) = 2\cos^{-1}(1 - x)$
Label clearly the coordinates of the end points. **2**

Question 4 (10 Marks) Start a new sheet of paper

(a) This is the graph of y=f(x)



Copy the graph onto your answer and draw the graph of $y = f^{-1}(x)$ on the same set of axes **2**

(b) What is the inverse function of $y = \frac{1}{4-x}$? **1**

(c) If $f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}(1 - x^2)$

(i) Find $f'(x)$ **2**

(ii) Hence or otherwise, show that $2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}(1 - x^2) = \frac{\pi}{2}$ **2**

(d) Water is pumped out of a 10 000 litre storage container through a valve such that the water is flowing out of the storage container at the rate of $2.4t$ litres per second $(t \geq 0)$.

(i) Show that if the tank was initially full, then the volume of water in the tank is given by $V = 10\,000 - 1.2t^2$. **2**

(ii) How long before the tank is only 30% full? Answer to the nearest second. **1**

Question 5 (10 Marks) Start a new sheet of paper

(a) Find the exact area bounded by the curve $y = 3\sin^{-1}(2x)$, the x-axis and the line $x = \frac{1}{2}$ **4**

(b) (i) Use the substitution $y = \sqrt{x}$ to find $\int \frac{dx}{\sqrt{x(1-x)}}$ **2**

(ii) Use the substitution $z = x - \frac{1}{2}$ to find another expression for $\int \frac{dx}{\sqrt{x(1-x)}}$ **2**

(iii) Use the results of parts (i) and (ii) to express $\sin^{-1}(2x-1)$ in terms of $\sin^{-1}\sqrt{x}$ for $0<x<1$ **2**

Question 6 (10 Marks) Start a new sheet of paper

(a) Let $(3+2x)^{20} = \sum_{r=0}^{20} a_r x^r$.

i) Write an expression for a_r **1**

ii) Show that $\frac{a_{r+1}}{a_r} = \frac{40-2r}{3r+3}$ **2**

iii) Hence find the greatest coefficient in the expansion of $(3+2x)^{20}$ **2**

(b) Find the exact value of $\int_1^2 \frac{4}{\sqrt{4-x^2}}\,dx$ **2**

(c) Find $\frac{d}{dx}(\tan^{-1}\frac{x}{3})^2$ and hence find the exact value

$$\int_0^{\sqrt{3}} \frac{\tan^{-1}\left(\frac{x}{3}\right)}{9+x^2}dx$$ **3**

END OF EXAMINATION

2017 Ext 1 Y. 11/12 Yrly Q1

a) $x^{12-2k} = x^4$

$\therefore k = 4$ 1

coeff of x^4 is $\binom{6}{4} 3^4 = \underline{1215}$ 1

must have 1215 to get 2nd mark

b) $(\sqrt{3}-1)^5 = (\sqrt{3})^5 - 5(\sqrt{3})^4 + 10(\sqrt{3})^3 - 10(\sqrt{3})^2 + 5\sqrt{3} - 1$ 1

must have 1 -5 10 -10 5 -1 to get 1st m

$= 9\sqrt{3} - 45 + 30\sqrt{3} - 30 + 5\sqrt{3} - 1$ 1

$= 44\sqrt{3} - 76$ 1

$\therefore x = -76 \quad y = 44$

c) $\frac{1}{2}\int t^2\sqrt{t^2+2}\,(2t\,dt)$ $\quad u = t^2+2$, $du = 2t\,dt$

$= \frac{1}{2}\int (u-2)\sqrt{u}\,du$ 1

$= \frac{1}{2}\int u^{3/2} - 2u^{\frac{1}{2}}\,du$

$= \frac{1}{5}u^{5/2} - \frac{2}{3}u^{3/2} + c$ 1

$= \frac{1}{5}(t^2+2)^{5/2} - \frac{2}{3}(t^2+2)^{3/2} + C$ 1

d) let $\alpha = \cos^{-1}x$, $\beta = \cos^{-1}3y$

$\therefore \cos\alpha = x \qquad \cos\beta = 3y$

$\sin^2\alpha + \cos^2\alpha = 1$, $\sin^2\beta + \cos^2\beta = 1$ 1

$\sin^2\alpha = 1 - \cos^2\alpha \qquad \sin^2\beta = 1 - \cos^2\beta$

Since $0 \le \alpha, \beta \le \pi$

$\therefore \sin\alpha = \sqrt{1-x^2}$, $\sin\beta = \sqrt{1-9y^2}$ 1

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= 3xy + \sqrt{1-x^2}\sqrt{1-9y^2}$

$\cos(\cos^{-1}x - \cos^{-1}3y) = 3xy + \sqrt{(1-x^2)(1-9y^2)}$ 1

Mathematics Ext1 Question 2.

Suggested Solutions	Marks	Marker's Comments
a) General Term for $(5+x)^4$ is ${}^4C_r 5^{4-r} x^r$		
General Term for $(1-\frac{2}{x})^3$ is ${}^3C_k (1)^{3-k}(-2x^{-1})^k$		1
$= {}^3C_k (-2)^k (x^{-k})$		
$\therefore$ General Term for product is ${}^4C_r 5^{4-r} x^r \, {}^3C_k (-2x)^{-k}$		
For constant: $x^r . x^{-k} = 1$		
$r = k$		
$\therefore$ when $r=k=0$: ${}^4C_0 5^4 \times {}^3C_0 = 625$		
$r=k=1$, ${}^4C_1 5^3 \times {}^3C_1(-2) = -3000$		
$r=k=2$, ${}^4C_2 5^2 \times {}^3C_2(-2)^2 = 1800$		
$r=k=3$, ${}^4C_3 5 \times {}^3C_3(-2)^3 = -160$		
$\therefore$ coefficient is $625 - 3000 + 1800 - 160$		
$= -735$		1
or		

Mathematics Ext 1 Question 2

Suggested Solutions	Marks	Marker's Comments

a) $(5+x)^4\left(1-\frac{2}{x}\right)^3$

$= \left(5^4 + {}^4C_1(5)^3x + {}^4C_2 5^2x^2 + {}^4C_3 5x^3 + x^4\right) \times$

$\left(1 - {}^3C_1\left(\frac{-2}{x}\right) + {}^3C_2\left(\frac{2}{x}\right)^2 - \left(\frac{2}{x}\right)^3\right)$ — 1

$\therefore$ constant term is :

$5^4 + {}^4C_1(5)^3\,{}^3C_1(-2) + {}^4C_2 5^2\,{}^3C_2(-2)^2 + {}^4C_3 5 \times (-2)^3$

$= -735$ — 1

b) $\frac{d}{dx}\left[x^2\cos^{-1}(3x)\right]$

$= x^2\frac{-1}{\sqrt{1-9x^2}} \times 3 + \boxed{2x\cos^{-1}(3x)}$ — 1

$= 2x\cos^{-1}(3x)\boxed{-\frac{3x^2}{\sqrt{1-9x^2}}}$ — 1

c) $\sin^{-1}(\sin\pi)$

$= \sin^{-1}(0)$

$= 0$ — 1

d) $\frac{dy}{dx} = \frac{1}{1+\sin^2x} \times \cos x$ — 1

$= \frac{\cos x}{1+\sin^2 x}$

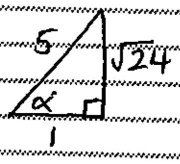
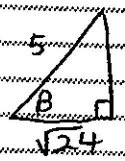
Note: $1+\sin^2x \neq \cos^2x$

When $x=\pi$, $\frac{dy}{dx} = \frac{-1}{1+0} = -1$ — 1

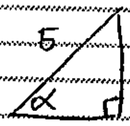
Mathematics Ext1 Question 2.

Suggested Solutions	Marks	Marker's Comments
e) $2\cos 3x \sin 4x + 2\cos 3x - \sin 4x - 1 = 0$ $2\cos 3x(\sin 4x + 1) - (\sin 4x + 1) = 0$ $(2\cos 3x - 1)(\sin 4x + 1) = 0$ $\therefore 2\cos 3x - 1 = 0$ or $\sin 4x + 1 = 0$ 1st: $\cos 3x = \frac{1}{2}$ $(n \in J)$ $3x = 2n\pi \pm \frac{\pi}{3}$ $x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ 2nd $\sin 4x = -1$ $4x = n\pi + (-1)^n\left(\frac{-\pi}{2}\right)$ $(n \in J)$ $x = \frac{n\pi}{4} + (-1)^n\left(\frac{-\pi}{8}\right)$ other general solutions 1st Mark — Factorise 2nd Mark — 1st correct general solution 3rd Mark — 2nd correct general solution and defining $n \in J$.		$\frac{n\pi}{4} + (-1)^n\frac{3\pi}{8}$, $\frac{n\pi}{2} + \left(\frac{-\pi}{8}\right)$ $\frac{n\pi}{2} + \left(\frac{3\pi}{8}\right)$

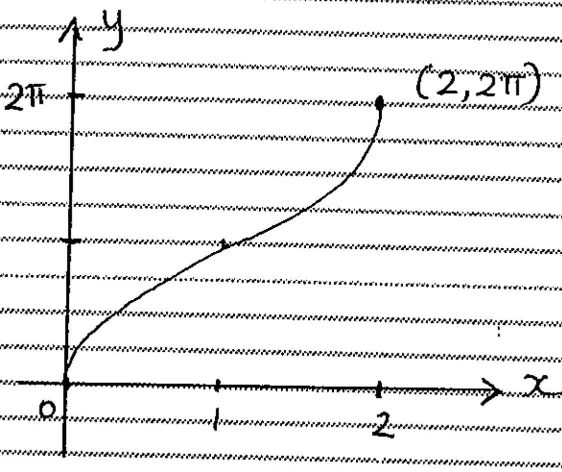
MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
a) $(1+ax)^n = 1 + {}^nC_1 ax + {}^nC_2 a^2x^2 + \dots$		
Coefficient of x : $na = -20$		
coefficient of x^2 : ${}^nC_2 a^2 = 180$		
$na = -20$ $\quad \dfrac{n!\,a^2}{2!(n-2)!} = 180$	1	coefficients with ${}^nC_2$ in factorial notation
$n(n-1)a^2 = 360$		
$a^2n^2 - a^2n = 360$		
$(-20)^2 - a(-20) = 360$	1	linear equation in a or n
$400 + 20a = 360$		
$20a = -40$		
$a = -2$		
$-2n = -20$		
$n = 10$	1	a and n
b) $\cos(2\cos^{-1}x + \sin^{-1}x)$, $x = 1/5$		
Method 1:		Note:
Let $\alpha = \cos^{-1} 1/5$ $\quad$ Let $\beta = \sin^{-1} 1/5$		$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos\alpha = 1/5$ $\quad$ $\sin\beta = 1/5$		$= 1 - 2\sin^2\alpha$
$\therefore 0 < \alpha < \pi/2$ $\quad$ $\therefore 0 < \beta < \pi/2$		$= \cos^2\alpha - \sin^2\alpha$
 		
$\therefore \sin\alpha = \dfrac{2\sqrt6}{5}$ $\quad$ $\cos\beta = \dfrac{2\sqrt6}{5}$	1	$\sin\alpha$ and $\cos\beta$
$\cos(2\alpha+\beta) = \cos 2\alpha\cos\beta - \sin 2\alpha \sin\beta$		
$= (\cos^2\alpha - \sin^2\alpha)\cos\beta - 2\sin\alpha\cos\alpha\sin\beta$	1	expansion with double angles
$= \left(\dfrac{1}{25} - \dfrac{24}{25}\right)\dfrac{2\sqrt6}{5} - 2 \times \dfrac{2\sqrt6}{5} \times \dfrac{1}{5} \times \dfrac{1}{5}$		
$= \dfrac{-23}{25} \times \dfrac{2\sqrt6}{5} - \dfrac{4\sqrt6}{125}$		
$= \dfrac{-50\sqrt6}{125} = \dfrac{-2\sqrt6}{5}$	1	answer

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
<u>Method 2</u>:		
$\cos(2\cos^{-1}x + \sin^{-1}x)$ $= \cos(\cos^{-1}x + \cos^{-1}x + \sin^{-1}x)$ $= \cos(\cos^{-1}x + \pi/2)$ since $\cos^{-1}x + \sin^{-1}x = \pi/2$	1	$\cos^{-1}x + \sin^{-1}x = \pi/2$ and $\cos(\cos^{-1}x + \pi/2)$
Let $\alpha = \cos^{-1}x$ $\cos\alpha = x$ $= 1/5$ $\therefore 0 < \alpha < \pi/2$		
 $\therefore \sin\alpha = 2\sqrt{6}/5$	1	$\sin\alpha$
$\cos(\cos^{-1}x + \pi/2)$ $= \cos(\alpha + \pi/2)$ $= \cos\alpha\cos\pi/2 - \sin\alpha\sin\pi/2$ $= \cos\alpha \times 0 - \sin\alpha \times 1$ $= -2\sqrt{6}/5$	1	answer
<u>Note</u>: Marks were not deducted for omission of domain of α and β, however the best answers showed why α and β are acute. Students should consider what their answers would have been had x been negative.		

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
c) $f(x) = 2\cos^{-1}(1-x)$		
i) Domain: $-1 \le 1-x \le 1$		
$1 \ge x-1 \ge -1$		
$2 \ge x \ge 0$		
Range: $0 \le \frac{y}{2} \le \pi$		
$0 \le y \le 2\pi$		
Domain: $\{x : 0 \le x \le 2\}$	1	domain
Range: $\{y : 0 \le y \le 2\pi\}$	1	range
ii) 	1	correct shape
	1	endpoints

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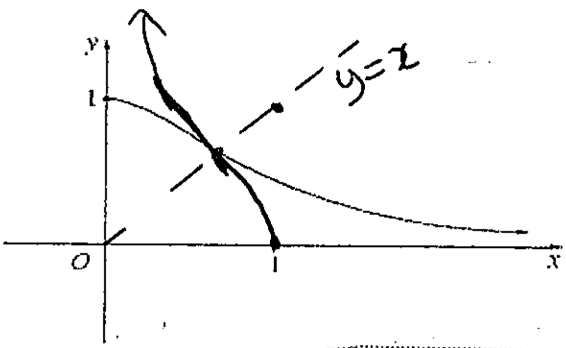
MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
a) 	2	1 for evidence of reflection in $y=x$ 1 for correct graph
b) $y = \frac{1}{4-x}$ $x = \frac{1}{4-y}$ $x(4-y) = 1$ $xy = 4x-1$ $y = \frac{4x-1}{x}$	1	well done had to be written in form $y =$ to gain mark
c) $f(x) = 2\cos^{-1}\frac{x}{\sqrt{2}} - \sin^{-1}(1-x^2)$ $f'(x) = \frac{-2}{\sqrt{2-x^2}} - (-2x)\frac{\cancel{2}}{\sqrt{1-(1-x^2)^2}}$ $= \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{\sqrt{2x^2-x^4}}$ $= \frac{-2}{\sqrt{2-x^2}} + \frac{2x}{x\sqrt{2-x^2}}$ if $x>0$ $= 0$	1	1 mark for correctly differentiated

MATHEMATICS Extension 1 : Question 4		
Suggested Solutions	**Marks**	**Marker's Comments**
(ii) Since $f'(x) = 0 \quad x > 0$ $f(x) = c$ or $f(x)$ is a horizontal line so testing a point	1	NB: A straight line is not a horizontal
$f(1) = 2\cos^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 1$ $= 2 \times \frac{\pi}{2} - \frac{\pi}{2}$ $= \frac{\pi}{2}$	1	
$\therefore f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}(1 - x^2)$ $= \frac{\pi}{2} \quad 0 < x < \sqrt{2}$		
(d) (i) $\frac{dv}{dt} = -2.4t$	1	students lost the negative
$v = -1.2t^2 + c$ when $t = 0 \quad v = 10{,}000$ $\therefore c = 10{,}000$ $v = -1.2t^2 + 10{,}000$	1	
(ii) $30\% \times 10{,}000 = 3000$ $1.2t^2 = 7000$ $t = \sqrt{\frac{7000}{1.2}}$ $= 76$ seconds accepted (77s)	1	$30\% \neq \frac{1}{3}$.

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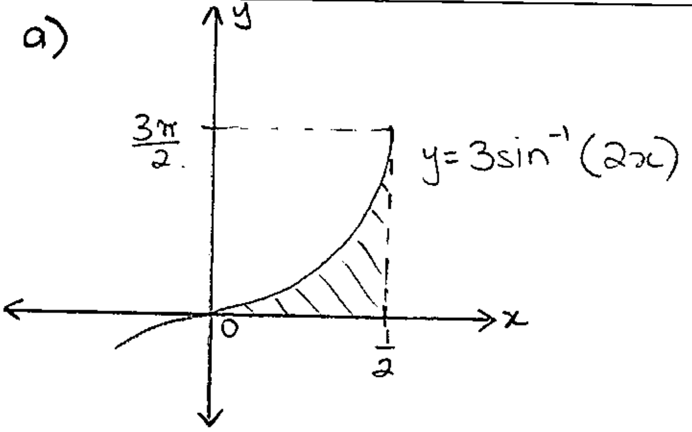
MATHEMATICS Extension 1: Question 5

Suggested Solutions	Marks	Marker's Comments
a)		
		
$y = 3\sin^{-1}(2x)$		
$\frac{y}{3} = \sin^{-1}(2x)$		
$2x = \sin\frac{y}{3}$		
$x = \frac{1}{2}\sin\frac{y}{3}$	1	
Area $= \frac{3\pi}{2} \times \frac{1}{2} - \int_0^{\frac{3\pi}{2}} \frac{1}{2}\sin\frac{y}{3}\,dy$		1 ⇒ rectangle
$= \frac{3\pi}{4} - \frac{1}{2}\left[-3\cos\frac{y}{3}\right]_0^{\frac{3\pi}{2}}$		1 ⇒ correct upper and lower bounds and correct integral
$= \frac{3\pi}{4} - \frac{1}{2}\left(-3\cos\frac{\frac{3\pi}{2}}{3} + 3\cos 0\right)$		
$= \frac{3\pi}{4} - \frac{1}{2}(0+3)$		
$= \frac{3\pi}{4} - \frac{3}{2}$	1	
b) i) $y=\sqrt{x} \Rightarrow x = y^2$		
$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$		
$\therefore 2dy = \frac{dx}{\sqrt{x}}$		
$\int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{2\,dy}{\sqrt{1-y^2}}$	1	
$= 2\sin^{-1}y + c$		
$= 2\sin^{-1}\sqrt{x} + c$	1	need to substitute $\sqrt{x}$ back.

MATHEMATICS Extension 1: Question 5

Suggested Solutions	Marks	Marker's Comments
ii) $z = x - \frac{1}{2} \Rightarrow x = z + \frac{1}{2}$		
$\frac{dz}{dx} = 1$		
$\therefore dz = dx$		
$\int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{dz}{\sqrt{(z+\frac{1}{2})(\frac{1}{2}-z)}}$	1	
$= \int \frac{dz}{\sqrt{\frac{1}{4} - z^2}}$		
$= \sin^{-1} 2z + c$	1	
$= \sin^{-1}(2x-1) + c$		
iii) from (i) & (ii)		
$\sin^{-1}(2x-1) = 2\sin^{-1}\sqrt{x} + c$	1	must have $+c$
when $x = \frac{1}{2}$, $0 < x < 1$		
$\sin^{-1}(0) = 2\sin^{-1}\frac{1}{\sqrt{2}} + c$	1	use correct x to find $c = -\frac{\pi}{2}$.
$\therefore c = -\frac{\pi}{2}$		
$\therefore \sin^{-1}(2x-1) = 2\sin^{-1}\sqrt{x} - \frac{\pi}{2}$		

MATHEMATICS Extension 1 : Question 6 1/4

Suggested Solutions	Marks Awarded	Marker's Comments

6ai) $\sum_{r=0}^{20} a_r x^r$

$$(3+2x)^{20} = \sum_{r=0}^{20} {}^{20}C_r\, 3^{20-r}(2x)^r$$

$$^{20}C_r\, 3^{20-r}(2)^r(x)^r$$

$$\therefore a_r = {}^{20}C_r\, 3^{20-r}\, 2^r$$ — 1

aii) $a_{r+1} = {}^{20}C_{r+1} 3^{20-(r+1)} 2^{(r+1)}$ *

common error not putting $(r+1)$ in brackets. so students got $(21-r)$ which is incorrect.

$$= \frac{20!}{(r+1)!(20-(r+1))!} \times 3^{19-r} \times 2^{r+1}$$

$$= \frac{20!}{(r+1)!(19-r)!} \times 3^{19-r} \times 2^{r+1}$$ — 1

must be in expanded form.

$$a_r = {}^{20}C_r\, 3^{20-r} \cdot 2^r$$

$$= \frac{20!}{r!(20-r)!} \times 3^{20-r} \times 2^r$$ — 1

$$\therefore \frac{a^{r+1}}{a^r} = \frac{\cancel{20!}}{(19-\cancel{r})!(r+1)!} \times \frac{(20-r)!\,\cancel{(r)}!}{\cancel{20!}} \times \frac{\cancel{3^{19-r}}}{3^{20-r}} \times \frac{2^{r+1}}{\cancel{2^r}}$$

*. match up so its easier to cancel off

$$= \frac{20-r}{r+1} \times \frac{2}{3} = \frac{40-2r}{3r+3}$$ Shown.

MATHEMATICS Extension 1 : Question……6 2/4

Suggested Solutions	Marks Awarded	Marker's Comments

aiii) for greatest coefficient (G.C)

$\frac{a_{r+1}}{a_r} > 1$

$\therefore$ then $C_{r+1} > C_r$.

$40 - 2r > 3r + 3$

$37 > 5r$

$r < \frac{37}{5}$ or $7\frac{2}{5}$ or 7.4 — (1)

check.

when $r = 7$ $a_8 > a_7$

$r = 6$ $a_7 > a_6$

$r = 0$ $a_1 > a_0$

$\therefore a_8 > a_7 > a_6 > \ldots > a_0$.

$\frac{a_{r+1}}{a_1} < 1$ then $a_8 > a_r > \ldots a_0$

$\frac{a^{r+1}}{a^r} > 1$ holds true

$T_0 < T_1 < T_2 \ldots < T_6 < T_7 < T_8 > T_9$

$\therefore G.C = a_8$

$= {}^{20}C_8\, 3^{12}\, 2^8$
or 1.71×10^{13} — (1)

MATHEMATICS Extension 1 : Question 6 3/4

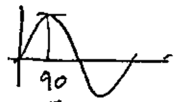
Suggested Solutions	Marks Awarded	Marker's Comments

b) $\int_1^2 \frac{4}{\sqrt{4-x^2}}\,dx$

$= 4\int_1^2 \frac{1}{\sqrt{2^2-x^2}}\,dx$

$= 4\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$ —— 1

$= 4\left[\sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right)\right]$



$= \frac{\pi}{2}$

$\sin 30 = \frac{1}{2}$

$\frac{\pi}{6}$

$= 4\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \frac{4\pi}{3}$ —— 1

c) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]^2$

Let $y = \left(\tan^{-1}\left(\frac{x}{3}\right)\right)^2$

$\frac{dy}{dx} = 2\left(\tan^{-1}\left(\frac{x}{3}\right)\right) \times \frac{3}{3^2+x^2}$

$= \frac{6\tan^{-1}\frac{x}{3}}{9+x^2}$ —— 1

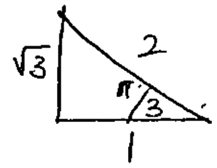
* Common error
→ not multiplying 2 & 3
→ not differentiating the inside.

Hence.

$\int_0^{\sqrt{3}} \frac{\tan^{-1}\left(\frac{x}{3}\right)}{9+x^2}\,dx.$ compare the two.

$\therefore = \frac{1}{6}\left[\tan^{-1}\left(\frac{x}{3}\right)^2\right]_0^{\sqrt{3}}$ —— 1

MATHEMATICS Extension 1 : Question…….. 4/4

Suggested Solutions	Marks Awarded	Marker's Comments
$= \frac{1}{6}\left(\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)^2 - \left(\tan^{-1}\frac{0}{3}\right)^2\right)$ $= \frac{1}{6}\left(\frac{\pi}{6}\right)^2 - 0$ $= \frac{\pi^2}{216}$	1	 *common error - not identifying the angle correctly - not multiplying by $\frac{1}{6}$ - not squaring