Question One **Marks**

a) In a factory that makes metal containers, it is found that 2% of containers have defects.

i) Find the probability that a random samples of 20 has no defective containers. **1**

ii) Find the probability that a random sample of 20 has no more than one defective containers. **2**

b) A committee of ten people, including three teenagers, sit around a round table. **2**
Find the number of different arrangements of the committee around the table if all the three teenagers want to sit together. Give reasons.

c) Initially when an object falls from rest through a resisting liquid, the rate of change of its velocity (v) at time t seconds is given by $\frac{dv}{dt} = -k(v - 600)$, where k is a positive constant.

i) Show that $v = 600 + Pe^{-kt}$ is a solution to the differential equation for some constant P. **1**

ii) If the velocity of the object at $t = 3$ is 25m/s, find the values of P and k. **2**

iii) Find the velocity of the object at $t = 10$. **1**

Question Two (Start a New Page)

The following information relates to parts a) and b).

a) Two particles are moving in a straight line. x is the displacement in metres from a fixed point O on the line, t is the time in seconds , v is the velocity in m/s and a is the acceleration in m/s^2 .

For the first particle: Its velocity is v m/s and acceleration is given by $a = x + \frac{3}{2}$.

Initially the particle is 5 m to the right of O and moving towards O with a speed of 6 m/s.

i) Is the particle initially speeding up or slowing down? Give reasons. **1**

ii) Show that $v^2 = x^2 + 3x - 4$. **2**

iii) Find where the particle changes direction. Give reasons. **2**

b) For the second particle: Its velocity v is given by $v = \sqrt{x}$ and acceleration is a m/s^2 .
Initially the particle is 1 m to the right of O .

i) Show that a is independent of x. **1**

ii) Express x in terms of t. **3**

Question Three (Start a New Page)

a) After t hours, the number of individuals in a population is given by $N = 500 - 400e^{-0.1t}$.

i) Show that $\frac{dN}{dt} = 0.1(500 - N)$. **1**

ii) Find the population size for which the rate of growth of the population is half the initial rate of growth. **2**

b) An isolated insect population (P) is attached to a single plant which has a carrying capacity of 100 insects at time t. Its logistic law of growth is given by $\frac{dP}{dt} = 0.001P(100 - P)$.

i) Show that $P = \frac{100}{1 + (\frac{1}{k})e^{-0.1t}}$. **3**

ii) If initially $P = 10$, find k. **1**

iii) Given that $P = 85.85$ when $t = 40$, sketch the graph of the function $P(t)$. **2**

Question Four (Start a New Page)

a) There are 3 red, 3 blue and 2 yellow cars. They are to be parked in line. **2**
If the yellow cars are not parked together, find the number of different arrangements (assuming cars of the same colour are identical).

b) Four people visit a town with four restaurants *A, B, C* and *D*. Each person chooses a restaurant at random.
Find the probability that they all choose different restaurants. **2**

c) From a deck of 40 cards, there are 10 yellow, 10 blue, 10 green and 10 red. **2**
10 cards are chosen at random.
Find the probability that there is at least one blue chosen if it is known that no yellow cards were chosen.

d) The displacement x metres from the origin after t seconds of a particle moving in a straight line is given by $x = \sin^2 3t, \quad t > 0$.
Show that the particle undergoes simple harmonic motion. **3**

Question Five (Start a New Page)

a) A particle is oscillating between A and B, 7m apart, in Simple Harmonic Motion. **3**
The time for the particle to travel from A to B and back is 3 seconds.
If O is the centre of AB and M is the mid-point of OB, find the speed of the particle at M.

b) The acceleration of a particle moving on the x-axis is given by $a = x - 2$ where x is the displacement from the origin in metres after t seconds and a is in m/s^2.
Initially the particle is at rest at the position $x = 3$.

i) If v is the velocity in m/s at any position x, show that $v^2 = (x-1)(x-3)$ **2**

ii) Describe the motion of the particle. **1**

iii) Find the acceleration of the particle when its velocity is $2\sqrt{6}$ m/s^2. **3**

Question Six (Start a New Page)

a) Ming has a tank containing 10 tagged fish and 50 untagged fish. Each day he selects 4 fish at random and places them together in a separate tank for observation.
They are returned to the original tank later on the same day.

i) What is the probability of selecting no tagged fish on a given day? **1**

ii) What is the probability of selecting at least one tagged fish on a given day? **1**

iii) Find the probability of selecting no tagged fish on exactly 3 days of the week. **2**

b) In a tidal river, the top of an anchorage post measured 0.8 metres below the water level at high tide and 0.2 metres above the river level at low tide. High tide occurred at 6.30am and low tide occurred at 12.35pm on the day that the measurements were taken.
The motion of the tide can be assumed to be simple harmonic.

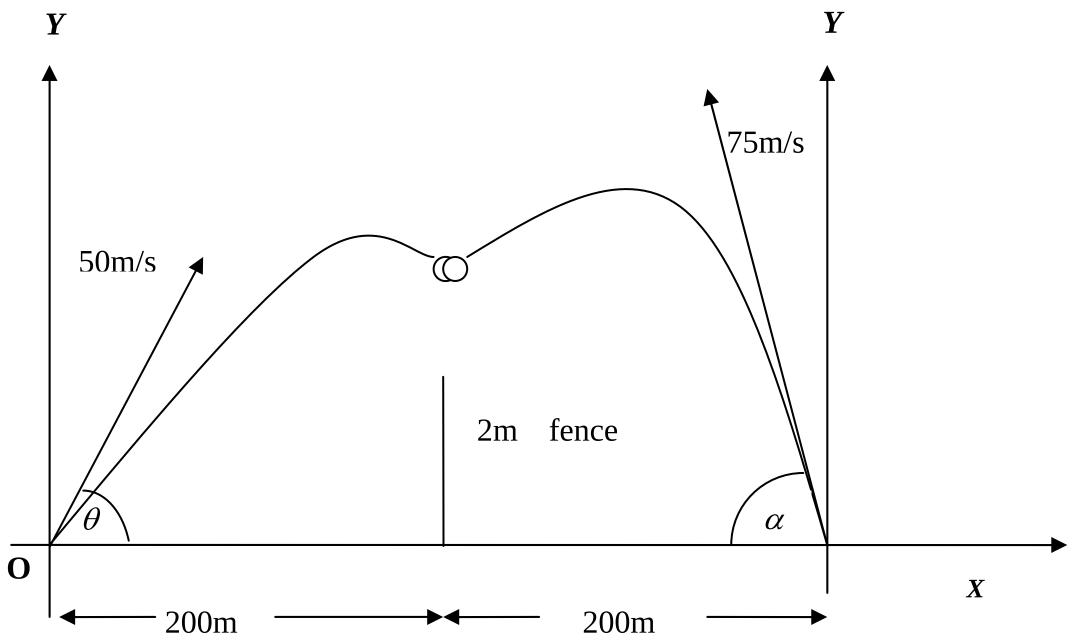
i) Show that the period of the motion is 730 minutes. **1**

ii) Assuming the day started at 12am, find the first time period of the day when there was at least 0.5 metres of water above the top of the old anchorage post. Answer to the nearest minute. **4**

Question Seven (Start a New Page)

One method to score a home run in a baseball game is to hit the ball over the boundary fence on the full.

Diagram not to scale



A ball is hit at 50 metres per second with an angle θ to the horizontal.
The fence 200 metres away is 2 metres high.

i) Write down the 6 equations of motion of the ball in the x and y directions (neglect air resistance and take acceleration due to gravity as 10m/s^2). **1**

ii) Show the ball would just clear the 2 metre high boundary fence when $40\tan^2\theta - 100\tan\theta + 41 = 0$, where θ is the angle of projection. **2**

iii) In what range must θ lie to score a home run by this method ? **2**

iv) In an adjacent field, another ball which is 200m away from the other side of the fence is hit at the same instant at 75m/s with an angle α to the horizontal. **2**
The two balls collide as shown in the above diagram.
Assuming $\theta = 30^\circ$, find the angle of projection α of the second ball .

v) Find the time when the balls collide. **2**

End of Paper

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MATHEMATICS Extension 1 : Question 1 11/12

Suggested Solutions	Marks	Marker's Comments
(a) $(0.98)^{20} = 0.6676$	1	Should calculate values.
(ii) $P(\text{not more than } 1) = P(0) + P(1)$ $= 0.6676 + \binom{20}{19}(0.98)^{19}(0.02)^{1}$ $= 0.94$ (2dp)	2	Students didn't read closely did 1 - incorrectly
(b) 3 teenagers is 3! ways around a table 8 groups is 7!	1	Incorrect number with explanation gained 1.
$\therefore 3! \times 7! = 30240$	1	
(c) (i) $v = 600 + Pe^{-kt}$ —(1) $\frac{dv}{dt} = -kPe^{-kt}$ but $Pe^{-kt} = v - 600$ from (i) $\therefore \frac{dv}{dt} = -k(v - 600)$	1.	without this, did not gain mark.
(ii) $25 = 600 + e^{-3k}$. ~~525~~ when $t = 0$ $v = 0$ $\therefore 0 = 600 + P$ $P = -600$	1	

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MATHEMATICS Extension 1 : Question ...1.... 2/2

Suggested Solutions	Marks	Marker's Comments
$-600e^{-3k} = -575$ $\therefore -3k = \ln\frac{23}{24}$ $k = \frac{1}{3}\ln\frac{24}{23}$	1	
(iii) $t=10$ $v = 600 - 600e^{\left(\frac{1}{3}\ln\frac{23}{24}\right)\times 10}$ $v = 79.4$ m/s.	1	Students lost −ve sign and so got −ve answer.

MATHEMATICS Extension 1 : Question 2		1/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
ai) $x = 5$, $v < 0$ $\quad a = 5 + \frac{3}{2} > 0$ $\therefore$ slowing down	1	
ii) $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = x + \frac{3}{2}$ $\frac{1}{2}v^2 = \frac{x^2}{2} + \frac{3x}{2} + C$	1	
$x = 5 \quad v = -6$ $\frac{1}{2}(-6)^2 = \frac{5^2}{2} + \frac{3(5)}{2} + C$ $C = -2$	1	
$v^2 = x^2 + 3x + 2(-2)$ $v^2 = x^2 + 3x - 4$		
OR $\int_{-6}^{v} d\,\frac{1}{2}v^2 = \int_5^x \left(x + \frac{3}{2}\right)dx$ $\frac{1}{2}\left[v^2\right]_{-6}^{v} = \left[\frac{x^2}{2} + \frac{3}{2}x\right]_5^x$ *	1	stating the initial limit conditions correctly.
$\frac{1}{2}(v^2 - 36) = \frac{x^2}{2} + \frac{3}{2}x - \left(\frac{25}{2} + \frac{15}{2}\right)$ $\frac{v^2}{2} - 18 = \frac{x^2}{2} + \frac{3}{2}x - 20$ $\frac{v^2}{2} = \frac{x^2}{2} + \frac{3}{2}x - 2$ $v^2 = x^2 + 3x - 4$	1	tidy up.

2/4

MATHEMATICS Extension 1 : Question 2		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
a iii) $v^2 = (x+4)(x-1)$ $\therefore v=0$ $\therefore x = 1, -4$	1	* if have not mentioned acceleration ONLY 1 mark ! is awarded.
at $x=1$ $a = 2.5 > 0$. turning right at $x=1$ speeding up & never reach $x=4$. $\therefore$ changes direction at $x=1$ only.	1	
OR at $x=1$ $v=0$, $a>0$ changes direction at $x=1$ because particle stops. Initially particle was moving left but $a>0$, the particle is being moved to the right so it changes direction.	2	
OR starts at $x=5$ so $x \geq 1$ at $x=1$ $v=0$ $a = 2.5$ (+ve) $\therefore$ stops & changes direction.	2.	

MATHEMATICS Extension 1 : Question 2		3/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

b i) $v = \sqrt{x}$

$v^2 = x$

$\frac{1}{2}v^2 = \frac{x}{2}$

$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{1}{2}$

$\therefore \frac{1}{2} = a$ — 1

a is independant of x.

ii) $v = \sqrt{x}$

$= \frac{dx}{dt}$

$dt = \frac{dx}{\sqrt{x}}$ — 1

$\int_0^t dt = \int_1^x \frac{1}{\sqrt{x}}\,dx$

$t = \left[2x^{\frac{1}{2}}\right]_1^x$

$t = 2\sqrt{x} - 2.$ — 1

$2\sqrt{x} = t + 2$

$x = \left(\frac{t+2}{2}\right)^2$

or $\frac{t^2+4t+4}{4}$

or $\frac{(t+2)^2}{4}$

or $\frac{t^2}{4} + t + 1$ — 1

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MATHEMATICS Extension 1 : Question 2		4/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

OR.

$$\int_1^x \frac{dx}{\sqrt{x}} = \int_0^t dt \quad 1$$

$2\sqrt{x} = t + k$. $\quad t=0 \quad x=1$

$k = 2$.

$$2\sqrt{x} = t+2. \quad 1$$

$$x = \left(\frac{t+2}{2}\right)^2 \quad 1$$

OR.

$a = \frac{1}{2}$

$\frac{dv}{dt} = \frac{1}{2}$

$$\int_1^v dv = \int_0^t \tfrac{1}{2}\, dt \qquad \left.\begin{array}{l} t = 0 \\ v = \sqrt{x} = 1 \\ \text{coz } x = 1 \end{array}\right\} \quad 1$$

$v - 1 = \frac{1}{2}t$

$v = \frac{1}{2}t + 1$

$$\int_1^x dx = \int_0^t \left(\tfrac{1}{2}t + 1\right) dt \quad 1$$

$[x]_1^x = \left[\frac{1}{4}t^2 + t\right]_0^t$

$x - 1 = \frac{1}{4}t^2 + t$

$$x = \frac{1}{4}t^2 + t + 1 \quad 1$$

MATHEMATICS Extension 1 : Question 3		
Suggested Solutions	**Marks**	**Marker's Comments**
a) i) Show that $\frac{dN}{dt} = 0.1(500-N)$ Given that $N = 500 - 400e^{-0.1t}$ $\frac{dN}{dt} = -400(0.1)e^{-0.1t}$ ✓ $= 0.1(400e^{-0.1t})$ $= 0.1(500-N)$ from $N = 500 - 400e^{-0.1t}$ $400e^{-0.1t} = 500 - N$	1 Mark	Marks given for correct differentiation and replacing $400e^{-0.1t}$ with $500-N$
(ii) Population size when rate of growth is halved $\frac{dN}{dt} = 0.1(500-N)$ Initially at $t=0$ $N = 500 - 400e^{-0.1(0)}$ $= 500 - 400$ $= 100$ $\therefore \frac{dN}{dt} = 0.1(500-100)$ $= 0.1 \times 400$ $= 40$ ✓ Find t when $\frac{dN}{dt} = 20$ $20 = 0.1(500-N)$ $200 = 500 - N$ $\therefore N = 500 - 200$ $N = 300$ ✓	2 Marks	1 mark given for getting $\frac{dN}{dt} = 40$ 1 Mark for halving 40 and arriving at the answer 300
b (i) $\frac{dP}{dt} = 0.001P(100-P)$ show $P = \frac{100}{1+\frac{1}{k}e^{-0.1t}}$	3 Marks	Two approaches • Differentiation of $P = \frac{100}{1+\frac{1}{k}e^{-0.1t}}$ • Integration of $\frac{dP}{dt} = 0.001P(100-P)$

MATHEMATICS Extension 1 : Question 3		
Suggested Solutions	**Marks**	**Marker's Comments**
Differentiation approach Quotient Rule Simplifying $100\left(1+\frac{1}{k}e^{-0.1t}\right)^{-1}$ Using Quotient Rule $u = 100k \rightarrow u' = 0$ $v = k + e^{-0.1t} \rightarrow v' = -0.1e^{-0.1t}$ $\frac{vu' - uv'}{v^2} = \frac{0(k+e^{-0.1t}) - 100k(-0.1e^{-0.1t})}{(k+e^{-0.1t})^2}$ $= \frac{10ke^{-0.1t}}{(k+e^{-0.1t})^2}$ ✓ $\frac{dP}{dt} = \frac{10k(k+e^{-0.1t}) - 10k^2}{(k+e^{-0.1t})^2}$ ✓ $= \frac{10k}{k+e^{-0.1t}} - 0.001\frac{10000k^2}{(k+e^{-0.1t})^2}$ $= 0.1P - 0.001P^2$ ✓ $= 0.001P(100-P)$		✓ 1 mark for getting correct $\frac{dP}{dt}$. Accepted • $\frac{10ke^{-0.1t}}{k+e^{-0.1t}}$ • $\frac{10ke^{-0.1t}}{1+\frac{1}{k}e^{-0.1t}}$ • $\frac{\frac{10}{k}e^{-0.1t}}{1+\frac{1}{k}e^{-0.1t}}$ • $\frac{100}{(1+\frac{1}{k}e^{-0.1t})^2} \times \frac{0.1e^{-0.1t}}{k}$ ✓ 1 mark correct manipulation ✓ 1 mark for writing in terms of P
Integration approach. $\frac{dP}{dt} = 0.001P(100-P) = \frac{P(100-P)}{1000}$ $\frac{dt}{dP} = \frac{1000}{P(100-P)}$ $\int dt = 1000\int \frac{1}{P(100-P)}\,dP$ $t = 10\int \frac{1}{P}\,dP + 10\int \frac{1}{100-P}\,dP$ $10\ln P - 10\ln(100-P) = 0.1t + C$, $\ln\left(\frac{P}{100-P}\right) = 0.1t + C$ $\frac{P}{100-P} = e^{0.1t+C_2}$ $\therefore P = C_3e^{0.1t}100 - PC_3e^{0.1t}$ $e^{0.1t+C_2}$ is same as $e^{0.1t}+e^{C_2}$ $e^{0.1t}+C_3$		1 mark for correct integration use $\frac{1}{P(100-P)} = \frac{A}{P} + \frac{B}{100-P}$ $1 = A(100-P) + BP$. 1 mark for correct manipulation to get $P = \frac{100}{1+\frac{1}{k}e^{-0.1t}}$

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments

$$P + PC_3 e^{0.1t} = 100e^{0.1t}.C_3$$

$$P(1 + e^{0.1t}.C_3) = 100e^{0.1t}C_3$$

$$P = \frac{100e^{0.1t}C_3}{1 + e^{0.1t}C_3}$$

Divide $e^{0.1t}C_3$

$$P = \frac{100}{1 + e^{-0.1t}.\frac{1}{C_3}}$$

$$= \frac{100}{1 + \frac{1}{K}e^{-0.1t}}$$

b(ii) Find K if P = 10

$$P = \frac{100}{1 + \frac{1}{K}e^{-0.1t}} = \frac{100K}{K + e^{-0.1t}}$$

when t = 10, P = 10

$$10 = \frac{100K}{K + e^{0}}$$

$$10 = \frac{100K}{K+1}$$

$$10K + 10 = 100K$$

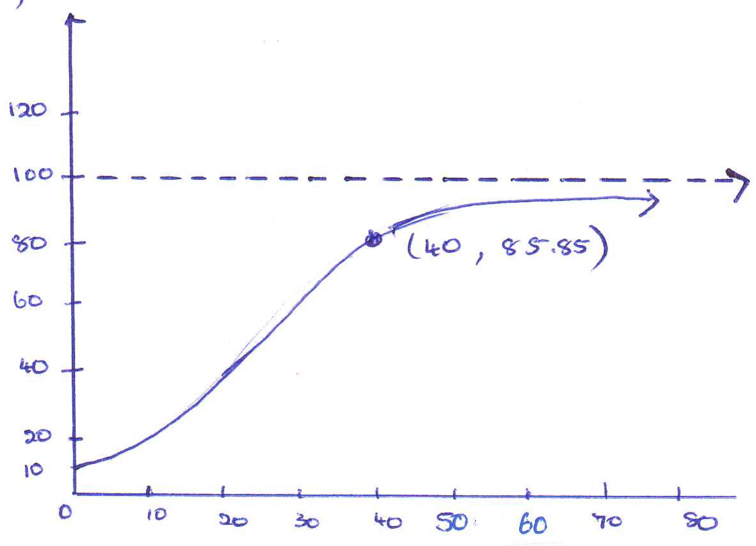
$$90K = 10$$

$$K = \frac{1}{9}$$ ✓

Marks: 1 mark

Marker's Comments: 1 mark for $K = \frac{1}{9}$ with correct working

(iii)



Marks: 2 marks.

Marker's Comments: 1 mark for showing asymptote at 100

1 mark for giving one point (40, 85.85) and shape

1 mark taken away if graph looks like touching the asymptote

MATHEMATICS Extension 1 : Question ...4...

Suggested Solutions	Marks	Marker's Comments
a) without restriction: $\dfrac{8!}{3!3!2!} = 560$ yellow together: $\dfrac{7!}{3!3!} = 140$	1	
$\therefore$ yellow not together: $560 - 140$ $= 420$	1	
OR ↓_ ↓_ ↓_ ↓_ ↓_ ↓_ ↓ ways of arranging red and blue: $\dfrac{6!}{3!3!} = 20$ placing yellow in the 'gaps': $\dfrac{7\times6}{2!}$ or ${}^7C_2 = 21$	1	
$\therefore$ yellow not together: 20×21 $= 420$	1	
b) all different : $4!$ without restriction: 4^4	1	
$\therefore$ P(all different) $= \dfrac{4!}{4^4}$ $= \dfrac{3}{32}$	1	
OR first person : $\frac{4}{4}$ second person: $\frac{3}{4}$ third person: $\frac{2}{4}$ last person: $\frac{1}{4}$	1	$\frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1$ is incorrect, since the people are not restricted by which restaurant they have to go to.
$\therefore$ P(all different) $= \frac{4}{4} \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4}$ $= \dfrac{3}{32}$	1	

MATHEMATICS Extension 1 : Question…4…

Suggested Solutions	Marks	Marker's Comments
c) $1 - P(\text{no blue}) = 1 - \dfrac{{}^{20}C_{10}}{{}^{30}C_{10}}$	1	this is a conditional probability question, hence it is over ${}^{30}C_{10}$ not ${}^{40}C_{10}$.
$= 0.99385\ldots$		
$= 0.99$ (2dp)	1	
d) $x = \sin^2 3t$		
$= \dfrac{1}{2} - \dfrac{\cos 6t}{2}$	1	
$\dot{x} = 3\sin 6t$		
$\ddot{x} = 18\cos 6t$	1	
$= 18(1 - 2\sin^2 3t)$		
$= 18(1 - 2x)$		
$= 36(\frac{1}{2} - x)$		
$= -36(x - \frac{1}{2})$		
$= -6^2(x - \frac{1}{2})$	1	must have conclusion and correct form of $\ddot{x}$
which is in the form $\ddot{x} = -n^2(x - x_0)$		
$\therefore$ particle is in simple harmonic motion		

Mathematics Ext1 Question 5

Suggested Solutions	Marks	Marker's Comments
b) i) $a = x - 2$		
$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 2$	1	
$\frac{1}{2}v^2 = \int x - 2 \, dx$		
$v^2 = \int 2x - 4 \, dx$		
$= x^2 - 4x + C$		
when $x = 3$, $v = 0$		
$\therefore 0 = 3^2 - 4(3) + C$		
$= -3 + C$		
$\therefore C = 3$		
$\therefore v^2 = x^2 - 4x + 3$	1	
$v^2 = (x-3)(x-1)$		
ii) Intially, the particle at rest will experience a positive acceleration giving it a positive velocity. as x increase, the particle will continue to accelerate towards the positive direction, never turning around.	1	
iii) Since the particle never changes direction,		
$v = \sqrt{(x-1)(x-3)}$		
$v = 2\sqrt{6} \longrightarrow 2\sqrt{6} = \sqrt{(x-1)(x-3)}$	1	
$(x-1)(x-3) = 24 \quad (x > 3)$		
$x^2 - 4x - 21 = 0$		
$\therefore x = \frac{4 \pm \sqrt{16 + 84}}{2} = \frac{4 \pm 10}{2} = 2 \pm 5$	1	
$x = 7 \ (x > 0)$, $\therefore a = 5 \text{ ms}^{-2}$	1	

Mathematics Ext 1 Question 5

Suggested Solutions	Marks	Marker's Comments

a) $a = 3.5 \quad T = 3 \rightarrow \frac{2\pi}{n} = 3, \quad \therefore n = \frac{2\pi}{3}$ — (1)

$x = \frac{7}{2}\sin\left(\frac{2\pi}{3}t + \alpha\right)$

Assuming that the particle starts at the origin,

then $0 = \frac{7}{2}\sin(\alpha)$.

$\therefore \alpha = 0$

$\therefore x = \frac{7}{2}\sin\left(\frac{2\pi}{3}t\right)$

$v = \frac{7\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$ — (1) Expression for v or finding $t = \frac{1}{4}$

when $x = \frac{7}{4}$, then $\frac{7}{4} = \frac{7}{2}\sin\left(\frac{2\pi}{3}t\right)$

$\sin\left(\frac{2\pi}{3}t\right) = \frac{1}{2}$

$\frac{2\pi}{3}t = \frac{\pi}{6}$ (just taking a value for when x is at M)

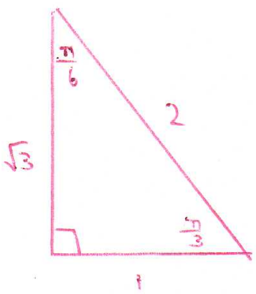
$\therefore t = \frac{1}{4}$

$\therefore$ speed $= |v| = \left|\frac{7\pi}{3}\cos\left(\frac{\pi}{6}\right)\right|$

$= \frac{7\pi}{3} \times \frac{\sqrt{3}}{2}$

$= \frac{7\pi}{2\sqrt{3}}$ — (1)

$= \frac{7\sqrt{3}\pi}{6}$



Mathematics Ext 1 Question 5

Suggested Solutions	Marks	Marker's Comments

Alternative Method

a) $a = 3.5$, $T = 3 \longrightarrow \frac{2\pi}{n} = 3$, $n = \frac{2\pi}{3}$ — 1

Since the motion is Simple harmonic

then $v^2 = n^2(a^2 - x^2)$

$$= \frac{4\pi^2}{9}\left(\frac{49}{4} - x^2\right)$$ — 1

when $x = \frac{7}{4}$

$$v^2 = \frac{4\pi^2}{9}\left(\frac{49}{4} - \frac{49}{16}\right)$$

$$= \frac{4\pi^2}{9}\left(\frac{196-49}{16}\right)$$

$$= \frac{4\pi^2}{9}\left(\frac{147}{16}\right)$$

$$= \frac{147\pi^2}{36}$$

speed $= |v| = \frac{\sqrt{147}\,\pi}{6}$

$$= \frac{7\pi}{2\sqrt{3}}$$ — 1

$$= \frac{7\sqrt{3}\pi}{6}$$

Ext 1 Q6

Q6.

(a)(i) $P(\text{no tagged}) = \frac{50}{60} \times \frac{49}{59} \times \frac{48}{58} \times \frac{47}{57}$

$= \frac{46060}{97527} = 0.4722794$ 1

(ii) $P(\text{at least one tagged}) = 1 - P(\text{no tagged})$

$= 1 - \frac{46060}{97527}$

$= \frac{51467}{97527} = 0.5277205$ 1

(iii) $P(\text{no tagged on 3 days of the week}) = {}^7C_3 (0.5277)^4 (0.47\ldots$ 1

$= 0.2859440132$ (

* Students who used binomial probability in all 3 parts scored a maximum of 3 out of 4.

(b)(i) period $= 6\text{hrs } 5\text{mins} \times 2$

$= 12\text{hrs } 10\text{mins}$

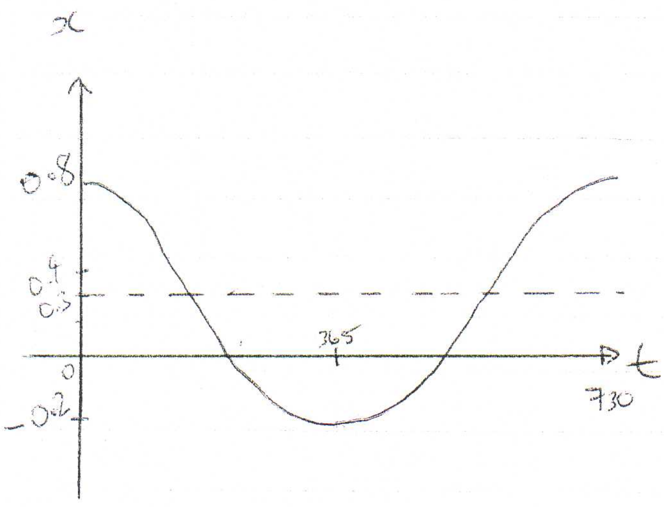
$= 730$ minutes 1

(ii) amplitude $= \frac{0.8 + 0.2}{2} = 0.5$

S.H.M.

$n = \frac{2\pi}{730} = \frac{\pi}{365}$ $b = 0.3$

1



$x = b + a\cos(nt)$

$x = 0.3 + 0.5\cos\left(\frac{\pi t}{365}\right)$

When $x = 0.5$ $t = ???$

1 $0.5 = 0.3 + 0.5\cos\left(\frac{\pi t}{365}\right)$

$2/5 = \cos\left(\frac{\pi t}{365}\right)$

$\frac{\pi t}{365} = 1.15928$

$t = \frac{1.15928 \times 365}{\pi}$

1 $= 135$ minutes

$= 2\text{hrs } 15\text{mins}$

1 Actual Time $= 6{:}30\text{am} - 2{:}15$

$= 4{:}15\text{am}$

JRAHS Y12 : T3 MATHEMATICS Extension 1 : Question 7		Task 3 : 2017
Suggested Solutions	**Marks**	**Marker's Comments**
(i) x-direction: $\ddot{x}=0$, $\dot{x}=50\cos\theta$, $x=50t\cos\theta$ y-direction: $\ddot{y}=-10$, $\dot{y}=-10t+50\sin\theta$, $y=-5t^2+50t\sin\theta$	1	For all 6 correct equations of motion (Must include $g=10$ and $v=50$)
(ii) To just clear the fence, $x=200$, $y=2$ from $x=50t\cos\theta$, $t=4\sec\theta$	1	For substituting t into y
Sub into $y=-5t^2+50t\sin\theta$ $2=-5\times16\sec^2\theta+50\times4\tan\theta$ $=-80(1+\tan^2\theta)+200\tan\theta$ $=-80-80\tan^2\theta+200\tan\theta$ i.e $\underline{40\tan^2\theta-100\tan\theta+41=0}$	1	For substituting (200, 2) and showing full simplification to given result.
(iii) $\tan\theta=\dfrac{100\pm\sqrt{3440}}{80}$ $\theta=27.33^\circ$ or 63.24°	1	2 extreme angles of projection in degrees
To score a home run, $(y>2)$ the range must be $\underline{27^\circ20'<\theta<63.14'}$	1	For correct range in degrees

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(iv) When the balls collide, their vertical heights are equal i.e. $y_1 = y_2$ $-5t^2+50t\sin\theta = -5t^2+75t\sin\alpha$	1	For equating 2 heights
$50t\sin 30° = 75t\sin\alpha$ $\therefore \sin\alpha = \frac{1}{3}$ $\therefore \underline{\alpha = 19.5°}$	1	For the projection angle
(v) When the balls collide, the sum of the horizontal displacements is 400 i.e. $x_1 + x_2 = 400$ $50t\cos 30° + 75t\cos\alpha = 400$	1	$\sin\alpha = \frac{1}{3}$, $\cos\alpha = \frac{\sqrt{8}}{3}$ For stating at $t=0$, then 2nd ball is at $x = 400$
$\therefore t = \dfrac{400}{25\sqrt{3} + 75 \times \frac{\sqrt{8}}{3}}$ $= \dfrac{400}{25\sqrt{3}+25\sqrt{8}}$ $= \underline{3.51\ s}$ Hence the balls collide 3.51 seconds after being hit.	1	For correct time Note:
		If assumed $x_1 = x_2 = 200$ then $\alpha = 54°$ and $t = \frac{8}{\sqrt{3}}$ s, then maximum 1 mark