Name:	
Class:	

Year 12
Assessment task 2
Term 1 2017

MATHEMATICS EXTENSION 1

Time Allowed - 120 minutes

(plus 5 minutes Reading Time)

General Instruction:

- All questions may be attempted
- All questions are equal value
- Reference Sheet will be supplied
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or badly arranged work

Questions 1-5 are to be completed on the multiple choice sheet, then Questions 6-12 to be completed on separate sheets of paper and handed in in separate bundles. Questions must show your Candidate Number (in the top right hand corner).

SECTION I
MULTIPLE CHOICE QUESTIONS

Attempt Questions 1 – 5 (1 mark each).

Allow approximately 8 minutes for this section

1. Find the fifth term in the expansion of $\left(3x^2 - \frac{1}{2x}\right)^8$.

   A) $\binom{8}{4}\left(\frac{3}{2}\right)^4 x^4$;

   B) $\binom{4}{8}\left(\frac{2}{3}\right)^4 x^{-4}$;

   C) $\binom{8}{4}\left(\frac{3}{2}\right)^4 x^{-4}$;

   D) $\binom{4}{8}\left(\frac{2}{3}\right)^4 x^4$;

2. By using substitution $u = tan^{-1}x$ evaluate $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

   A) $e^{x^2} + c$;

   B) $e^{\tan^{-1}x} + c$;

   C) $e^x + c$;

   D) $e^{1+x^2} + c$.

3. Which one of these functions has an inverse relation that is not a function?

   A) $y = x^3$;

   B) $y = \ln x$;

   C) $y = \sqrt{x}$;

   D) $y = |x|$.

4. If $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to:

   A) $x - 2\pi$

   B) $\pi + x$

   C) $\frac{\pi}{2} - x$

   D) $\pi - x$

5. Evaluate $\lim_{n\to-\infty} \frac{(n^4+3n^2)\tan^{-1}(2^n)}{n^4-1}$:

   A) 1;

   B) $\frac{\pi}{2}$;

   C) $-\frac{\pi}{2}$;

   D) 0.

END OF SECTION I

SECTION II
EXTENDED RESPONSE (77 marks)

77 marks
Attempt Questions 6 – 12
Allow approximately 1 hour and 52 minutes for this section

MARKS:

QUESTION 6. (11 marks) COMMENCE A NEW PAGE

(a) Evaluate

$$\lim_{x \to 0} \left(\frac{\sin 4x}{5x} + \cos x \right).$$

2

(b) Use the binomial theorem to find the term independent of x in the expansion of

$$\left(\frac{4}{x} + x^2 \right)^{15}$$

2

(c) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3, for all positive integers n.

3

(d) Sketch the graph of the function $f(x) = 3\cos^{-1}(2x - 1)$.
Clearly indicate the domain and range of the function.

2

(d) A spherical balloon is expanding so that its volume, V m^3, increases at a constant rate of 72 m^3 per second. What is the rate of increase of the surface area when the radius is 12 metres? You may use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere and $S = 4\pi r^2$ for its surface area.

2

QUESTION 7. (11 marks) COMMENCE A NEW PAGE

(a) If

$$\int \frac{\ln x}{x\sqrt{1 + lnx}} dx = F(x) + C,$$

using substitution $u = 1 + \ln x$

(i) find $F(x)$,

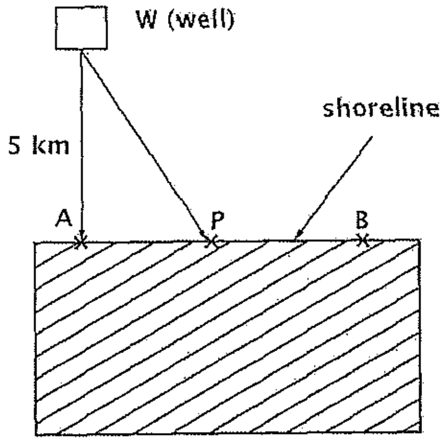
2

(ii) hence, find $F(e^3) - F(1)$

2

MARKS:

(b) An offshore oil well is located at a point W, which is 5 km from the closest shorepoint, A, on a straight shoreline. The oil is to be piped to a shorepoint B that is 8 km from A by piping it on a straight line underwater from W to some shorepoint, P, between A and B and then on to B via a pipe along the shoreline.



The cost of laying the pipe is \$125 000 per km underwater and \$75 000 per km over land. Let x km be the distance between A and P, and C (in thousands of dollars) be the cost of the entire pipeline.

(i) Show that the cost is given by $C = 125\sqrt{x^2+25} + 75(8-x)$ 2

(ii) Find the domain of x. 1

(iii) Find where the point P should be located to minimise the cost of laying the pipe, then find this minimised cost. 4

QUESTION 8. (11 marks) COMMENCE A NEW PAGE

(a) Use mathematical induction to prove that, for all integers n with $n \geq 1$

$$3 \cdot 2! + 7 \cdot 3! + 13 \cdot 4! + \cdots + (n^2+n+1)(n+1)! = n(n+2)!$$ 4

(b) Find the angle that $y = \tan^{-1}(2x)$ makes with the <u>y-axis</u> at the origin. Give your answer to the nearest degree. 3

(c) For a given function

$$f(x) = x^2 - (m - \frac{2}{3})x^3 - mx + 1,$$

find the real number, m, for which the function does not have a minimum or maximum? 4

MARKS:

QUESTION 9. (11 marks) COMMENCE A NEW PAGE

(a) Let $x = 3\sin\theta$ and find the following integral

$$\int \frac{x^2}{\sqrt{9-x^2}}dx$$ 4

(b) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of the tangent to the parabola at P is $y = px - ap^2$. 2

(ii) The tangent at P and the line through Q parallel to the y axis intersect at T. Find the coordinates for T. 2

(iii) Write down the coordinates of M, the midpoint of PT. 1

(iv) Following information given above, draw corresponding diagram, showing coordinates of all points. 1

(iv) Determine the locus of M when $pq = -1$ 1

QUESTION 10. (11 marks) COMMENCE A NEW PAGE

(a) Let $u = \sin x$, find the following indefinite integral

$$\int \sin^3 x \cos x dx$$ 2

(b) Isabella invests \$P at 8% per annum compounded annually. She intends to withdraw \$3000 at the end of each of the next six years, immediately after the interest is paid, to cover school fees.

(i) Write down an expression for the amount \$A1 remaining in the account following the withdrawal of the first \$3000. 1

(ii) Find an expression for the amount \$A2 remaining in the account after the second withdrawal. 1

(iii) Calculate the amount \$P that Isabella needs to invest so that she is just able to cover the school fees. 3

(c) Prove by induction that

$$\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \dots - \frac{1}{x^n} = \frac{1}{x^n(x-1)}$$

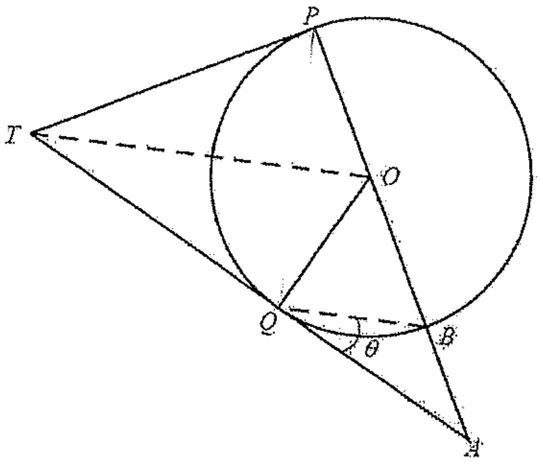
for all positive integers $n \geq 1$, $x \neq 0, 1$. 4

MARKS:

QUESTION 11. (11 marks) COMMENCE A NEW PAGE

(a) Find the area bounded by the curves $y = \cos x$ and $y = \sin(2x)$ and the x-axis for $0 \le x \le \frac{\pi}{2}$ 3

(b)



From an external point T, tangents are drawn to a circle with centre O, touching the circle at P and Q. Angle PTQ is acute. The diameter PB produced meets the tangent TQ at A. Let $\angle AQB = \theta$.

(i) Copy the diagram above into your answer booklet.

(ii) Prove that $\angle PTQ = 2\theta$. 3

(iii) Prove that $\triangle PBQ$ and $\triangle TOQ$ are similar. 3

(iv) Hence show that $BQ \times OT = 2 \times OP^2$. 2

QUESTION 12. (11 marks) COMMENCE A NEW PAGE

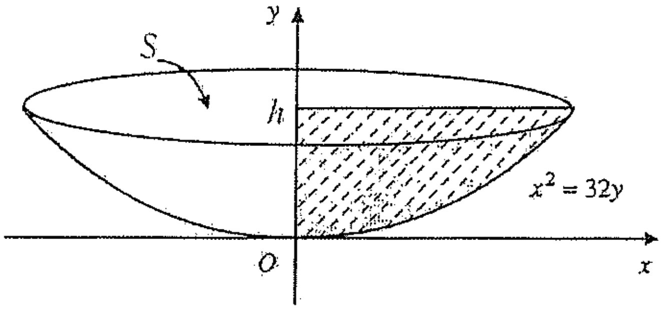
(a) For the binomial expansion

$$\left(\frac{a\sqrt[3]{a}}{b} + \frac{1}{\sqrt[15]{a^{28}}}\right)^n, \quad n \in \mathrm{N}, \quad a, b \neq 0, \quad a, b \quad \text{are} \quad \text{variables}$$

the sum of the coefficients of the first three terms is 79.
Find the term independent of a. 4

MARKS:

(b)



(i) The part of the curve $x^2 = 32y$ between $y = 0$ and $y = h$ is rotated about the y axis. Show that the volume enclosed is given by $V = 16\pi h^2$.

1

(ii) The diagram represents the water in a dam on a farm. The depth of the water is h metres, the volume of the water in the dam is V m^3, and the area of the surface of the water is S m^2. The water in the dam evaporates according to the rule $\frac{dV}{dt} = -kS$, where k is a positive constant, and t is the time in hours.
Show that $\frac{dh}{dt} = -k$.

3

(iii) Initially the dam contains 64π m^3 of water. Calculate how long it will take for the dam to empty by evaporation when $k = 0.001$.

3

Extension 1 Mathematics.

1.

$$T_{k+1} = \binom{n}{k} a^{n-k} (b)^k$$

$$T_5 = \binom{8}{4} (3)^4 \left(\frac{1}{2x}\right)^4$$

$$= \binom{8}{4} \left(\frac{3}{2}\right)^4 x^{-4}$$

part C.

2. $u = \tan^{-1} x$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\therefore \int e^u \, du.$$

$$= e^u + c$$

$$= e^{\tan^{-1} x} + c.$$

part B

3 part D

4. $\sin^{-1}(\sin x)$

x is in 2nd and third quadrant.

$$\sin^{-1}\left(\sin \frac{7\pi}{6}\right) =$$

$$\therefore \pi - x.$$

$$\sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \frac{\pi}{6}$$

$$\underline{\pi - x}.$$

$\therefore$ part D.

5. $\lim_{n \to -\partial}$

$$\frac{(1+0)\tan^{-1} 1/\infty}{1-0}$$

$$= \frac{(1+0)\tan^{-1} 0}{1-0}$$

$$= \underline{0}$$

part D

1/2

MATHEMATICS Extension 1 : Question 7

Suggested Solutions | **Marks** | **Marker's Comments**

(a)(i) $\int \frac{\ln x\,dx}{x\sqrt{1+\ln x}}$ using $u = 1+\ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$= \int \frac{(u-1)\,du}{\sqrt{u}}$

$= \int (\sqrt{u} - u^{-1/2})\,du$ ①

$= \frac{2}{3}u^{3/2} - 2u^{1/2} + c$

$= \frac{2}{3}(1+\ln x)^{3/2} - 2(1+\ln x)^{1/2} + c$ ①

A lot of students forgot to substitute back!! :(

or $\frac{2}{3}(1+\ln x)^{1/2}(\ln x - 2)$

(ii) $F(e^3) - F(1)$

$F(e^3) - F(1) = \frac{2}{3}(1+\ln e^3)^{3/2} - 2(1+\ln e^3)^{1/2} - \frac{2}{3}(1+\ln 1)^{3/2} + 2(1+\ln 1)^{1/2}$ ①

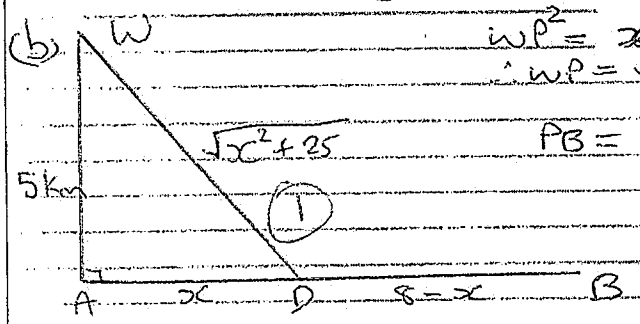
$= \frac{2}{3}(4)^{3/2} - 2(4)^{1/2} - \frac{2}{3}(1)^{3/2} + 2(1)^{1/2}$

$= \frac{16}{3} - 4 - \frac{2}{3} + 2$

$= \frac{8}{3}$ ①

* If students didn't sub back in (a) then they made the question easier for (b) so they only got one mark.

(b)



$WP^2 = x^2 + 25^2$ (by Pythagoras)

$\therefore WP = \sqrt{x^2+25}$

$PB = 8 - x$

* disappointing that for 2 marks students did very little work.

$\text{Cost} = 125\,000\sqrt{x^2+25} + 75000(8-x)$ ①

$\therefore C = 125\sqrt{x^2+25} + 75(8-x)$

$0 < x \leq 8$ scored no marks.

(ii) $x \geq 0$ as its a distance

$8 - x \geq 0$

$\therefore x \leq 8$

$\therefore 0 < x \leq 8$ or $0 \leq x \leq 8$ ①

(iii) $C = 125(x^2+25)^{1/2} + 75(8-x)$

$\frac{dC}{dx} = 125 \times \frac{1}{2}(x^2+25)^{-1/2} \times 2x + -75$

MATHEMATICS Extension 1 : Question……7 continued

Suggested Solutions | **Marks** | **Marker's Comments**

$\therefore \frac{dc}{dx} = \frac{125x}{\sqrt{x^2+25}} - 75$ ①

stat. points when $\frac{dc}{dx} = 0$

$\therefore \frac{125x}{\sqrt{x^2+25}} = 75$

$\frac{5x}{\sqrt{x^2+25}} = 3$

$25x^2 = 9(x^2+25)$

$16x^2 = 225$

$x = \pm\frac{15}{4}$

$x = \frac{15}{4}$ (as $0 < x \leq 8$) ①

x	3	$^{15}/_4$	4
$\frac{dc}{dx}$	−10.69	0	3.09
grad.	\	—	/

$\therefore$ rel. min at $x = {}^{15}/_4$ ①

*needed to test properly.

$\therefore$ minimum cost is:

$C = 125\sqrt{(\frac{15}{4})^2 + 25} + 75(8 - {}^{15}/_4)$

$= 125 \times \frac{25}{4} + 75 \times \frac{17}{4}$

$= 1100$ thousands

$\therefore$ minimum cost is \$1100 000 ①

*A lot of students forgot to actually find the minimum cost ☹

Ext 1 Task 2 Term 1 2017 (HSC)

MATHEMATICS Extension 1 : Question…….. 8 1/4

Suggested Solutions | **Marks Awarded** | **Marker's Comments**

8a) Step 1 Prove true for $n=1$

$LHS = (1+1+1)(1+1)!$

$= 3 \times 2!$

$= 6$

$RHS = n(n+2)!$

$= 1(1+2)!$

$= 3!$

$= 6$

$LHS = RHS$ true for $n=1$ — ①

Need to show LHS & RHS separately then say $LHS = RHS$ & true for $n=1$ to gain 1st mark.

step 2

Assume true for $n=k$

$3\cdot 2! + 7\cdot 3! + 13\cdot 4! + \dots + (k^2+k+1)(k+1)! = k(k+2)!$

Step 3.

Prove true for $n=k+1$ $n \in \mathbb{Z}$

RTP

$3\cdot 2! + 7\cdot 3! + 13\cdot 4! + \dots + ((k+1)^2+(k+1)+1)(k+1+1)!$

$= (k+1)(k+1+2)$

$\downarrow$

$(k+1)^2+(k+2)(k+2)! = (k+1)(k+3)!$

LHS

$$\underbrace{3\cdot 2! + 7\cdot 3! + 13\cdot 4! + \dots + (k^2+k+1)(k+1)!}_{k(k+2)!} + (k+1)^2 + (k+2)(k+2)!$$

by assumption

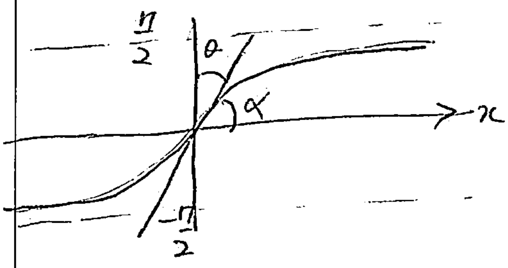
$k(k+2)! + [(k^2+2k+1)+(k+2)](k+2)!$

$k(k+2)!\ (k^2+3k+3)(k+2)!$

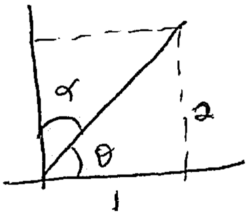
① mark awarded to state by assumption

2/4

MATHEMATICS Extension 1 : Question…8….

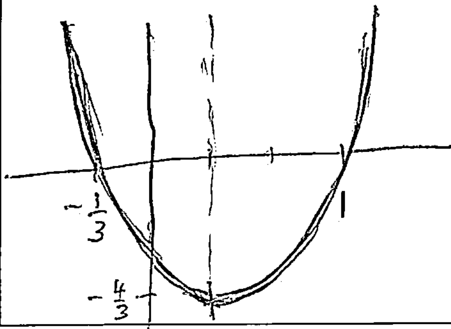
Suggested Solutions	Marks Awarded	Marker's Comments
$(k+2)!\,(k+k^2+3k+3)$ $(k+2)!\,(k^2+4k+3)$ $(k+2)!\,(k+3)(k+1)$ $k \times 3$, $k \times 1$	①	tidy up
* $(k+3)(k+2)! = (k+3)!$ $\therefore (k+3)!\,(k+1)$ $= RHS$	①	to prove = RHS
Hence if true for $n=k$, then true for $n=k+1$. ∴ Result true by mathematical induction for all integers $n \geq 1$		
b) $y = \tan^{-1} 2x$  α - angle with x axis θ - angle with y axis.		
$\tan y = 2x$ $x = \frac{1}{2}\tan y$ $\frac{dx}{dy} = \frac{1}{2}\sec^2 y$ $= \frac{1}{2}\left(\frac{1}{\cos^2 y}\right)$	①	able to differentiate

MATHEMATICS Extension 1 : Question…8…. 3/4

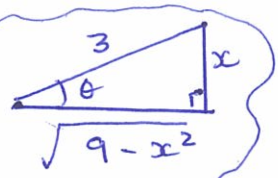
Suggested Solutions	Marks Awarded	Marker's Comments
For $\frac{dy}{dx} = 2\cos^2 y$ m at 0		
$m = 2\cos^2(0)$		
$= 2(1)$		
$= 2$		
$\therefore \tan\alpha = 2$		
$\alpha = \tan^{-1}(2)$		
$= 63\cdot43^\circ$ or $63^\circ 26'$	— ①	
∴ angle with y axis is		
$90 - 63^\circ 26'$	— ①	
$= 27^\circ$ (nearest degree)		
OR.		
$\frac{dy}{dx} = \frac{1}{1+(2x)^2} \times 2.$		
$= \frac{2}{1+4x^2}$ when $x=0$		
$\tan\theta = \frac{2}{1}$		
		
$\theta = 63^\circ$		
$\alpha = 90 - 63$		
$= 27^\circ$		

4/4

MATHEMATICS Extension 1 : Question 8

Suggested Solutions	Marks Awarded	Marker's Comments
c) $f(x)=x^2-(m-\frac{2}{3})x^3-mx+1$ $f'(x)=2x-3x^2(m-\frac{2}{3})-m$ $=-3(m-\frac{2}{3})x^2+2x-m$	①	know that have to differentiate
$a=-3(m-\frac{2}{3})$ $b=2$ $c=-m$ $\Delta=b^2-4ac$ $=(2)^2-4(-3(m-\frac{2}{3})(-m)$ $=4-4(3m^2-2m)$ $=4-12m^2+8m$ To not have min or max ie no roots $\Delta<0$ & $f'(x)\neq 0$	①	if haven't mentioned $\Delta<0$ no marks awarded
$4-12m^2+8m<0$ $4(1-3m^2+2m)<0$ $-(3m^2-2m-1)<0$ $3m^2-2m-1>0$ $3m$ $+1$ m m -1 $3m$ $(3m+1)(m-1)>0$ $m=-\frac{1}{3}, 1$		
 $\therefore m>1$ or $m<-\frac{1}{3}$	① ①	cannot write inequations of $m\geq 1$ $m\leq -\frac{1}{3}$ coz if doesn't include those 2 values.

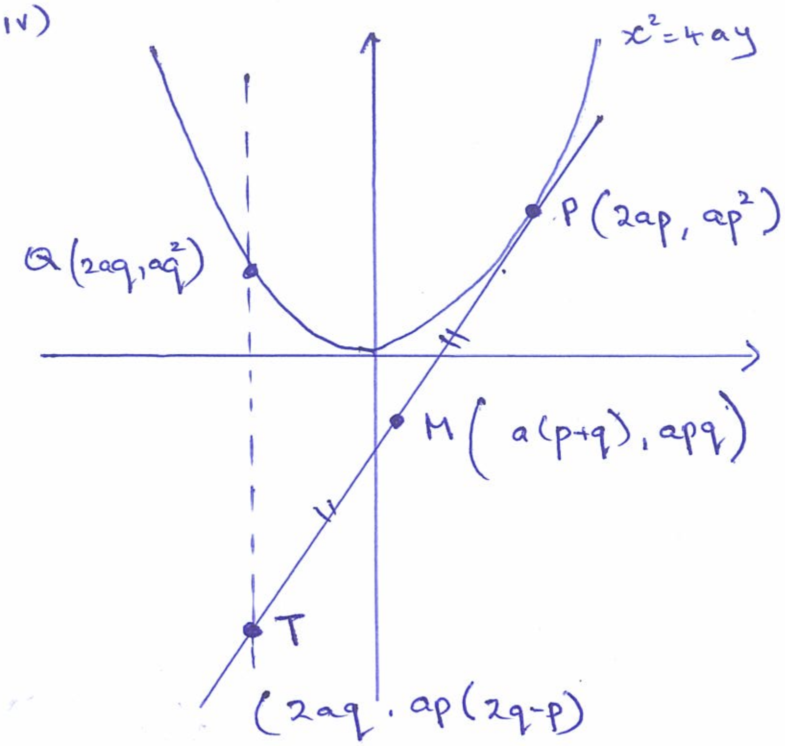
MATHEMATICS Extension 1 : Question 9……

Suggested Solutions	Marks Awarded	Marker's Comments
a) $\int \frac{x^2}{\sqrt{9-x^2}}\, dx = \int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot dx$	4 marks	let $x = 3\sin\theta$ $\frac{dx}{d\theta} = 3\cos\theta$ $\therefore dx = 3\cos\theta . d\theta$
Replace dx with $3\cos\theta . d\theta$ and simplify denominator $= \int \frac{9\sin^2\theta}{3\sqrt{1-\sin^2\theta}} \cdot 3\cos\theta . d\theta$		
$= \int \frac{9\sin^2\theta}{3\cos\theta} \times 3\cos\theta\, d\theta$		✓1 mark for replacing dx with $3\cos\theta . d\theta$
$= \int 9 \sin^2\theta . d\theta$ we know $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $= \frac{9}{2}\int 1 - \cos 2\theta . d\theta$		✓ 1 mark for simplifying to $\frac{9}{2}\int 1-\cos 2\theta . d\theta$
$= \frac{9}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$ $= \frac{9\theta}{2} - \frac{9\sin 2\theta}{4} + C$		✓ 1 mark for integrating to $\frac{9\theta}{2} - \frac{9\sin 2\theta}{4} + C.$
we know $\sin\theta = \frac{x}{3}$  $\therefore \cos\theta = \frac{\sqrt{9-x^2}}{3}$		✓ Simplifying to answer given
$= \frac{9\theta}{2} - \frac{2\cos\theta \sin\theta \times 9}{4} + C$	as	$\sin 2\theta = 2\cos\theta\sin\theta$ and $\theta = \sin^{-1}\left(\frac{x}{3}\right)$
$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - 2 \times \frac{\sqrt{9-x^2}}{3} \times \frac{x}{3} \times \frac{9}{4} + C$		
$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C$		
Also accepted : $\frac{9}{2}\sin^{-1}(\frac{x}{3} - \frac{3x}{2}\sqrt{\frac{1-x^2}{9}} + C.$		

MATHEMATICS Extension 1 : Question……9b

Suggested Solutions	Marks Awarded	Marker's Comments
b(i) $P(2ap, ap^2)$ and $Q = 2aq, aq^2)$ $x^2 = 4ay \Rightarrow \therefore y = \frac{x^2}{4a}$ and $\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$ at point P $x = 2ap$ $\therefore \frac{dy}{dx} = \frac{2ap}{2a} = p$ Equation : $y - y_1 = m(x - x_1)$ $y - ap^2 = p(x - 2ap)$ $y - ap^2 = px - 2ap^2$ $\therefore y = px - ap^2$	2 marks	Well done ✓1 mark for getting gradient P ✓ 1 mark for using equation formula correctly.
b(ii) Substitute $x = 2aq$ to get y equation $\therefore y = px - ap^2$ $= p(2aq) - ap^2$ $= 2apq - ap^2$ $\therefore$ Coordinates of T is $(2aq, 2apq - ap^2)$ or $(2aq, ap(2q-p))$	2 marks	As T is on parallel line from Q the x value is the same as 2aq ✓1 mark for recognising $x = 2aq$ ✓ 1 mark for getting y value
b(iii). Midpoint of PT. $P(2ap, ap^2)$ and $T(2aq, ap(2p-q))$ $\left(\frac{2ap+2aq}{2}, \frac{ap^2+2apq-ap^2}{2}\right)$ $= (ap+aq, apq)$ $= (a(p+q), apq)$	1 mark.	✓ 1 mark for getting both x and y values correctly.

MATHEMATICS Extension 1 : Question…9…. Page 3

Suggested Solutions	Marks Awarded	Marker's Comments
iv) 	1 mark.	All coordinates of P, Q, M and T had to be written Few students forgot to label M and lost a mark.
(v) For M $(ap+aq , apq)$ $x = a(p+q)$ and $y = apq$ $y = a(-1)$ $y = -a$ Since the y coordinate of M is $-a$ and which is independent of the variables p and q then the locus M is the line $y = -a$. This is also the directrix of the parabola. $x^2 = 4ay$	1 mark	One had to have $y=-a$ as the answer. Some had $y=-a$ in their working out but used that to do other calculations. They used $y=-a$ but did not conclude $y=-a$ is locus. No marks awarded

Task 2 : 2017 **MATHEMATICS Extension 1 : Question** 10 JRAHS

Suggested Solutions	**Marks**	**Marker's Comments**
10a) $I = \int \sin^3 x(\cos x\,dx)$		
$u = \sin x$ $du = \cos x\,dx$		
$\therefore I = \int u^3\,du = \frac{u^4}{4} + C$	①	integrand in u
$= \frac{\sin^3 x}{4} + C$	①	final correct answer in terms of x.
b) (i) $A_1 = (1.08)^1 P - 3000$	①	For correct expression
(ii) $A_2 = (1.08^1)A_1 - 3000$		
$= 1.08(1.08P - 3000) - 3000$	①	for correct expression
$= 1.08^2 P - 3000(1 + 1.08)$		
(iii) $A_3 = 1.08^2 P - 3000(1 + 1.08 + 1.08^2)$ $\vdots$		
$A_6 = 1.08^6 P - 3000(1 + 1.08 + \dots + 1.08^5)$	①	For A_6 or A_n
$= 1.08^6 P - 3000 \times \left[\frac{1(1.08^6 - 1)}{1.08 - 1}\right]$	GP	with $a = 1, r = 1.08$ $n = 6$
	①	for S_n for GP
$= 1.08^6 P - 37500(1.08^6 - 1)$		
But $A_6 = 0$		
$\Rightarrow 1.08^6 P = 37500(1.08^6 - 1)$		
i.e $P = \frac{37500(1.08^6 - 1)}{1.08^6}$	①	Final investment amount
$= \$13\,868.64$ (nearest cent)		

* If $n = 5$, max 4
* If $A_6 = 18000$, max 4
* If regular investments, max 3

Task 2 : 2017 **MATHEMATICS Extension 1 : Question** 10 JRAHS

Suggested Solutions	**Marks**	**Marker's Comments**
c) Let the proposition be $P(n)$ i.e $P(n): \frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}-\dots-\frac{1}{x^n}=\frac{1}{x^n(x-1)}$		
Prove $P(n)$ true for $n=1$ LHS (of $P(n)$) $=\frac{1}{x-1}-\frac{1}{x^1}=\frac{1}{x(x-1)}$ RHS (of $P(n)$) $=\frac{1}{x^1(x-1)}=$ LHS $\therefore$ $P(n)$ is true for $n=1$	1	For base case
Assume $P(n)$ is true when $n=k\in Z^+$ i.e $\frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}-\dots\frac{1}{x^k}=\frac{1}{x^k(x-1)}$		
Prove $P(n)$ true when $n=k+1$ i.e. To prove: $\frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}-\dots-\frac{1}{x^k}-\frac{1}{x^{k+1}}=\frac{1}{x^{k+1}(x-1)}$	1	Equation for Inductive step for $n=k+1$
Now LHS $=\left(\frac{1}{x-1}-\frac{1}{x}-\frac{1}{x^2}-\dots-\frac{1}{x^k}\right)-\frac{1}{x^{k+1}}$ $=\frac{1}{x^k(x-1)}-\frac{1}{x^{k+1}}$ $=\frac{x-(x-1)}{x^{k+1}(x-1)}=\frac{1}{x^{k+1}(x-1)}=$ RHS	1	Correct algebra in proving LHS = RHS
$\therefore$ $P(n)$ is true when $n=k+1$	1	For overall. Structure including appropriate conclusion
Hence by the principle of Mathematical Induction $P(n)$ is true $\forall n\geq 1$		

* If $n=2$ ... max 3

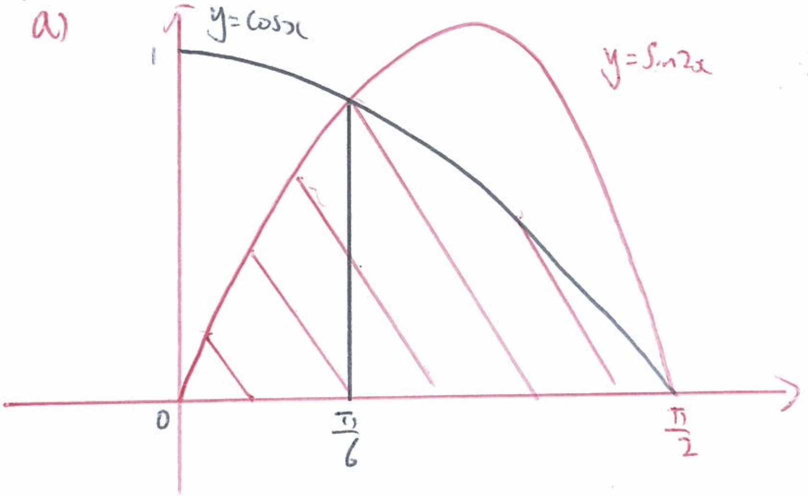
* If $x=2$ ... max 2

* If no equation for inductive step - max.

* If $\frac{1}{x^k}$ missing from inductive step

Suggested Solutions	Marks	Marker's Comments

11. a)



Solve $\cos x = \sin 2x$

$\cos x = 2\sin x \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0 \rightarrow \cos x = 0$ or $\sin x = \frac{1}{2}$

$\therefore x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$ ——— ①

$\therefore A = \int_0^{\frac{\pi}{6}} \sin 2x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$ ——— ①

$= \left[\frac{-1}{2}\cos 2x\right]_0^{\frac{\pi}{6}} + \left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \frac{-1}{2}\left(\frac{1}{2} - 1\right) + \left(1 - \frac{1}{2}\right)$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4}u^2$ ——— ①

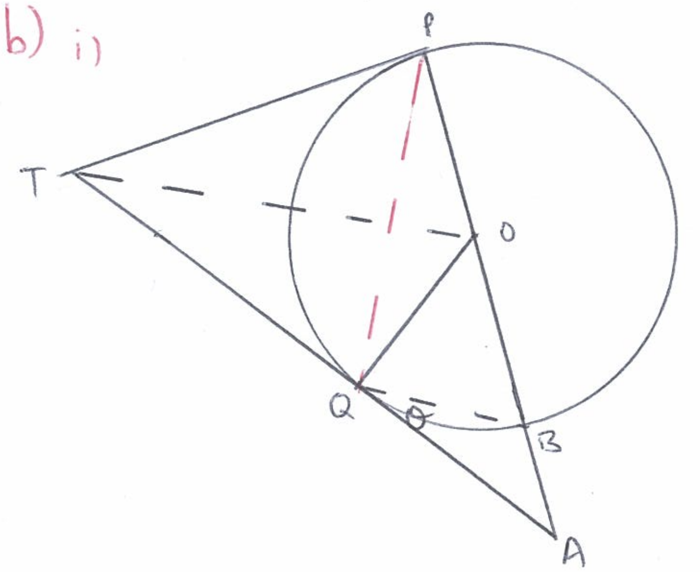
Mathematics Ext 1 Question 11.

Suggested Solutions

Marks

Marker's Comments

b) i)



ii) Construct PQ

$\angle QPB = \theta$ (angle between chord and tangent is equal to angle in the alternate segment) — ①

$\angle TPO = 90°$ (radius OP is perpendicular to tangent TP at point of contact).

$\angle TQO = 90°$ (radius OQ is perpendicular to tangent TQ at the point of contact)

$\therefore \angle TPO + \angle TQO = 180°$

$\therefore$ TPOQ is cyclic — ①

$\angle QOB = 2\theta$ (angle at the centre is twice the angle at the circumference standing on the same arc)

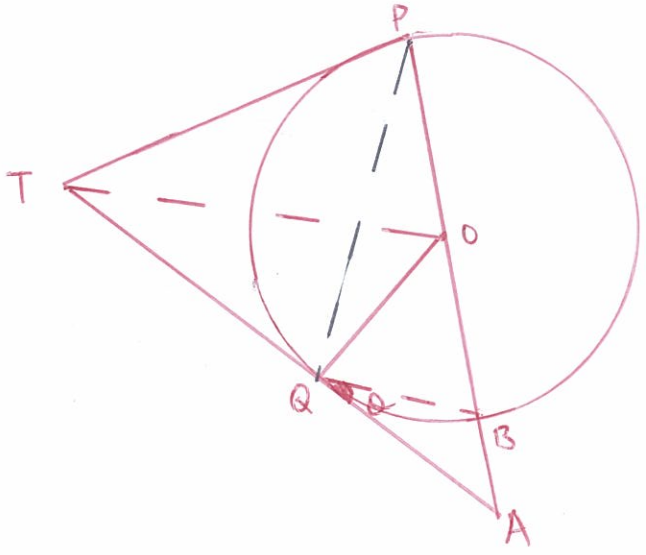
$\therefore \angle PTQ = 2\theta$ (exterior angle of a cyclic quadrilateral is equal to the interior opposite angle) — ①

Mathematics ____ Question ____

Suggested Solutions	Marks	Marker's Comments

b)

i)



ii) In ΔPTO and ΔQTO,

$\angle OQT = 90°$ (angle between radius OQ and tangent TQ)

$\angle OPT = 90°$ (angle between radius OP and tangent PT)

$\therefore \angle OQT = \angle OPT = 90°$

TO is common

$PO = QO$ (radii of a circle)

$\therefore \Delta PTO \equiv \Delta QTO$ (RHS)

Construct PQ,

$\angle QPB = \theta$ (angle between chord and tangent is equal to the angle in the alternate segment)

$\therefore \angle QOB = 2\theta$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$\angle POQ = 180 - 2\theta$ (angle sum of a straight angle)

$\angle POQ = \angle POT + \angle QOT$

$= 2\angle QOT$ (corresponding angles, $\Delta POT \equiv \Delta QOT$)

Mathematics ____ Question ____

Suggested Solutions	Marks	Marker's Comments
$\therefore \angle QOT = 90 - \theta$ $\therefore \angle QTO = 180 - \angle OQT - \angle QOT$ (angle sum ΔTOQ) $= 180 - (90-\theta) - 90°$ $= \theta$ $\therefore \angle PTO = \theta$ (corresponding angles $\Delta PTO \equiv \Delta QTO$) $\therefore \angle PTQ = 2\theta$.		

Mathematics ____ Question ____		
Suggested Solutions	Marks	Marker's Comments
iii) $QO = BO$ (radii of a circle) $\therefore \angle OQB = \angle OBQ$ (equal angles are opposite to equal sides in ΔBOQ) $\angle QOB + \angle QBO + \angle BQO = 180°$ (angle sum of ΔBQO) $\therefore \angle QOB + 2\angle OBQ = 180°$ (proven above) $\therefore \angle OBQ = \frac{180 - QOB}{2}$ $\angle QOB = 2\theta$ (proven above) $\therefore \angle OBQ = 90 - \theta$ $\therefore$ In ΔPBQ and ΔTOQ, $\angle PQB = 90°$ (angle in a semi-circle) $\angle OQT = 90°$ (proven above) $\therefore \angle PQB = \angle OQT$ $\angle PBQ = 90 - \theta = \angle TOQ$ (proven above) $\therefore \Delta PBQ \parallel\!\mid \Delta TOQ$ (equiangular) Using Equiangular: 1 - for proving each pair of angles equal 1 - for $\Delta PBQ \parallel\!\mid \Delta TOQ$ (equiangular) *Students receive 1/3 if any part of the proof is invalid		

Mathematics ____ Question ____

Suggested Solutions | Marks | Marker's Comments

iv) $\frac{BQ}{QO} = \frac{PB}{TO}$ (corresponding sides are in the same ratio $\triangle PBQ \; ||| \; \triangle TOQ$) ①

$\therefore BQ \times TO = PB \times QO$

$= 2 \times OP \times OQ$ (diameter is twice the radius in a circle)

$= 2 \times OP^2$ (QO and OP are radii of the circle) ①

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
a) $T_{r+1} = {}^nC_r\left(\frac{a\sqrt[3]{a}}{b}\right)^{n-r}\left(\frac{1}{\sqrt[15]{a^{28}}}\right)^r$		
$= {}^nC_r\ \frac{a^{\frac{4}{3}(n-r)-\frac{28}{15}(r)}}{b^{n-r}}$		
${}^nC_0 + {}^nC_1 + {}^nC_2 = 79$		
$1 + \frac{n!}{(n-1)!} + \frac{n!}{2!(n-2)!} = 79$		
$1 + n + \frac{1}{2}(n)(n-1) = 79$	1	if used 'guess and check' students lost this mark.
$2 + 2n + n^2 - n = 158$		
$n^2 + n - 156 = 0$		
$(n+13)(n-12) = 0$		
$n = 12, -13$		
$\therefore n = 12,\ n \geq 0$	1	
for term independent of a:		
$\frac{4}{3}(n-r) - \frac{28}{15}(r) = 0$		
$\frac{4}{3}n - \frac{4}{3}r - \frac{28}{15}r = 0$		
$\frac{4}{3}n = \frac{16}{5}r$		
$r = \frac{5}{12}n$	1	
when $n = 12$		
$r = 5$		
$\therefore$ Term independent of a is:		
$T_6 = {}^{12}C_5\ b^{12-5}$		
$= \frac{792}{b^7}$	1	
b) i) $x^2 = 32y$		
$V = \pi\int_0^h x^2\,dy$		
$= \pi\int_0^h 32y\,dy$	1	
$= \pi\left[16y^2\right]_0^h$		
$= \pi(16h^2 - 0)$		
$= 16h^2\pi$		

MATHEMATICS Extension 1 : Question..12..

Suggested Solutions	Marks	Marker's Comments
ii) $S = \pi r^2$		
$x^2 = 32y$		
x = radius y = height	1	
$\therefore r^2 = 32h$		
$\therefore S = 32h\pi$		
$\frac{dV}{dh} = 32h\pi$		
$\therefore \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$	1	
$= \frac{1}{32h\pi} \times -kS$		
$= \frac{1}{32h\pi} \times -k(32h\pi)$	1	
$= -k$		
iii) $\frac{dh}{dt} = -k$		
$dh = -k\,dt$		Cannot use $\frac{dV}{dt} = -k(32h\pi)$ since h is also a variable
$h = -kt + c$	1	
when $t = 0$, $V = 64\pi$		
$\therefore 64\pi = 16h^2\pi$		
$h^2 = 4$		
$h = 2$, $h \geq 0$	1	
$\therefore 2 = -k(0) + c$		
$c = 2$		
$\therefore h = -kt + 2$		
when $h = 0$, $k = 0.001$		
$0 = -0.001t + 2$		
$t = 2000$ hours	1	

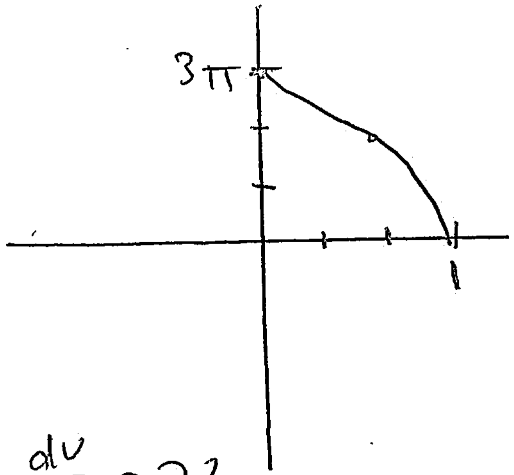
MATHEMATICS Extension 1 : Question..6.....

Suggested Solutions	Marks	Marker's Comments
(a) $\lim_{x\to0} \frac{4}{5} \frac{\sin 4x}{4x} + \lim_{x\to0} \cos x$		
$= \frac{4}{5} + 1$	1 for $\frac{4}{5}$	
$= \frac{9}{5}$	1 for 1	
(b) $T_{k+1} = \binom{15}{k}\left(\frac{4}{x}\right)^{15-k}\left(x^2\right)^k$		
independent of		
$x^{-15+k}x^{2k} = x^0$		
$-15+3k = 0$		
$3k = 15$	①	
$\underline{k = 5}$		
$\binom{15}{5}4^{10} = 3148873728$	1	Should have calculated answer, not left in unexpanded form

MATHEMATICS Extension 1 : Question…6…

Suggested Solutions	Marks	Marker's Comments
(6) Prove true for n=1		
$1^3+2\times1 = 3$ which is divisible by 3	1	
Therefore true for n=1		
Assume true for n=k		
$k^3+2k=3M$ $\quad M \in Z^+$ $k \in Z^+$		
Prove true for n=k+1		
$(k+1)^3+2(k+1)$		
$= k^3+3k^2+3k+1+2k+2$	1	
$= k^3+2k+3k^2+3k+3.$		
by assumption		
$= 3M+3k^2+3k+3$		
$= 3(M+k^2+k+3)$		
$= 3I$ where I is an integer	1	Needed to state integer

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
d) $f(x) = 3\cos^{-1}(2x-1)$ $-1 \leq 2x-1 \leq 1$ $0 \leq 2x \leq 2$ $0 \leq x \leq 1$ domain $0 \leq y \leq 3\pi$ range	2	2 correct facts 1 mark all correct 2 marks
		there is no horizontal point of inflexion.
(e) $\frac{dV}{dt} = 72$ $S = 4\pi r^2 \quad \frac{dS}{dr} = 8\pi r$ $V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2$ $\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dV} \cdot \frac{dV}{dt}$	1	Students who substituted V in terms of S needed to use implicit differentiation as both depended on r, so received 0 ~~Answers~~ if they didn't.
$= 8\pi r \times \frac{1}{4\pi r^2} \times 72$ $= \frac{2}{r} \times 72 \quad r = 12$ $= 12$ The rate of increase $12m^2/s$.	1	ignored units